Course “Algorithmic Foundations of Sensor Networks”
Lecture 6: Energy-aware Routing Algorithms

Sotiris Nikoletseas
Professor

Department of Computer Engineering and Informatics
University of Patras, Greece

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Overview

A. The energy balance problem
B. A distance-based probabilistic energy balance protocol
C. Another energy balance problem (residual-energy based)
D. Other methods
    - Adjusting transmission ranges
    - An offline approach
    - Power aware routing
    - Lifetime maximizing routing
    - Load-balanced energy-aware routing

- Directed Diffusion (DD): a *tree-structure* protocol (suitable for low dynamics)
- LEACH: *clustering* (suitable for small area networks)
- Local Target Protocol (LTP): *greedy, single-path optimization* (best for dense networks)
- Probabilistic Forwarding Protocol (PFR): *redundant optimized* transmissions (good efficiency / fault-tolerance trade-offs, best for sparse networks)
- Energy Balance Protocol (EBP): guaranteeing *same per sensor energy* (prolong network life-time)

The Energy Balance Problem (II)

All protocols *tend to "strain" some specific nodes* in the network.

- In a *hop-by-hop* scheme the nodes *closer to the sink* tend to be overused.
- In a *direct transmission* scheme the *distant nodes* tend to be overused.

"How can we achieve *equal energy dissipation per node* in order to prolong the network lifetime by avoiding early network disconnection?"

B. A Probabilistic Energy Balance Protocol
(distance-based)

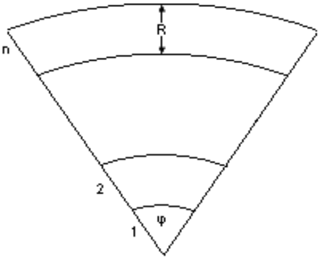
- Direct transmission cost is much larger than that of one-hop transmission and exhausts distant nodes.
- One-hop transmissions are cheap but tend to overuse nodes that lie closer to the sink.

Solution: “Each node *chooses randomly* whether to propagate the event *one-hop closer to the sink* or to transmit it *directly to the sink*”.

The *goal* is to “*balance*” the two types of transmissions and achieve *equal average energy dissipation per sensor.*

The EBP Protocol (I)

- **Network Partition:**
  - *Partition* the network into n sectors, *“slices"*, of width R (the transmission range).



The EBP Protocol (II)

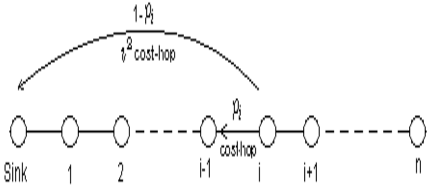
- **Data Propagation:**

  Each node in sector i propagates its messages according to the following rule:

  - Propagate the message to *sector* $i-1$ with *probability* p_i.
  - Propagate the message *directly to the sink* with *probability* $1-p_i$.

The *choice* of p_i is made such as the *average per sensor energy dissipation is the same* for all sensors in the network.

There is a message on node *i*.



Balanced Average per Sensor Energy Dissipation

- S_i: the *area size* of sector i.
- $\mathcal{E}_i$: the *total energy* dissipated in sector i.
- Sensor nodes are spread *random uniformly* in the network area. We want:

$$\frac{E[\mathcal{E}_i]}{S_i} = \frac{E[\mathcal{E}_j]}{S_j} \quad \forall i, j \in \{1, \ldots n\}$$

This guarantees the balanced energy dissipation per sensor throughout the network.

The Model & Assumptions

The events occur in random uniform positions in the network.

Let ϵ_{ij} a *random variable* that measures the *energy that dissipates* the sector i so as to handle message j.

$$\epsilon_{ij} = \begin{cases} cR^2 & \text{with probability } p_i \\ c(iR)^2 & \text{with probability } 1 - p_i \end{cases}$$

Clearly,

$$E[\epsilon_{ij}] = [i^2 - p_i(i^2 - 1)]cR^2$$

Computation of p_i (message handling)

- Let h_i the number of messages that *handles* sector i.
- Let f_i the number of messages that *were forwarded* to sector i.
- Let g_i the number of messages that *were generated* in sector i.

Clearly:

$$h_i = f_i + g_i$$

By linearity of expectation:

$$E[h_i] = E[f_i] + E[g_i]$$

even though h_i, f_i, g_i are not independent.

Computation of p_i

Lemma

The following relationship holds:

$$E[f_i] = p_{i+1} E[h_{i+1}] = p_{i+1} \cdot (E[f_{i+1}] + E[g_{i+1}])$$

Clearly:

$$p_{i+1} = \frac{E[f_i]}{E[f_{i+1}] + E[g_{i+1}]}$$

The Energy Balance Property:

$$E\left[\frac{\sum_{k=1}^{h_i} \epsilon_{ik}}{S_i}\right] = E\left[\frac{\sum_{k=1}^{h_j} \epsilon_{jk}}{S_j}\right] \quad \forall i, j \in \{1, \ldots n\}$$

A recurrence relation for p_i

$$a(i+1)E[f_{i+1}] - (d(i)+a(i))E[f_i] + d(i-1)E[f_{i-1}] = a(i)E[g_i] - a(i+1)E[g_{i+1}]$$

where

$$a(i) = \frac{i^2}{2i-1} \qquad d(i) = \frac{(i+1)^2 - 1}{2i+1}$$

Initial conditions:

$$E[f_n] = 0 \quad E[f_0] = n$$

Computation of p_i

Structure of the rest of solution evaluation.

- First we transform the recurrence into a *simpler recurrence* with only *two successive terms* (whose coefficient is 1).
- Having solved the latter recurrency, we solve the initial one.
- Having computed the values of $E[f_i]$ for $i = 1, \ldots, n$, we compute the exact values of probabilities p_i.

The exact solution

- The solution:

$$E[f_i] = -\sum_{k=1}^{n-i} \frac{\prod_{j=k}^{n-i+1} a(n-j)}{\prod_{j=k}^{n-i} d(n-j)} \cdot \left(\sum_{j=1}^{n-k} (a(j)E[g_j] - a(j+1)E[g_{j+1}]) + a(1) \cdot E[f_1] \right)$$

where

$$\prod_{i}^{i-1} a(i) = 1$$

- Easily computed in a repetitive manner
- Thus we can calculate p_i's:

$$p_{i+1} = \frac{E[f_i]}{E[f_{i+1}] + E[g_{i+1}]}$$

A closed approximate form for p_i

If $E[f_i] \simeq E[f_{i-1}]$ then

$$p_i = 1 - \frac{3x}{(i+1)(i-1)} \qquad \text{for } 3 \leq i \leq n$$

$p_2 = x$ and can be set to $\frac{1}{2}$

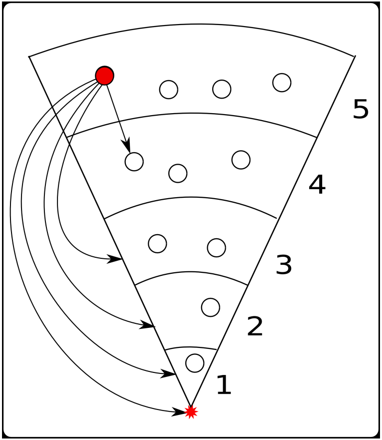
$E[f_i] \simeq E[f_{i-1}]$ is realistic because:

- $S_i \simeq S_{i-1}$
- $p_i \simeq p_{i-1}$.

Comments on p_i

- when i is large, p_i is large, i.e. when far away from the sink it is better to move hop-by-hop, to avoid spending too much energy.
- when i becomes small, p_i is small, i.e. when we approach the sink it is better to transmit directly in order to bypass the critical region (and since energy consumption is small).

C. Another Energy Balance Protocol
(based on residual energy)

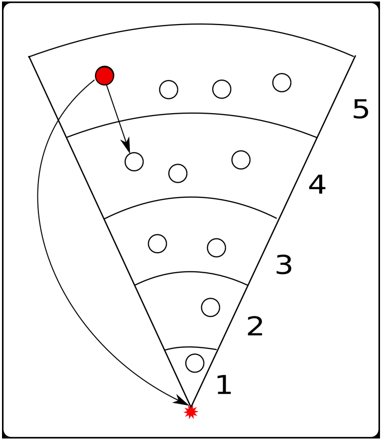


This new algorithm balances energy consumption:

- Slicing of the network
- Generalized data propagation algorithm, allows to jump over bottle-neck nodes

Note: do hops to "intermediate" slices help?

Lifespan Maximization



Main Theoretical Result

Lifespan is maximized by a *mixed* data propagation algorithm

Application

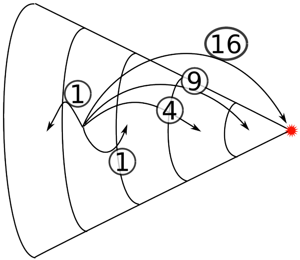
We use this fact to propose a distributed optimal data propagation algorithm

- mixed propagation: only single hops and direct to sink transmissions
- mixed strategies beat every other possible strategy (wrt. lifespan)

Model

1. *Energy cost:*
   Sending a message from slice i to j costs $(i-j)^2 \mathrm{E/msg}$
2. $f_{i,j}$ is the *message rate* from slice i slice to slice j $\mathrm{msg/t}$

	sink	1	2	3	4
sink	0	1	4	9	16
1	1	0	1	4	9
2	4	1	0	1	4
3	9	4	1	0	1
4	16	9	4	1	0



Model

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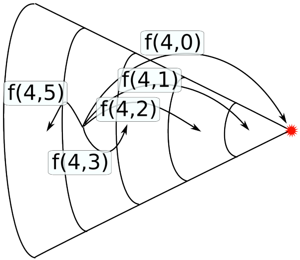
	sink	1	2	3	4
sink	$f_{0,0}$	$f_{0,1}$	$f_{0,2}$	$f_{0,3}$	$f_{0,4}$
1	$f_{1,0}$	$f_{1,1}$	$f_{1,2}$	$f_{1,3}$	$f_{1,4}$
2	$f_{2,0}$	$f_{2,1}$	$f_{2,2}$	$f_{2,3}$	$f_{2,4}$
3	$f_{3,0}$	$f_{3,1}$	$f_{3,2}$	$f_{3,3}$	$f_{3,4}$
4	$f_{4,0}$	$f_{4,1}$	$f_{4,2}$	$f_{4,3}$	$f_{4,4}$



Generalized-flow maximization

Problem

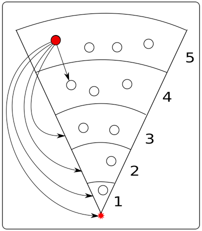
Given a WSN: Maximize the *generalized* network flow

- Given the detection rates $f_{0,i} := g_i$
- Given the available energy: b_i

LP to maximize the generalized-flow:

Maximize T in:

1. $f_{0,i} = Tg_i$ *Detection rates*
2. $\sum_{j=0}^{N} f_{i,j}(i-j)^2 \leq b_i$ *Energy constraint*
3. $\sum_{j=0}^{N} f_{i,j} = \sum_{j=0}^{N} f_{j,i}$ *Flow equations*



- g_i: event generation rates (i.e. data injected at i)
- maximize time $T \Rightarrow$ lifespan maximization

Mixed-flow maximization

Problem

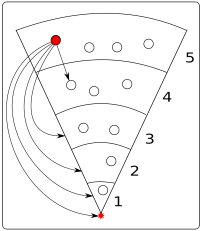
Given a WSN: Maximize the *mixed* network flow

- Given the detection rates $f_{0,i} := g_i$
- Given the available energy: b_i

LP to maximize the mixed-flow:

Maximize T in:

1. $f_{0,i} = Tg_i$ *Detection rates*
2. $\sum_{j=0}^{N} f_{i,j}(i-j)^2 \leq b_i$ *Energy constraint*
3. $\sum_{j=0}^{N} f_{i,j} = \sum_{j=0}^{N} f_{j,i}$ *Flow equations*
4. $f_{i,j} = 0 \text{ if } j \notin \{0, i-1\}$ *mixed flow constraint*



Mixed-flow maximization

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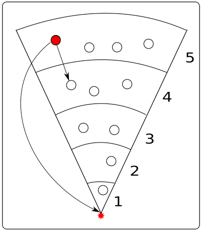


(4) guarantess no hops to “intermediate” slices

(3) guarantees flow preservation

Mixed-flows are optimal

Result

If there exists an NRG-balanced mixed flow

1. it maximizes the *mixed-flow*
2. it maximizes the *generalized-flow*

Application

We can maximize the generalized-flow problem using:

- a mixed-flow
- a local property (energy-balance)

Inputs

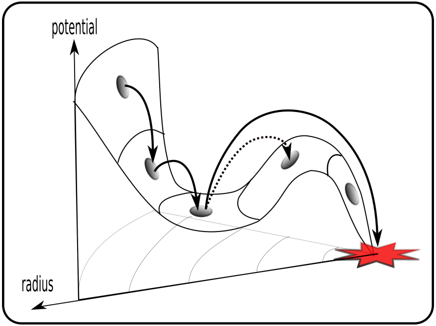
1. Each node has a potential $\mathit{potential}(n) \simeq \mathit{EnergySpent}(n)$
2. Each node knows its list of neighbours $\mathcal{V}_n$
3. Each node knows the potential of its neighbours

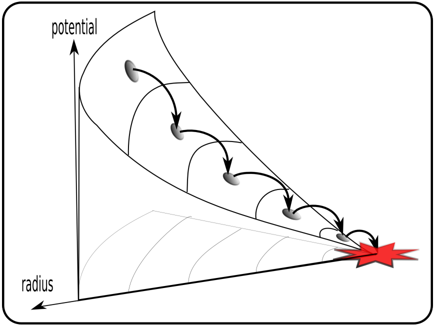
Algorithm `Propagate Data`

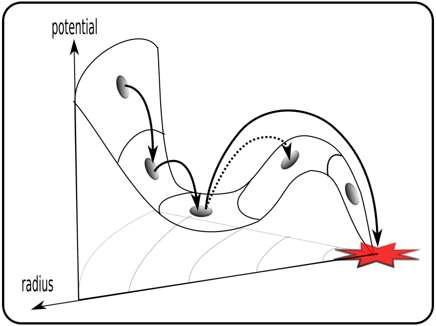
- `Find` m: the lowest potential neighbour
- `If` $\mathit{potential}(m) < \mathit{potential}(\mathit{self})$ `then` send data to m
- `Else` send data directly to the sink

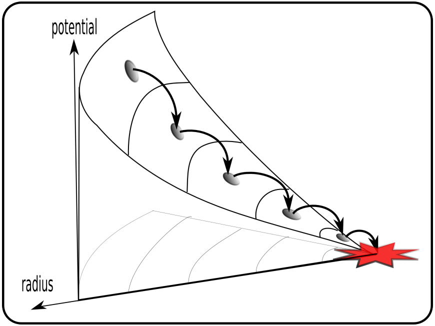
A distributed algorithm

Illustration









Remark

- The algorithm produces a mixed flow
- The algorithm balances energy

Inputs

- Let λ_i be the probability that the event occurs in slice i
- $X_i(t)$ is the energy spent by slice i at time $t = 1, 2, 3, \ldots$

We consider

$$X(t) = \begin{pmatrix} X_N(t) - X_{N-1}(t) \\ X_{N-1}(t) - X_{N-2}(t) \\ \vdots \\ X_2(t) - X_1(t) \end{pmatrix}$$

Remarks

- $\{X(t)\}_{t \geq 0}$ is a Markov Chain*
- If $\lambda_i > 0$, the Markov chain is irreducible**

We would like to have $X(t) \xrightarrow{t} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$

* a memorylessness stochastic process
** the process can move from any state to any state. Such processes converge to a stable distribution.

Note: $\lambda_i > 0$ is necessary for irreducibility but not always realistic

Thus, we give sufficient conditions for stability

Theorem

If $i^2\lambda_i > (i-1)^2\lambda_{i-1}$, the Markov chain $X(t)$ is stable around $\mathcal{A}$, with

$$\mathcal{A} = \left\{ X \in \mathbb{R}^{N-1} \; : \; |X_i(t) - X_{i-1}(t)| \leq \frac{i^2}{2}, \; i = 2, ..., N \right\}$$

In other words, quite similar energy dissipation in adjacent sectors $i, i-1$.

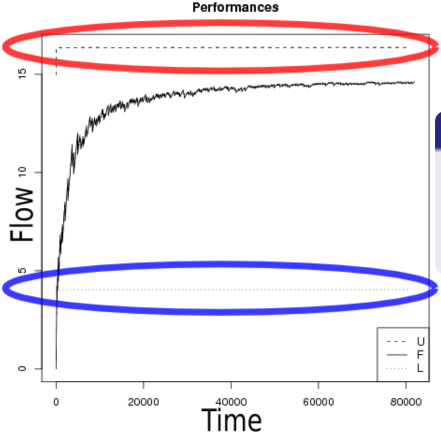
Simulations

- Scatter nodes randomly over a region
- Events are generated randomly
- Data is propagated according to the algorithm

First Simulation

- 1000 sensors randomly dispersed over a 10m disc
- Sink at the center of the disc
- Potential function $potential(n) = Energy\_Spent(n)$

First Simulation



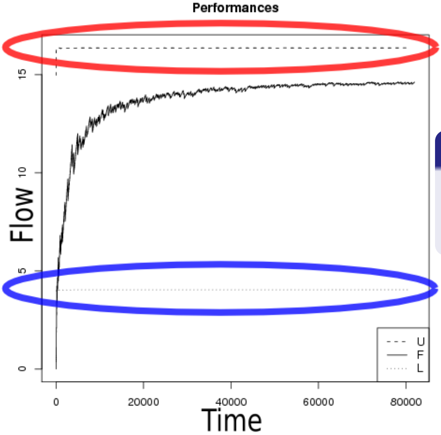
Network flow

$$\frac{\textit{Messages received by sink}}{\max_{\{nodes\}}(\textit{EnergySpent})}$$

F: the flow of the algorithm

U: maximum possible flow (offline, computed by an LP)

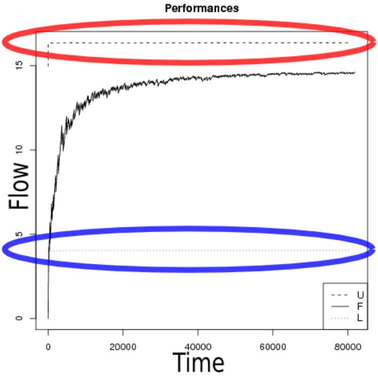
L: maximum possible flow without direct transmissions

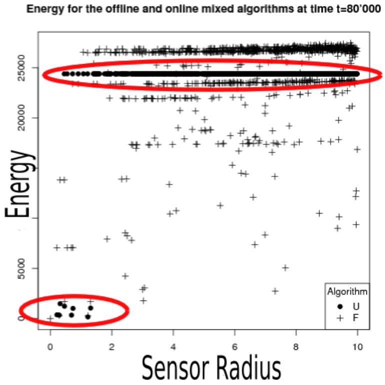


Remarks

- Close to optimal
- Distributed and on-line!

First Simulation

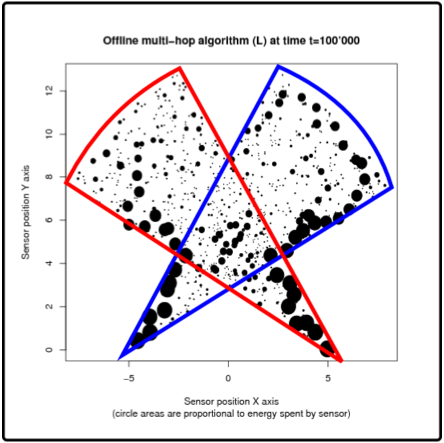




- off-line ideal flow *U* balances the energy load
- the online algorithm performs very well

Second Simulation

- 600 sensors randomly dispersed over 2 intersecting 30^osector graphs of 10m diameter with one sink at the narrow end of each sector
- events can be reported to either sink



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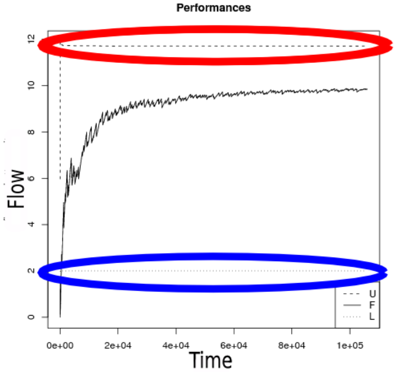

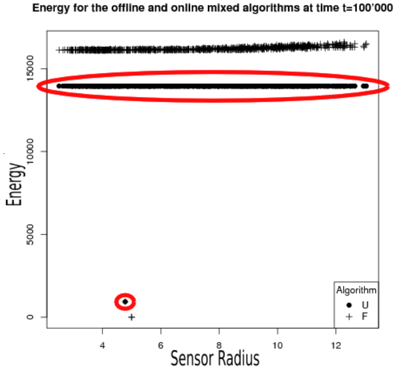

Summary

1. Prove that optimal solution belongs to a subset of realistic data propagation algorithms: a *mixed strategy* which is *energy balanced*
2. Propose a distributed algorithm based on theoretical results
3. Show that the algorithm is efficient (Markov chain context and simulations)

D. Other Methods

- Adjusting transmission ranges
- An offline approach
- Power aware routing
- Lifetime maximizing routing
- Load-balanced energy-aware routing

D1. Adjusting transmission ranges to avoid energy holes

Design Guidelines for Maximizing Lifetime and Avoiding Energy Holes in Sensor Networks with Uniform Distribution and Uniform Reporting
S. Olariu, I. Stojmenovic
in INFOCOM 2006

- Uniformly distributed sensors, each sending roughly the same number of reports to the sink
- They prove that to minimize energy spent, all sectors must have the same width (which they evaluate)
- This choice, however, leads to uneven energy depletion. Towards energy balance, sector widths must be fine-tuned
- As expected, the width of sectors in energy-balanced network decreases as we near the sink

D2. An offline approach

On the Energy Hole Problem of Nonuniform Node Distribution in Wireless Sensor Networks
X. Wu, G. Chen, S. K. Das
in MASS 2006

- They find that in a circular sensor network with a nonuniform node distribution and constant data reporting, the unbalanced energy depletion among the nodes in the whole network is unavoidable
- A suboptimal energy efficiency among the inner parts of the network is possible if the number of nodes increases with geometric proportion from the outer parts to the inner ones
- They also present a routing algorithm with this node distribution strategy

D3. Energy balance under mobility

Energy balanced data propagation in wireless sensor networks with diverse node mobility
D. Efstathiou, I. Kotsogiannis, S. Nikoletseas
in MOBIWAC 2011

- This work is the first studying the energy balance property in wireless networks where the nodes are highly and dynamically mobile
- They propose a new diverse mobility model which is easily parametrized
- They also present a new protocol which tries to adaptively exploit the inherent node mobility in order to achieve energy balance in the network in an efficient way

D4. Power-Aware Routing

- In its basic version, the method selects routes in such a way as to *prefer nodes with longer remaining battery lifetime* (residual energy).
- Let R_i the remaining energy of an intermediate node i. The following link metric is used for all links out of node i:

$$C_{ij} = \frac{1}{R_i}$$

- A shortest-cost path algorithm (such as Dijkstra's or Bellman-Ford) is used to determine a path $\mathcal{P}$, minimizing

$$\sum_{i \in \mathcal{P}} \frac{1}{R_i}$$

This way, nodes with residual energy R_i are favored.

D5. Lifetime-Maximizing Routing (I)

- The previous method (avoiding low-energy nodes) avoids early node failures. On the other hand, methods minimizing per-hop transmission costs minimize the total energy spent. To optimize the system lifetime globally, *both goals* should be addressed simultaneously.
- A possible balance is to select *the minimum energy path at the beginning* (when all nodes have high energy) and *avoiding the low residual energy nodes later* during the protocol evolution.

Lifetime-Maximizing Routing (II)

- This can be implemented by the following link metric:

$$C_{i,j} = T_{i,j}^{a} \cdot R_i^{-b} \cdot E_i^{c}$$

  where $T_{i,j}$ the transmission cost on the (i,j) link, R_i the residual energy of node i and E_i the initial energy of node i.
- The choice of the a, b, c parameters allows addressing *different performance priorities and their combinations*:
  - If $(a,b,c) = (0,0,0)$ we get a minimum number of hops protocol.
  - If $(a,b,c) = (1,0,0)$ we get the minimum total energy per packet protocol.
  - If $b = c$, we have normalized residual energies (e.g. based on initial energy at the node), while $c = 0$ implies absolute residual energies.
  - If $(a,b,c) = (0,1,0)$ we get the power-aware routing method described previously.
- Simulation results suggest that a non-zero a and a relatively large $b = c$ terms (e.g. $(1,50,50)$) provide best performance.