Lecture 2: Introduction / Efficient Data propagation in Wireless Sensor Networks (I)

Sotiris Nikoletseas
Professor

CEID - ETY Course
2017 - 2018
Routing can be made robust and energy efficient by taking into account some pieces of state information available locally in the network, such as:

- **link quality**: e.g. packet reception rates obtained by periodic monitoring
- **link distance**: if link monitoring is problematic/expensive (e.g. in highly dynamic, rapidly fading environments) link distances can be alternatively used as indicators of link quality and energy consumption
- **residual energy**: avoid nodes with low energy, to prolong the network lifetime
- **location information**: if such information is available, geographic routing techniques can be used
- **mobility information** can also be exploited; sensor/sink mobility is both a challenge and an opportunity
Overview

A. Link quality based methods (ETX, MOR)
B. Link distance based methods
   • Routing with relay diversity (ExOR)
   • Greedy, local neighbor selection (LTP)
A. Link Quality Methods

• **Main goal:** Providing robustness via selecting routes that minimize *end-to-end retransmissions or failure probabilities*.

• **Motivation:** If wireless links were (ideally) error-free, then a shortest hop-count path could be chosen. Such paths however require high distance links, which in realistic settings are likely to be error-prone. Thus, the link quality should be taken into account (via simple ACK signals for each successfully delivered packet).
A1. The ETX metric

- Let $d_f$ be the packet reception rate (probability of successful delivery) over a link in the forward direction, and $d_r$ the probability that the corresponding ACK is received in the reverse direction.

- Assuming each packet transmission can be abstracted as a Bernoulli trial, the expected number of retransmissions required for successful delivery of a packet on this link is:

$$ETX = \frac{1}{d_f \cdot d_r}$$

- This metric for a single link can then be incorporated into any routing protocol, so that chosen end-to-end paths minimize the sum of ETX over all links on the path, i.e. the total expected number of transmissions on the path.
ETX routing - example

This example shows 3 routes from A to B, with the forward probabilities on each link (for simplicity, all reverse probabilities are taken $d_r = 1$).

- The direct A-B transmission would require an expected number of 10 retransmissions.
- The path via C,D,E would require $4(1/0.9) \approx 4.44$ retransmissions (1.1 per link) on average.
- The A-F-B path would require $2(1/0.8) \approx 2.5$ retransmissions (1.25 per link) on average.

$\Rightarrow$ the ETX-minimizing path is A-F-B.
ETX routing - discussion

- As demonstrated in the example, ETX neither favors long paths with many short-distance, high-quality links, nor very short paths with a few long-distance, low-quality links. It actually takes an “in-between” approach.

- **ETX advantages:** It minimizes the number of transmissions required, as well as it improves energy efficiency. Also, it directly addresses the potential link asymmetries, by taking into account packet delivery probabilities in both directions.

- **ETX limitation:** It assumes knowledge of the $d_r, d_f$ packet reception probabilities. In quite static networks these values may be periodically obtained via link monitoring, however in dynamic networks the obtained values may become obsolete very soon and the link monitoring overhead may be prohibitively high.
A2. The Minimum Outage Route (MOR) Metric (I)

- Under *high dynamics* (mobile nodes, mobile objects) the link quality fluctuates very rapidly and *ETX is not feasible*. In such cases, *analytic models* of link quality can be used. They explicitly model the wireless channel as having multi-path fading with Rayleigh statistics (fluctuations over time).

- Let $d$ the distance between transmitter and receiver, $h$ the path-loss exponent, $SNR$ the signal-to-noise ratio without fading, $f$ the fading state of the channel. Then, the *instantaneous capacity* of the channel is:

$$C = \log \left( 1 + \frac{|f|^2}{d^h \cdot SNR} \right)$$

- The *outage probability* $P_{out}$ is defined as the probability that the instantaneous channel capacity falls below the transmission rate $R$. It is shown that:

$$P_{out} = 1 - \exp \left( \frac{-d^h}{\mu \cdot SNR^*} \right)$$

where $\mu = E[|f|^2]$ is the mean of the Rayleigh fading and $SNR^*$ a normalized $SNR$. 
Then, end-to-end reliability of a route is defined as the probability that none of the intermediate links suffers outage, and the most reliable route between two nodes is one that minimizes the following path metric:

\[
\sum_{i} d^h_i
\]

where \( d_i \) is the distance of the \( i^{th} \) hop in the path. The metric \( d^h_i \) for each link is called the minimum outage route (MOR) metric.

The MOR approach does not require collection of link quality metrics like ETX, neither ACK messages are used. However, its practicality is limited by the basic abstraction that the channel fading follows a certain distribution (the Rayleigh distribution).
• In general, nodes *closer to the data destination* are chosen/favored.

• The relay diversity provided by the fact that wireless transmissions are *broadcast to multiple nodes* is exploited.
B1. Routing With Relay Diversity

- An example:

![Diagram showing routing with relay diversity](image)

- In traditional routing, the reliability of the AC path depends on whether both AB and BC transmissions have been successful.
- However, if C is also allowed to accept packets directly from A, then the reliability can be further increased without much additional energy cost.
- Allowing such 2-hop packet reception, in the high SNR regime, the end-to-end outage probability decays as \((SNR)^{-2}\). When nodes within \(L\) hops can communicate with each other with high \(SNR\), this probability can become as small as \((SNR)^{-L}\).
- A weakness of this method is that it requires a larger number of receivers to be actively overhearing each message, which may incur a radio energy penalty.
Main idea: the identity of the node which will eventually forward a packet, is not predetermined before the packet is transmitted. Instead, the method tends to ensure that the node closest to the destination that receives a given packet will forward the packet further.

The protocol has three stages:

1. Priority ordering: At each step, the transmitter includes in the packet a schedule with the priority order candidate receivers should forward the packet.

Priority of candidate receivers

A \rightarrow C: 0.9
C \rightarrow D: 0.9
D \rightarrow B: 0.9
E \rightarrow C: 0.6
E \rightarrow B: 0.6

B D E C
2. **Transmission acknowledgements**: A MAC scheme is used so that each candidate receiver sends the ID of the highest-priority successful recipient known to it. All nodes listen to all ACKs, so they distributively determine which node, among those who received the packet successfully, has highest priority.

3. **Forwarding decision**: After listening to all ACKs, the nodes that have not heard of any IDs with priorities greater than their own will transmit.

- **Features of the ExOR protocol**
  - *Nodes further away to the current node (yet closer to the destination)* are less likely to successfully receive the packet, but, whenever they receive it, they are favored to act as forwarders!
  - This *tends to make good progress towards the data destination*, without many transmissions and delays.
  - As with relay diversity, ExOR requires *a larger number of receivers to be active*. Also, the priority evaluation necessitates some inter-node packet delivery ratios to be tracked and maintained.
We call network nodes “particles”.

a) Each particle may have *two communication modes*: a *broadcast* (radio) *beacon mode* and a *directed to a point* transmission mode (laser beam). However, a radio broadcast suffices.

b) Each particle may alternate between a *sleeping* and an *awake* mode. During sleeping periods particles cease any communication.

c) Particles *do not move*.

d) The particles are spread in a *two-dimensional* area (plane).
e) A receiving wall $\mathcal{W}$ is a line in the plane. The wall represents the control center (multiple/mobile sinks).

f) Each particle is aware of the direction toward $\mathcal{W}$.

g) No geolocation abilities assumed

**Definitions:** Let $d$ (in numbers of particles /$m^2$) be the density of the cloud.

Let $\mathcal{R}$ be the maximum (beacon/laser) transmission range of each particle.
The Local Target Protocol (LTP)

Let $d(p_i, p_j)$ the (vertical) distance of $p_i$, $p_j$ and $d(p_i, \mathcal{W})$ the (vertical) distance of $p_i$ from $\mathcal{W}$. Let $\text{info}(\mathcal{E})$ the info to be propagated. Each $p'$ receiving $\text{info}(\mathcal{E})$ does the following:

- **Search Phase**: It uses a low energy broadcast of a beacon (angle $\alpha$ above and below the vertical line) to discover a particle closer to $\mathcal{W}$ (i.e. a $p''$ where $d(p'', \mathcal{W}) < d(p', \mathcal{W})$).

- **Direct Transmission Phase**: If found, $p'$ sends $\text{info}(\mathcal{E})$ to $p''$ via a direct line (laser) transmission.

- **Backtrack Phase**: If repetitions of the search phase fail to discover a particle nearer to $\mathcal{W}$, then $p'$ sends $\text{info}(\mathcal{E})$ to the particle it received the information from.
Example of the Search Phase

Example of Data Propagation

- beacon circle
- $p'_1$
- $p_0$
- $p_1$
- $p_2$
- $p_3$
- $a_0$
- $a_1$
- $a_2$
Efficiency

Definitions: Let $h_{opt}$ the (optimal) number of “hops" (vertical to W transmissions) needed to reach $W$, if particles always exist in pair-wise distances $R$ towards $W$.

Let $h$ the actual number of hops (transmissions) taken to reach $W$. The “hops” efficiency of the data propagation protocol is the ratio

$$C_h = \frac{h}{h_{opt}}$$

where $h_{opt} = \left\lceil \frac{d(p, W)}{R} \right\rceil$
Why studying $h, C_h$?

When a particle $p$ “looks around" for a particle as close to $W$ as possible to pass information, it may not get any particle in the perfect direction (on the line vertical to $W$ passing from $p$), mainly because:

a) There might never have been any particles in that direction.

b) Particles of sufficient remaining battery power may not be available anymore.

c) Particles available may temporarily “sleep" to save energy.
Simplifying Assumptions for a Rigorous Analysis

- The search phase always finds a \( p'' \) in the semicircle of center \( p' \) and radius \( R \) towards \( \mathcal{W} \). This assumption can be relaxed: (a) by repetitions of the search phase (b) we may consider a cyclic sector defined by circles of radii \( R - \Delta R, R \) (c) if a search phase ultimately fails, the protocol backtracks.

- The position of \( p'' \) is random uniform in the arc of angle \( 2a \).

- Each target selection is stochastically independent of the others.
Lemma

The expected “hops” efficiency of LTP in the $\alpha$-uniform case is $E(C_h) \approx \frac{\alpha}{\sin \alpha}$, for large $h_{opt}$. Also, $1 \leq E(C_h) \leq \frac{\pi}{2} \approx 1.57$.

Proof: A sequence of points is generated, $p_0 = p, p_1, p_2, \ldots, p_{h-1}, p_h$ where $p_{h-1}$ is the first particle found within $W$’s range and $p_h$ is beyond $W$. Let $\alpha_i$ be the (positive or negative) angle of $p_i$ w.r.t. $p_{i-1}$’s vertical line to $W$. It is:

$$\sum_{i=1}^{h-1} d(p_{i-1}, p_i) \leq d(p, W) \leq \sum_{i=1}^{h} d(p_{i-1}, p_i)$$
The (vertical) progress toward $\mathcal{W}$ is $\Delta_i = d(p_{i-1}, p_i) = R \cos \alpha_i$. We get:

$$\sum_{i=1}^{h-1} \cos \alpha_i \leq h_{opt} \leq \sum_{i=1}^{h} \cos \alpha_i$$

From Wald’s equation, then

$$E(h - 1) \cdot E(\cos \alpha_i) \leq E(h_{opt}) \leq E(h) \cdot E(\cos \alpha_i) \Rightarrow$$

$$\frac{\alpha}{\sin \alpha} \leq \frac{E(h)}{h_{opt}} = E(C_h) \leq \frac{\alpha}{\sin \alpha} + \frac{1}{h_{opt}}$$

since

$$E(\cos \alpha_i) = \int_{-\alpha}^{\alpha} \cos x \cdot \frac{1}{2\alpha} \cdot dx = \frac{\sin \alpha}{\alpha}$$

Assuming large values for $h_{opt}$ and since for $0 \leq \alpha \leq \frac{\pi}{2}$ it is $1 \leq \frac{\alpha}{\sin \alpha} \leq \frac{\pi}{2}$ we get the result.
The “min-two uniform targets” (M2TP) Protocol

We assume that the search returns two points \( p'' \), \( p''' \) each uniform in \((-\alpha, \alpha)\) and that the protocol selects the best.
Let \( \alpha_{i1}, \alpha_{i2} \) the angles of the particles found and let \( \alpha_i = \min \{|\alpha_{i1}|, |\alpha_{i2}|\} \). Then,

\[
P\{\alpha_i > \phi\} = P\{|\alpha_{i1}| > \phi \cap |\alpha_{i2}| > \phi\} = \left(\frac{\alpha - \phi}{\alpha}\right)^2
\]

Thus, the distribution function of \( \alpha_i \), is

\[
F_{\alpha_i}(\phi) = P\{\alpha_i \leq \phi\} = \frac{2\alpha\phi - \phi^2}{\alpha^2}
\]
and the probability density function is,

\[
f_{\alpha_i}(\phi) = \frac{d}{d\phi} P\{\alpha_i \leq \phi\} = \frac{2}{\alpha} \left(1 - \frac{\phi}{\alpha}\right)
\]

The expected local progress is:

\[
E(\cos \alpha_i) = \int_0^\alpha \cos \phi \cdot f_{\alpha_i}(\phi) \, d\phi = \frac{2(1 - \cos \alpha)}{\alpha^2}
\]
Lemma

The expected “hops” efficiency of the “min-two uniform targets” protocol in the $\alpha$-uniform case is $E(C_h) \approx \frac{\alpha^2}{2(1-\cos \alpha)}$, for $0 \leq \alpha \leq \frac{\pi}{2}$ and for large $h$.

We remark that

$$\lim_{\alpha \to 0} E(C_h) = \lim_{\alpha \to 0} \frac{2\alpha}{2 \sin \alpha} = 1$$

and

$$\lim_{\alpha \to \frac{\pi}{2}} E(C_h) = \frac{(\pi/2)^2}{2(1-0)} = \frac{\pi^2}{8} \approx 1.24$$

Lemma

The expected “hops" efficiency of the min two uniform targets protocol is $1 \leq E(C_h) \leq \frac{\pi^2}{8} \approx 1.24$ for large $h$ and for $0 \leq \alpha \leq \frac{\pi}{2}$. 
Consider $p$ at distance $x$ from $\mathcal{W}$. We assume that when $p$ searches in the sector $S$ defined by $(-\alpha, \alpha)$ and $R$, a particle $p'$ is returned in the sector with some probability density $f(p')dA$.

**Definition:** (Horizontal progress) Let $\Delta x$ be the projection of the line segment $(p, p')$ on the line from $p$ vertical to $\mathcal{W}$. Assume each search returns such a $p'$, with independent and identical distribution $f()$.

**Definition:** (Significant progress)
Let $m > 0$ be the least integer such that $\Pr\{\Delta x > \frac{R}{m}\} \geq p$, where $0 < p < 1$ is a given constant.
**Definition:** Let the stochastic process $P$ where with probability $p$ the horizontal progress is $\mathcal{R}/m$ and with probability $q = 1 - p$ it is 0.

**Lemma:**

Let $Q$ the actual process. Then $\mathbb{P}_P\{h \leq h_0\} \leq \mathbb{P}_Q\{h \leq h_0\}$ (stochastic dominance).

Now let $t = \left\lceil \frac{x}{\mathcal{R}/m} \right\rceil = \left\lceil \frac{mx}{\mathcal{R}} \right\rceil$. Consider the integer r.v. $H$ such that $\mathbb{P}\{H = i\} = q^i (1 - q)$ for any $i \geq 0$. Then $H$ is geometrically distributed. Let $H_1, \ldots, H_t$ be $t$ random variables, i.i.d. according to $H$. Clearly then:

**Lemma**

$\mathbb{P}_P\{\text{# of hops is } h\} = \mathbb{P}\{H_1 + \cdots + H_t = h\}$
Theorem

\[ P \{ \text{the number of hops is } h \} = \binom{-t}{h} p^t (-q)^h = \binom{t+h-1}{h} p^t q^h \]

(since \( h \) is negative binomial because it is the number of failures until the \( t_{th} \) success)

**Corollary:** For the process \( P \), the mean and variance of the number of hops are:

\[
E(h) = \frac{tq}{p} \quad \text{Var}(h) = \frac{tq}{p^2} \]

The method above finds a distribution that upper bounds the number of hops. Since for all \( f() \) it is \( h \geq \frac{x}{R} = h_{opt} \) we get

**Theorem**

The process \( P \) estimates the expected number of hops with a guaranteed ratio \( \frac{(m+1)(1-p)}{p} \) at most.

**Example:** When for \( p = 0.5 \) we have \( m = 2 \) and the efficiency ratio is 3, i.e. the overestimate is 3 times the optimal number of hops.
Summary evaluation of LTP

- local, simple, greedy protocol
- no global structure (set of paths) maintained
- good for dense networks
- performance drops in sparse / faulty networks