Design Guidelines for Maximizing Lifetime and Avoiding Energy Holes in Sensor Networks with Uniform Distribution and Uniform Reporting

Stephan Olariu
Department of Computer Science
Old Dominion University
Norfolk, VA 23529-0162
Email: olariu@cs.odu.edu

Ivan Stojmenović
SITE, University of Ottawa
Ottawa, Ontario, K1N 6N5
CANADA
Email: ivan@site.uottawa.ca

***Abstract*— This paper investigates theoretical aspects of the uneven energy depletion phenomenon recently noticed in sink-based wireless sensor networks. We consider uniformly distributed sensors, each sending roughly the same number of reports toward the closest sink. We assume an energy consumption model governed by the relation $E = d^\alpha + c$ where $d,\ (d \leq t_x)$, is the transmission distance, $\alpha \geq 2$ is the power attenuation, c is a technology-dependent positive constant, and t_x is the maximum transmission range of sensors. Our results are multifold. First, we show that for $\alpha > 2$, all sensors whose distance to the sink is $\min\{t_x, (\frac{2c}{\alpha-2})^{\frac{1}{\alpha}}\}$ should transmit directly to the sink. Interestingly, this limit does not depend on the size of the network, expressed as the largest distance R from a sensor to the closest sink. Next, we prove that in order to minimize the total amount of energy spent on routing along a path originating at a sensor in a corona and ending at the sink, all the coronas must have the same width, equal to the above expression. This choice, however, leads to uneven energy depletion and to the creation of energy holes. We show that for $\alpha > 2$ the uneven energy depletion can be prevented by judicious system design, resulting in balanced energy expenditure across the network. We describe an iterative process for determining the sizes of coronas. Their optimal sizes (and corresponding transmission radii) and the number of coronas depend on R. As expected, the width of coronas in energy-balanced sensor network increases. Finally, we show that for $\alpha = 2$, the uneven energy depletion phenomenon is intrinsic to the system and no routing strategy can avoid the creation of an energy hole around the sink.**

I. INTRODUCTION

Aggregating tiny sensors into sophisticated communication infrastructures, called wireless sensor networks (*sensor networks*, for short) is expected to have a significant impact establishing ubiquitous networks that will pervade society redefining the way in which we live and work. The novelty of sensor networks and their potential applications have triggered a well-deserved flurry of activity in both industry and academia. We refer the reader to [1]–[3], [15] for a summary of recent applications.

We assume that the sensor network interfaces with the outside world via one or several *sinks*. The sensory data collected by the sensors is routed to the closest sink where it is further aggregated. Recently, it was noticed that the sensors closest to the sink tend to deplete their energy budget faster than other sensors [6], [7], [10], [11], [16], [26]. This uneven energy depletion is apt to drastically reduce the useful lifespan of sensor networks and should be prevented to the largest extent possible. In fact, [26] argue that by the time the sensors one hop away from the sink exhaust their energy budget, sensor farther away still have up to 93% of their initial energy budget. These studies assume that each sensor uses the same fixed transmission radius for reporting.

The main contribution of this work is to provide a theoretical explanation of the uneven energy depletion phenomenon noticed in sink-based wireless sensor networks.

We prove that in order to minimize the total amount of energy spent on routing along a path originating at a sensor in a corona and ending at the sink, all the coronas must have the same width, equal to $(\frac{2c}{\alpha-2})^{\frac{1}{\alpha}}$ or the maximal transmission radius t_x, whichever is smaller. However, we show that this choice necessarily leads to uneven energy depletion and to the creation of energy holes, regardless of the value of α.

To remedy the energy imbalance problem, we assume that the transmission radii of sensors are adjustable, and

1-4244-0222-0/06/$20.00 (c)2006 IEEE
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the Proceedings IEEE Infocom.

attempt to balance the energy expenditure among sensors by selecting proper sizes of coronas around the sink. The transmission radius of a given sensor is assumed to be equal to the width of the corona containing it. Our most interesting result is to show that, for $\alpha > 2$, all sensors whose distance to the sink is $\min\{t_x, \leq \frac{2c}{\alpha-2})^{\frac{1}{\alpha}}\}$ should transmit directly to the sink. Surprisingly, this limit does not depend on the size of the network, expressed as the maximum distance R from a sensor to the closest sink. However, such balancing is possible as long as the required corona widths do not exceed the maximum transmission radius. There also exists a maximum network radius around the sink that allows for energy balancing, and that radius depends solely on α, c and t_x.

We describe an iterative process for determining the sizes of coronas. Their optimal sizes are obtained by equating the energy expenditure in a given corona with the expenditure in the rink containing the sink. The width of coronas in an energy-balanced sensor network increases with the distance to the sink.

Finally, we show that for $\alpha = 2$, the uneven energy depletion phenomenon is intrinsic to the system and no routing strategy can avoid the creation of an energy hole around the sink.

The remainder of the paper is organized as follows. Our literature review is presented in section II. Section III introduces the system assumptions used throughout the work. Section IV establishes general formulas for energy expenditure. In Section V these results will be used to evaluate the energy expenditure for individual sensors. In Section VI we show that by insisting on the energy-optimality of each path to the sink we, in fact, are guaranteed to create an energy hole around the sink drastically curtailing the lifetime of the network. In Section VII we show that the energy hole problem can be avoided by carefully tailoring the coronas. In this case we avoid uneven energy depletion at the cost of suboptimal routing. Finally, Section X offers concluding remarks and points out directions for further work.

II. LITERATURE REVIEW

Mhatre and Rosenberg [16] consider sensor networks containing two types of sensors. Regular sensors use either single or multi-hop communication to send their data to their respective cluster-heads (CHs), have a smaller energy budget, and the same transmission radius. The other type of sensors have more energy, and can serve as CHs for regular sensors. CH sensors send data directly to a helicopter, therefore requiring the same energy. They aggregate received data (energy needed for aggregating is proportional to number of incoming reports) before transmitting to the helicopter. The problem is to find design parameters so that both types of sensors loose energy at about the same time, network life exceeds a threshold set in advance, and total cost of the network is minimized. The total cost includes the cost to build sensors and the energy spent by them, combined into a linear function. The total number of sensors of both types is fixed. The authors of [16] conclude that the number of regular sensors is proportional to the square of the number of CHs. They analyze two modes of communicating between sensors and base stations, and derive conditions under which single-hop transmission by all nodes is best. One of the conclusions reached is that for $\alpha = 2$, there is no benefit from multi-hop communication. When multi-hop communication is better, each CH is assumed to be at the center of a circle divided into equal width rings (the width is equal to the transmission radius). Therefore they assume that each of multiple hops is approximately of equal length and find the optimal forwarding distance for each hop. The authors [16] do not actually prove that it is indeed optimal to use each hop of equal length (that is, that the rings indeed all need to have equal width for optimality). Their result is based on minimizing energy in a ring that is found to be critical. However, other rings may not be critical at that time. In this article, we prove that the minimal energy consumption per path is achieved for equal width rings.

Mhatre *et al.* [17] considered a heterogeneous sensor network in which the nodes are to be deployed over a unit area for the purpose of surveillance. An aircraft visits the area periodically and gathers data about the activity in the area from the sensor nodes. There are two types of nodes that are distributed over the area uniformly, but each with its own densities and battery energy. Type 0 nodes do the sensing while type 1 nodes act as the cluster heads besides doing sensing. Nodes use multi-hopping to communicate with their closest cluster heads. The authors of [17] determine the optimum node intensities and node energies that guarantee a lifetime of at least T units, while ensuring connectivity and coverage of the surveillance area with a high probability. They minimize the overall cost of the network under these constraints. Lifetime is defined as the number of successful data gathering trips (or cycles) that are possible until connectivity and/or coverage are lost. Conditions for a sharp cutoff are also taken into account, to ensure that almost all the nodes run out of

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the Proceedings IEEE Infocom.

energy at about the same time so that there is very little energy waste due to residual energy.

Node clustering is commonly considered as one of the most promising techniques for dealing with maximizing network lifetime and has been addressed by many researchers. However, very few, if any, provide explicit analysis of node clustering in sensor networks and/or manage to prove its actual effectiveness. Vlajic and Xia [25] take a close analytical look at clustered sensor networks They prove that these networks do not necessarily outperform non-clustered sensor networks. The condition that ensures superior performance of clustered sensor networks, with absolute certainty, is that their resultant clusters lie within the isoclusters of the monitored phenomenon. They also show that in clustered sensor networks that satisfy the given condition, cluster sizes do not need to match the sizes of their respective underlying isocluster. Instead, simple 5-hop clusters can ensure near-optimal network performance under a wide range of cluster-to-sink and cluster-to-isocluster spatial arrangements.

Perillo, Cheng, and Heinzelman [20] study the energy imbalance among sensors. When each node has a fixed transmission range, the amount of traffic that sensor nodes are required to forward increases dramatically as the distance to the data sink becomes smaller. Thus, sensors closest to the data sink tend to die early, leaving areas of the network completely unmonitored and causing network partitions. Alternatively, if all sensors transmit directly to the data sink, the furthest nodes from the data sink will die much more quickly than those close to the sink. While it may seem that network lifetime could be improved by use of a more intelligent transmission power control policy that balances the energy used in each node by requiring nodes further from the data sink to transmit over longer distances, such a policy can only have a limited effect. In fact, this energy balancing can be achieved only at the expense of gross energy inefficiencies. In this paper, we investigate the transmission range distribution optimization problem and show where these inefficiencies exist when trying to maximize the lifetime of many-to-one wireless sensor networks. Soro and Heinzelman [23] proposed unequal clustering size model for network organization, which can lead to more uniform energy dissipation among the cluster head nodes, thus increasing network lifetime. The approach is applied for both homogeneous and heterogeneous sensor networks.

Lian, Naik and Agnew [11] also recognize the energy imbalance problem and propose a non-uniform sensor distribution strategy. The density of sensors increases when the distance to the sink decreases. Simulation results show that for large dense networks, the non-uniform sensor distribution strategy can increase the total data capacity by an order of magnitude. Lian et all [12] proposed a Broadcasting-Based query Scheme (BBS) that reduces the energy depletion rate of sensors near the sink, builds different localized routing trees for different query types, and eliminates the flooding cost of query distribution. In [13], Lian, Naik and Agnew proposed to use mobile sinks in sensor networks with uniform distribution. In [14], Lian *et all* propose a strategy that employs uniform energy distribution but applies broadcasting to a specific sensor (local root) before the aggregated report is sent to the sink.

While most recent work has focused on the deployment of large numbers of cheap homogeneous sensor devices, in practical settings it is often feasible to consider heterogeneous deployments of devices with different capabilities. Lee, Krishnamachari and Kuo [9] introduce cost constraints, and analyze such heterogeneous deployments both mathematically and through simulations, and show how they impact the coverage aging process of a sensor network, i.e., how it degrades over time as some nodes become energy-depleted. They derive expressions for the heterogeneous mixture of devices that optimizes the lifetime sensing coverage in a single-hop direct communication model. They also investigate a multi-hop communication model through simulations, and examine the impact of heterogeneity on lifetime sensing coverage and coverage aging both with and without data aggregation.

Sheldon, Chen, Nixon and Mok [22] seek the ways to deploy the network so that the workload is evenly distributed, thus the network overall behavior degrades in a smooth fashion. Assuming that the sensors should be evenly deployed within the monitored area, they look at the approach where a set of more powerful nodes are designated for data relaying. In particular, they select subregions to deploy relaying nodes at calculated density, and propose a simple method where the density is simply based on the size of the area whose data will be relayed by these nodes.

Li and Mohapatra [10] study the following problem: given the required lifetime of the sensor network, the initial energy of each sensor node, and the area to be covered, what is the minimum number of nodes needed to construct such a network and what is the corresponding placement scheme? They describe a solution for linear network, using a model with $c = 0$.

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the Proceedings IEEE Infocom.

III. System assumptions

A. The sensor model

We assume that individual sensors operate subject to following fundamental constraints. (1) Once deployed, the sensors must work *unattended* as it is either impractical or infeasible to devote attention to individual sensors; (2) Sensors are *anonymous* – they do not have fabrication-time identities. In particular, point-to-point routing cannot be based on IDs of neighboring sensors; (3) Each sensor has a non-renewable energy budget – when the on-board energy supply is exhausted, the sensor becomes in-operational; (4) Each sensor has a maximum transmission range, denoted by t_x, assumed to be much smaller than R, the furthest possible distance from a sensor to its closest sink.[1] This implies that messages sent by a sensor can reach only the sensors in its proximity, typically a small fraction of the sensors deployed; (5) In order to save energy, the sensors can adjust their transmission range.

B. The sensor network model

Throughout the work we assume a sensor network endowed with one or more sinks as illustrated in Figure 1. We assume that each sink is equipped with a steady energy supply and with a powerful radio that can cover a disk of radius R centered at the sink. In our model, each sink pushes requests (referred to as *tasks*) targeted at a subset of the sensors in its disk of radius R and collects the sensory data generated.

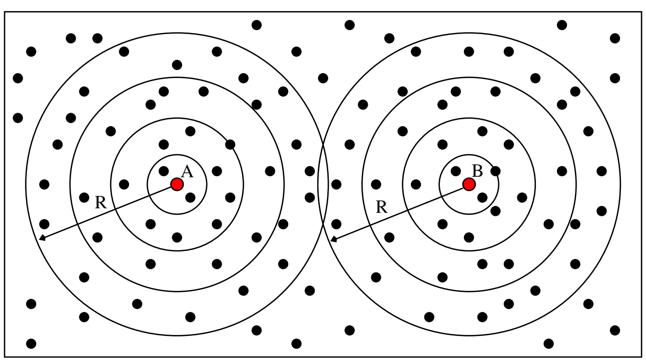

Fig. 1. *A multi-sink sensor network.*

The sink organizes the sensors around it into a dynamic infrastructure as illustrated in Figure 2. This task is referred to as *training* [18], [26] and involves partitioning the disk D of radius R into disjoint concentric sets termed *coronas* obtained as follows. Consider k concentric circles of radii $0 < r_1 < r_2 < \cdots < r_k = R$ centered at the sink. To handle boundary conditions we take $r_0 = 0$. Now, for every i, $(1 \leq i \leq k)$, corona C_i is the subarea of D delimited by the circles of radii r_{i-1} and r_i. The width of each corona is at most t_x, the maximum transmission range of a sensor. For example, in Figure 2, $k = 4$ and the area is partitioned into four coronas C_1, C_2, C_3 and C_4.

Importantly, the massive deployment of sensors, combined with the fact that the width of each corona does not exceed the maximum transmission range t_x, guarantees communication between sensors in adjacent coronas. The width of corona C_i is $r_i - r_{i-1}$. These widths may differ, as the sensors are allowed to adjust their transmission radii. To simplify, we assume that a sensor in corona C_i uses a transmission radius of $r_i - r_{i-1}$ to reach a sensor in corona C_{i-1}.

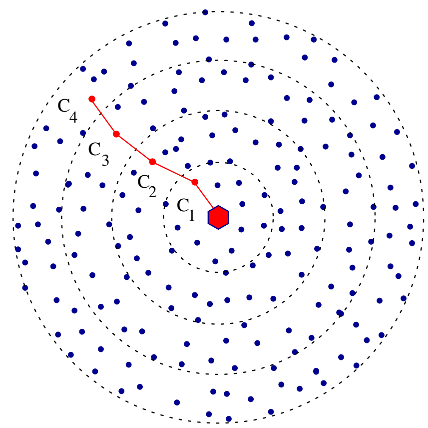

Fig. 2. *Concentric coronas and routing the result of a task to the sink.*

In the remainder of this work we consider a *generic* sink and the disk of radius R around it. For ease of exposition, we refer to the sensors in this disk as the *sensor network*. We adopt a task-based model compatible with [18], [26] whereby the sensor network is subjected to a set $\overline{T}$ of tasks. Each task involves performing local sensing by a subset of the sensors in a corona followed by (local) data aggregation and sending the resulting information (the answer to the query) to the sink. Thus, we associate each of the $\overline{T}$ tasks with a sensor-to-sink path. It follows that there is a one-to-one map between tasks and paths to the sink.

We define the *longevity* of the network as the maximum number of tasks that can be performed by the individual sensors [26].

[1] Of course, t_x is a system parameter that depends on the particular type of sensors deployed. Under present-day technology, t_x is about 50m for micro-sensors.

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the Proceedings IEEE Infocom.

C. Routing

In the dynamic infrastructure discussed above, routing is relatively straightforward. As we have already mentioned, the sink is pushing queries targeting sensors in one corona. Having collected and aggregated locally the sensory data, as discussed in [18], the answer is routed to the sink. Figure 2 illustrates a possible path along which the result of the task (i.e. the answer to the corresponding query) performed by a subset of the sensors in the outermost corona is routed to the sink. Notice that each hop involves sensors from adjacent coronas.

IV. ROUTING-RELATED ENERGY EXPENDITURE

Since our results involve reasoning about the energy spent by various sensors, it is important to begin by specifying the energy consumption model assumed throughout this paper. Specifically, we assume that the amount of energy expended to transmit a message of unit length a distance d away from the sender is

$$E_d = ad^\alpha + b \tag{1}$$

where $2 \le \alpha \le 6$ and a and b are positive constants. By normalizing in the obvious way we obtain the equivalent form

$$E_d = d^\alpha + c \tag{2}$$

where c is a technology-dependent positive constant. While c is a function that depends on a large number of parameters and whose exact evaluation is quite challenging [21], under present-day sensor technology a reasonable approximation of c, valid for values of α in the range above is about 4500 [5], [6], [10]. With the exception of Subsection VII-C where a numerical example is offered, we consider c to be, simply, a positive constant.

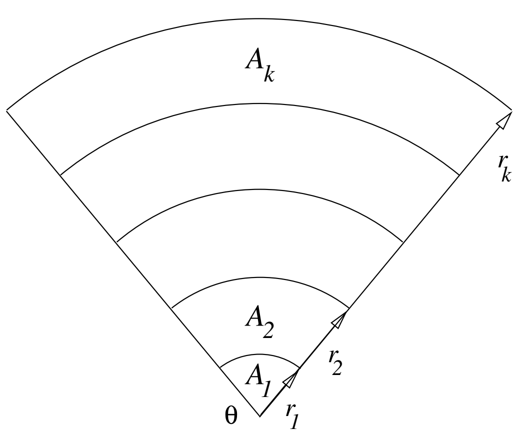

Fig. 3. *A wedge W and the associated sectors.*

Consider an arbitrary wedge W subtended by an angle of θ and refer to Figure 3. W is partitioned into k sectors $A_1, A_2, \ldots, A_k$ by its intersection with k concentric circles, centered at the sink, and of monotonically *increasing* radii $r_1 < r_2 < \cdots < r_k = R$. It is important to recall that $r_k = R$ is a system parameter and, thus, a constant.

For convenience of notation we write $r_0 = 0$ and interpret A_0 as the sink itself.

Let n denote the total number of sensors deployed in wedge W. We assume a uniform deployment with density ρ. In particular, with A standing for the area of wedge W, we can write

$$n = \rho A = \frac{\rho\theta}{2} r_k^2 \tag{3}$$

Let $n_1, n_2, n_3, \ldots, n_k$ stand for the number of nodes deployed in the sectors $A_1, A_2, A_3, \ldots, A_k$, respectively. Since the deployment is uniform, it is easy to confirm that for every $i,\ (1 \le i \le k)$,

$$n_i = \rho A_i = \rho \int_0^\theta \int_{r_{i-1}}^{r_i} x \; dx \; d\theta = \frac{\rho\theta}{2}(r_i^2 - r_{i-1}^2). \tag{4}$$

Let T denote the number of sector-to-sink paths (henceforth, simply denoted by *paths*) that the wedge W sees during the lifetime of the sensor network. By our previous discussion there is a one-to-one map between paths and tasks. Thus, T equals the total number of tasks that the wedge can handle during the lifetime of the network.

We make the following assumptions motivated by the uniformity of the deployment:

- each sensor in W is equally likely to be the source of a path to the sink
- for $2 \le i \le k$, each sensor in sector A_{i-1} is equally likely to serve as the next hop for a path that involves a node in A_i.

By virtue of the first assumption, the expected number of paths originating at a node in W is

$$\frac{T}{n}. \tag{5}$$

Consider sector A_1. Since the T paths have the sink as their destination, the nodes in sector A_1 must collectively participate in all the T paths. Since A_1 contains n_1 nodes, the expected number of transmissions per node is $\frac{T}{n_1}$. By (2), the energy expended by a node in A_1 per path served is $r_1^\alpha + c$. Thus, the total energy E_1 consumed by a node in A_1 to fulfill its routing obligations is

$$E_1 = \frac{T}{n_1}\left[r_1^\alpha + c\right]$$

which, by (4), can be written as

$$E_1 = \frac{T}{n_1}\left[r_1^\alpha + c\right] = \frac{2T}{\rho\theta r_1^2}\left[r_1^\alpha + c\right] = \frac{2T}{\rho\theta}\frac{r_1^\alpha + c}{r_1^2}. \quad (6)$$

Let $\overline{T}$ denote the total number of tasks performed by the entire wireless sensor network (not just wedge W) during its lifetime. Assuming that the $\overline{T}$ tasks are uniformly distributed throughout the sensor network, and recalling the one-to-one map between tasks and sensor-to-sink paths, we can write

Using

$$\frac{\overline{T}}{2\pi} = \frac{T}{\theta}. \quad (7)$$

By (6) and (7) combined, the total energy needed by a node in A_1 to handle its routing duties is

$$E_1 = \frac{\overline{T}}{\rho\pi}\frac{r_1^\alpha + c}{r_1^2}. \quad (8)$$

At this point it is important to note that given α and c, we can determine r_1 in such a way that expression of E_1 in (8) is minimized. Indeed, simple manipulations (finding the value of r_1 for which the derivative of this function is $= 0$ for $\alpha > 2$, and noting that the function is decreasing for $\alpha = 2$) show that the *optimal* values of r_1 (as a function of α and c) are given by

$$r_1 = \begin{cases} \left(\frac{2c}{\alpha-2}\right)^{\frac{1}{\alpha}} & \text{for } \alpha > 2 \\ t_x & \text{if } \alpha = 2 \end{cases} \quad (9)$$

It follows that

- If $\alpha = 2$ then
$$E_1^{min} = \frac{\overline{T}}{\rho\pi}\left[1 + \frac{c}{t_x^2}\right];$$
- If $2 < \alpha \leq 6$ then
$$E_1^{min} = \frac{\overline{T}}{\rho\pi}\frac{\alpha c}{\alpha - 2}\left(\frac{2c}{\alpha - 2}\right)^{-\frac{2}{\alpha}}.$$

Therefore, we showed that for $\alpha > 2$, all sensors whose distance to the sink is $\min\{t_x, (\frac{2c}{\alpha-2})^{\frac{1}{\alpha}}\}$ should report directly to the sink. It is important to note that (9) implies that the optimal choice for r_1 does not depend on R, the size of the network. That is, the optimal transmission radius for the sensors in the first corona (closest to the sink) and the corresponding width of that corona do not depend on the overall size of the network. This is a counterintuitive and somewhat surprising conclusion since one expected that the need to help other sensors, when R increases, would imply that the sensors in the first corona should reduce their transmission radius. This is not the case for the optimal choice.

We also note that the optimal value for r_1 is also bounded by the maximum transmission radius t_x. That is, the exact formula is $r_1 = \min\{t_x, \left(\frac{2c}{\alpha-2}\right)^{\frac{1}{\alpha}}\}$. However, in this and other formulas that will similarly follow in this article, we decided to keep expressions simple by eliminating the t_x bound. The bound, however, needs to be observed and will have its straightforward impact on derived conclusions.

Further, we note that the conclusions we made did not depend on the selected angle θ for the wedge that was taken for convenience only. That is, we could have considered only one full angle ($\theta = 2\pi$), and all derived results remain the same. This means that the conclusions are not bound to the particular training method in [18], [26] but to any training that initially divides sensors into coronas.

V. EVALUATING THE PER-SENSOR ENERGY EXPENDITURE

In this section we turn to the task of evaluating the energy expenditure per sensor in an arbitrary sector A_i with $i \geq 1$. Since the case $i = 1$ was handled in Section IV we now assume $i \geq 2$.

Observe that nodes in a generic sector A_i, $(2 \leq i \leq k)$, are called upon to serve two kinds of paths:

- paths originating in a sector A_j with $i < j \leq k$, and
- paths originating at a node in A_i.

It is easy to confirm that the number of paths involving nodes in A_i includes all paths except those originating in one of the sectors $A_1, A_2, \ldots, A_{i-1}$. Therefore, by (5), the total number of paths that the nodes in A_i must handle is

$$T - \frac{T}{n}(n_1 + n_2 + \cdots + n_{i-1}).$$

By (3) and (4) combined with elementary manipulations, this expression can be written as

$$T\left[1 - \frac{\sum_{j=1}^{i-1}(r_j^2 - r_{j-1}^2)}{r_k^2}\right] = T\left[1 - \frac{r_{i-1}^2}{r_k^2}\right]. \quad (10)$$

Recall that sector A_i contains n_i nodes. This implies that each node in A_i must participate in

$$\frac{T}{n_i}\left[1 - \frac{r_{i-1}^2}{r_k^2}\right]$$

paths. Using (4), the number of paths handled by each node in A_i can be written as

$$\frac{2T}{\rho\theta}\left[1-\frac{r_{i-1}^2}{r_k^2}\right]\frac{1}{r_i^2-r_{i-1}^2}. \quad (11)$$

Observe that the width of sector A_i is $r_i - r_{i-1}$. It follows that the transmission range needed to send information between A_i and A_{i-1} is $r_i - r_{i-1}$. Using (2), the energy expended by a node in A_i to send information to sensors in A_{i-1} is

$$(r_i - r_{i-1})^\alpha + c.$$

Let the total amount of energy expended by a node in A_i be E_i. By (7) and (11), we have

$$E_i = \frac{\overline{T}}{\pi\rho}\left[1-\frac{r_{i-1}^2}{r_k^2}\right]\frac{(r_i-r_{i-1})^\alpha + c}{r_i^2-r_{i-1}^2}. \quad (12)$$

VI. Optimizing the Energy Spent per Path

The main goal of this section is to derive a relation between the various radii $0 < r_1 < r_2 < \cdots < r_k = R$ in such a way that *total* energy spent per routing path is minimized. For this purpose, let $\mathcal{E}_i$ denote the total amount of energy expended by the sensors along a generic path transferring data from sector A_i to the sink. As before, write $r_0 = 0$ and assume that A_0 is the sink itself. Since in transmitting from A_j to A_{j-1} $(2 \le j \le i)$, the amount of energy spent is $(r_j - r_{j-1})^\alpha + c$, it follows that

$$\mathcal{E}_i = \sum_{j=1}^{i}\left[(r_j - r_{j-1})^\alpha + c\right]. \quad (13)$$

Recall the Lagrange identity [4] (page 64)

$$\sum_{1\le p<q\le i}(a_p b_q - a_q b_p)^2 = \left(\sum_{p=1}^{i} a_p^2\right)\left(\sum_{p=1}^{i} b_p^2\right) - \left(\sum_{p=1}^{i} a_p b_p\right)^2.$$

For every j, $(1 \le j \le i)$, write $a_j = (r_j - r_{j-1})^{\frac{\alpha}{2}}$ and $b_j = 1$. Noticing that

$$\sum_{p=1}^{i} a_p^2 = \mathcal{E}_i - ic, \quad (14)$$

and that

$$\sum_{p=1}^{i} b_p^2 = i, \quad (15)$$

and substituting (14) and (15) in Lagrange's identity, we obtain

$$\sum_{1\le p<q\le i}(a_p - a_q)^2 = i(\mathcal{E}_i - ic) - \left(\sum_{p=1}^{i} a_p\right)^2.$$

Thus, we can write

$$i(\mathcal{E}_i - ic) = \left(\sum_{p=1}^{i} a_p\right)^2 + \sum_{1\le p<q\le i}(a_p - a_q)^2. \quad (16)$$

Clearly, the left-hand side of the above equality is minimized whenever

$$\sum_{1\le p<q\le i}(a_p - a_q)^2 = 0$$

which occurs if and only if

$$a_1 = a_2 = a_3 = \cdots = a_i$$

Thus, for some positive number d we have

$$\text{for every } j,\ (1 \le j \le i),\ r_j - r_{j-1} = d. \quad (17)$$

It is easy to see that equation (17) implies

$$r_i = id. \quad (18)$$

and so, substituting in (13) we obtain

$$\mathcal{E}_i = i\left(d^\alpha + c\right).$$

To summarize, we have proved the following result.

Theorem 6.1: In order to minimize the total amount of energy spent on routing along a path originating at a sensor in corona A_i and ending at the sink, all the coronas must have the same width d and the optimal amount of energy is i times the energy needed to send the desired information between adjacent coronas. □

Observe that the optimal value of d in Theorem 6.1 is suggested by (9). Thus, we have

$$d = \begin{cases} \min\{t_x, \left(\frac{2c}{\alpha-2}\right)^{\frac{1}{\alpha}}\} & \text{for } \alpha > 2 \\ t_x & \text{if } \alpha = 2. \end{cases}$$

We now argue that the conditions of path optimality captured by Theorem 6.1 translate into excessive energy consumption around the sink, leading to the creation of an energy hole. To see that this is the case, recall that by (8), (12), (17) and (18), combined, we can write

$$E_i = E_1 \times \frac{k^2 - (i-1)^2}{k^2} \times \frac{1}{2i-1}. \quad (19)$$

To get an idea of how much imbalance there is between the sensors in various coronas, consider $R = 240$m and $\alpha = 3.5$. Using the formula above we obtain $d = 12$m and, consequently, $k = 20$. Table I summarizes the energy ratio $\frac{E_i}{E_1}$ for various values of i between 1 and 20. It is instructive to note that for $i = 10$ we obtain $E_{10} = E_1 \times \frac{319}{7600} = E_1 \times 0.04197...$ Consequently, the energy expended by a sensor in the 10-th corona is only about 4.197% of the energy spent by a sensor in the first corona. Worse, yet, by taking $i = k = 20$, we obtain $E_{20} = E_1 \times \frac{1}{400}$ implying that the energy expenditure of a sensor in the last corona is 400 times smaller than the expenditure of a sensor in the first corona.

Corona	Energy ratio
1	1
2	0.3325...
3	0.1980...
4	0.1396...
5	0.1066...
6	0.0852...
7	0.0700...
8	0.0585...
9	0.0494...
10	0.0419...
11	0.0357...
13	0.0256...
15	0.0157...
17	0.0109...
19	0.0051...
20	0.0025...

TABLE I

Illustrating various energy ratios.

VII. Balancing the Energy Expenditure

The goal of this section is to tailor the coronas in such a way that the energy expenditure is balanced across all the coronas. In other words, we require that

$$E_1 = E_2 = \cdots = E_k. \tag{20}$$

In order to achieve this goal, we propose to determine every r_i, $2 \le i \le k$, as a function of r_1 and R. This will be done by setting for all i, $2 \le i \le k$,

$$\Delta_i = r_i - r_{i-1}.$$

For uniformity of notation we write $\Delta_1 = r_1 - r_0$, where $r_0 = 0$. Observe that for every i, $1 \le i \le k$, Δ_i is the *width* of the i-th corona. It is intuitively clear that in order to balance the energy expenditure across the entire disk of radius R, the widths of the coronas must satisfy the following inequality.

$$r_1 = \Delta_1 < \Delta_2 < \cdots < \Delta_i < \cdots < \Delta_k \le t_x. \tag{21}$$

Notice that the inequalities in (21) must be strict for otherwise, by Theorem 6.1, the energy expenditures $E_1, E_2, \ldots, E_k$ cannot satisfy equation (20).

A. The iterative process

As it turns out, the Δ_is can be determined iteratively in a natural way. As we shall see shortly, Δ_2 is obtained as a result of writing $E_2 = E_1$; Δ_3 is obtained from r_2 and $E_3 = E_1$. More generally, Δ_i is obtained from r_{i-1} together with $E_i = E_1$. Clearly, once Δ_i is available, r_i can be determined immediately from $r_i = \Delta_i + r_{i-1}$.
The iterative process is straightforward; the details are presented next. To begin, by insisting that $E_2 = E_1$, we obtain

$$\frac{\overline{T}}{\pi\rho}\left[1 - \frac{r_1^2}{r_k^2}\right]\frac{(r_2 - r_1)^\alpha + c}{r_2^2 - r_1^2} = \frac{\overline{T}}{\pi\rho}\frac{r_1^\alpha + c}{r_1^2}. \tag{22}$$

Noticing that

$$\frac{(r_2 - r_1)^\alpha + c}{r_2^2 - r_1^2} = \frac{\Delta_2^\alpha + c}{\Delta_2(\Delta_2 + 2r_1)}$$

and replacing in (22) we obtain

$$\frac{\Delta_2^\alpha + c}{\Delta_2(\Delta_2 + 2r_1)} = \frac{r_1^\alpha + c}{r_1^2} \times \frac{r_k^2}{r_k^2 - r_1^2}.$$

Now, writing

$$a_1 = \frac{r_1^\alpha + c}{r_1^2} \times \frac{r_k^2}{r_k^2 - r_1^2}$$

the previous relation becomes

$$\Delta_2^\alpha - a_1\Delta_2^2 - 2r_1a_1\Delta_2 + c = 0. \tag{23}$$

Solving for Δ_2 in (23) allows one to determine $r_2 = r_1 + \Delta_2$ as a function of r_1 and $r_k = R$.

Now, assume that we have obtained $\Delta_3, \ldots, \Delta_{i-1}$ in a form similar to (23) where, for $2 \le j \le i-1$,

$$a_{j-1} = \frac{r_1^\alpha + c}{r_1^2} \times \frac{r_k^2}{r_k^2 - r_{j-1}^2}.$$

Having determined $\Delta_3, \ldots, \Delta_{i-1}$ one can determine $r_3, r_4, \ldots, r_{i-1}$.
To determine r_i we insist that $E_i = E_1$. Proceeding as above we obtain the equation

$$\frac{\Delta_i^\alpha + c}{\Delta_i(\Delta_i + 2r_{i-1})} = \frac{r_1^\alpha + c}{r_1^2} \times \frac{r_k^2}{r_k^2 - r_{i-1}^2}$$

which becomes

$$\Delta_i^\alpha - a_{i-1}\Delta_i^2 - 2r_{i-1}a_{i-1}\Delta_i + c = 0 \tag{24}$$

as soon as we write

$$a_{i-1} = \frac{r_1^\alpha + c}{r_1^2} \times \frac{r_k^2}{r_k^2 - r_{i-1}^2}.$$

The above derivation was presented generically and involved determining $r_2, r_3, \dots r_k$ as a function of r_1 and, of course, $r_k = R$. This then also, indirectly, determines the number of coronas k.

Note that we have derived in [24] optimal corona sizes by a different iterative method. After determining the size of the first corona, the optimal size of the last corona is determined. The obtained equation again require applying some numerical analysis method, e.g. bisection method, to be solved, which is the case with other equations as well. The corona size determination process then continues from the last ring towards the first one until the cumulative obtained corona sizes exceed R; at this point the optimal number of coronas is also found.

Notice that in the above derivations r_1 is a parameter. One would be tempted to chose the optimal value of r_1 suggested by (9). Unfortunately, this is not always possible. The details of the selection of r_1 are discussed next.

B. Determining r_1

Observe that all the equations developed in the previous subsection contained r_1 as a parameter. Ideally, then, we should set r_1 to the value determined in (9) and determine $r_2, r_3, \cdots, r_k$ accordingly. Unfortunately, things are not that simple. While the optimal value for r_1 derived in (9) does *not* depend on R, it does depend implicitly on t_x. Thus, if we start with too large a value for r_1 in the iterative process of determining the widths of the coronas, it may happen that the last inequality in (21) is violated.

Consequently, we need to choose a value for r_1 in such a way that last inequality in (21) holds. This, in turn, will specify whether or not the optimal value of r_1 in (9) may be used or else a sub-optimal value must be chosen. We shall obtain the limiting inequality for r_1 by expanding the equality

$$E_k = E_1.$$

By using (12) with $i = k$ and $i = 1$ we obtain

$$E_k = \frac{\overline{T}}{\rho\pi}\left[1 - \frac{r_{k-1}^2}{r_k^2}\right] \times \frac{(r_k - r_{k-1})^\alpha + c}{r_k^2 - r_{k-1}^2} = \frac{\overline{T}}{\rho\pi}\frac{r_1^\alpha + c}{r_1^2}$$

which, after some mechanical manipulations, can be written as

$$\frac{(r_k - r_{k-1})^\alpha + c}{r_k^2} = \frac{r_1^\alpha + c}{r_1^2}.$$

Noticing that $r_k - r_{k-1}$ is precisely Δ_k and replacing r_k by R we obtain

$$\frac{\Delta_k^\alpha + c}{R^2} = \frac{r_1^\alpha + c}{r_1^2}$$

and, finally,

$$\Delta_k^\alpha = \frac{R^2(r_1^\alpha + c)}{r_1^2} - c. \qquad (25)$$

Since we must have $\Delta_k \leq t_x$, (25) implies the following *limiting* inequality

$$r_1^\alpha - \frac{t_x^\alpha + c}{R^2} r_1^2 + c \leq 0. \qquad (26)$$

To summarize, the value of r_1 is dictated by both (9) and by inequality (26). In practice, the closest value to the optimal is selected subject to (26) being satisfied.

C. A numerical example

The goal of this subsection is to illustrate, by a numerical example, the generic computation in Subsections VII-A and VII-B above.

The assumed system parameters are: $R = 225$m, $c = 4500$, $t_x = 55$m, $\alpha = 4$. Using (9) we obtain the initial value $r_1 = 8.19$m. In order to check whether this value can be used, we first write down the limiting inequality (26) for r_1. With the parameters above, this inequality reads:

$$r_1^4 - 180.842 * r_1^2 + 4500 \leq 0$$

It is easy to see that the optimal value $r_1 = 8.19$ does satisfy the limiting inequality and, therefore, we can proceed.

First Iteration: $a_1 = 134.16$ and the equation that yields Δ_2 is

$$\Delta_2^4 - 134.16\Delta_2^2 - 2200.26\Delta_2 + 4500 = 0.$$

whose solution is $\Delta_2 = 15.95$, and thus $r_2 = 24.14$m.
Second Iteration: $a_2 = 135.65$ and the equation is

$$\Delta_3^4 - 135.65\Delta_3^2 - 6549.39\Delta_3 + 4500 = 0.$$

whose solution is $\Delta_3 = 20.95$, and so $r_3 = 45.09$m.
Third Iteration: a_3 is 139.52 and the equation to solve is

$$\Delta_4^4 - 139.52\Delta_4^2 - 12582.49\Delta_4 + 4500 = 0.$$

whose solution is $\Delta_4 = 25.15$, from which we obtain $r_4 = 70.24$m.
Fourth Iteration: $a_4 = 147.96$ and the equation is

$$\Delta_5^4 - 147.96\Delta_5^2 - 20785.31\Delta_5 + 4500 = 0.$$

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the Proceedings IEEE Infocom.

whose solution is $\Delta_5 = 29.22$ and, further, $r_5 = 99.46\text{m}$.
Fifth Iteration: $a_5 = 165.01$ and the equation to solve is

$$\Delta_6^4 - 165.01\Delta_6^2 - 32825.46\Delta_6 + 4500 = 0.$$

whose solution is $\Delta_6 = 34.69$. Now, $r_6 = 134.15\text{m}$.
Sixth Iteration: $a_6 = 201.78$ and the equation is

$$\Delta_7^4 - 201.78\Delta_7^2 - 53735.86\Delta_5 + 4500 = 0.$$

whose solution is $\Delta_7 = 40.50$, and thus $r_7 = 174.65\text{m}$.
Seventh Iteration: $a_7 = 307.23$ and the equation is

$$\Delta_8^4 - 307.23\Delta_8^2 - 106102.12\Delta_5 + 4500 = 0.$$

whose solution is $\Delta_8 = 49.98$, and so $r_8 = 224.63\text{m}$.

Consequently, the disk around the sink is partitioned into 8 coronas whose width are summarized in Table II below.

Corona	Width (m)
1	8.19
2	15.95
3	20.95
4	25.15
5	29.22
6	34.69
7	40.50
8	49.98

TABLE II
Illustrating the width of various coronas.

VIII. A CLOSER LOOK AT THE CASE $\alpha = 2$

The main goal of this section is to prove the following negative result.

Theorem 8.1: In $\alpha = 2$ then $E_1 = E_2 = \cdots = E_k$ cannot hold, regardless of the value of R, t_x and c.

Proof. We propose to show that regardless of the parameters R, t_x and c, it is impossible to have $E_2 = E_1$. Indeed, suppose that $E_2 = E_1$ does hold. This is equivalent to writing

$$E_2 = \frac{\overline{T}}{\rho\pi}\left[1 - \frac{r_1^2}{r^k}\right] \times \frac{(r_2 - r_1)^2 + c}{r_2^2 - r_1^2} = \frac{\overline{T}}{\rho\pi}\frac{r_1^2 + c}{r_1^2} = E_1$$

which, after some mechanical manipulations, can be written as

$$\frac{\Delta_2^2 + c}{\Delta_2(\Delta_2 + 2r_1)} = \frac{r_1^2 + c}{r_1^2} \times \frac{R^2}{R^2 - r_1^2}.$$

Now writing

$$a_1 = \frac{r_1^2 + c}{r_1^2} \times \frac{R^2}{R^2 - r_1^2} \tag{27}$$

and performing simple algebra we obtain the equation

$$(a_1 - 1)\Delta_2^2 + 2a_1 r_1 \Delta_2 - c = 0. \tag{28}$$

Case 1: $a_1 = 1$
In this case, we have

$$\Delta_2 = \frac{c}{2a_1 r_1} = \frac{c(R^2 - r_1^2)r_1}{2(r_1^2 + c)R^2}.$$

By insisting that $r_1 = \Delta_1 < \Delta_2$ as in (21), we obtain

$$\frac{c(R^2 - r_1^2)}{2(r_1^2 + c)R^2} > 1.$$

However, this is impossible since $R^2 - r_2^2 < R^2$ and $c < r_1^2 + c$.

Thus, Case 1 cannot occur.

Case 2: $a_1 \neq 1$
In this case, solving for Δ_2 in (28) yields

$$\Delta_2 = \frac{-a_1 r_1 + \sqrt{a_1 r_1 + c(a_1 - 1)}}{a_1 - 1}.$$

By (21), we insist that $r_1 = \Delta_1 < \Delta_2$. Since $a_1 \neq 1$, this amounts to writing

$$-a_1 r_1 + \sqrt{a_1 r_1 + c(a_1 - 1)} > (a_1 - 1)r_1$$

or, equivalently,

$$\sqrt{a_1 r_1 + c(a_1 - 1)} > (2a_1 - 1)r_1$$

After simple algebra we obtain

$$c(a_1 - 1) > (3a_1 - 1)(a_1 - 1)r_1^2$$

which is equivalent to

$$c > (3a_1 - 1)r_1^2.$$

Now, replacing in this latter inequality the value of a_1 from (27) and letting x stand for r_1^2 we have

$$c > \frac{x^2 + 2xR^2 + 3R^2 c}{R^2 - x}$$

which is equivalent to

$$x^2 + (2R^2 + c)x + 2R^2 c < 0$$

an impossibility. This concludes the proof of the theorem □

Theorem 8.1 suggests that for $\alpha = 2$ there is no way to balance the energy load and, consequently, sooner or later an energy hole will appear around the sink, drastically curtailing the useful lifetime of the networks. Nonetheless, from an energy consumption standpoint, the best strategy is to design all the coronas to be of the same width. In this case (9) implies that the best value is $r_1 = t_x$.

IX. Reasoning about the system parameters

Let E denote the total *energy budget* of a sensor at deployment time. Since the sensors in A_1 must have sufficient energy to handle their routing duties, by using (8) we can write

$$\frac{\overline{T}}{\rho\pi}\left[r_1^{\alpha-2}+\frac{c}{r_1^2}\right]\leq E. \tag{29}$$

Inequality (29) can be interpreted in several ways, each expressing a different view of the limiting factors inherent to the sensors deployed. The goal of this subsection is to look at some of possible interpretations of (29).

Network longevity We interpret $\overline{T}$, the number of transactions that the system can sustain during its lifetime as the *network longevity*. Thus, (29) allows us to write

$$\overline{T}<\frac{\rho\pi E r_1^2}{r_1^\alpha+c} \tag{30}$$

which tells us that the longevity of the system is upper bounded by the ratio (30). More specifically, the longevity is directly proportional to the deployment density and to the reciprocal of $r_1^\alpha+c$. Consequently, if we wish to design a wireless sensor network that must sustain a given number $\overline{T}$ of transactions, we must select the deployment density as well as the radius of the first corona accordingly. We also need to chose sensors packing an amount of energy compatible with (30).

Maximum transmission range close to the sink First, assuming a *known* deployment density[2] ρ, (29) shows that for a given energy budget E, in order to guarantee a desired network longevity of $\overline{T}$ tasks, the (maximum) transmission radius of sensors deployed in close proximity to the sink must satisfy

$$r_1^{\alpha-2}+\frac{c}{r_1^2}<\frac{\pi\rho E}{\overline{T}} \tag{31}$$

with the additional constraint that $r_1 \leq t_x$ where, recall, t_x stands for the *maximum* transmission range of a sensor.

Deployment density Likewise, for a selected radius r_1, $(t_x \geq r_1)$, and for a given energy budget E, in order to guarantee a network *longevity* of $\overline{T}$ tasks, the deployment density ρ must satisfy the inequality

$$\rho>\frac{\overline{T}\left[r_1^\alpha+c\right]}{E\pi r_1^2}. \tag{32}$$

This latter equation can also be used (perhaps in conjunction with (30) to plan future re-deployments as the existing sensors exhaust their energy budget.

[2]It is important to note that given the deployment area, the density can be engineered beforehand by simply deploying a suitable number of sensors uniformly at random.

X. Concluding remarks

This paper investigated theoretical aspects of the uneven energy depletion phenomenon recently noticed in sink-based wireless sensor networks. We assumed an energy consumption model governed by the relation $E = d^\alpha + c$ where d is the transmission distance and c is a system-dependent positive constant. First, we showed that for $\alpha = 2$, the uneven energy depletion phenomenon is intrinsic to the system and no routing strategy can avoid the creation of an energy hole around the sink. Second, we argued that for larger values of α the uneven energy depletion phenomenon is an artifact of designing *energy-efficient* sensor-to-sink routes. This is rather counter-intuitive. We also showed that for larger values of α the uneven depletion can be prevented by judicious system design. In such a system, the energy expenditure is balanced across the network, but suboptimal.

There are several other ways to counter the uneven energy depletion problem. Perhaps the most obvious strategy is to mandate the sinks to move around in such a way that some load balancing is obtained across the deployment area [13]. This solution works especially well in autonomous sensor networks [19]. Yet another solution involves establishing *temporary* sinks that act as ad-hoc aggregation points. Finally, as discussed in [8], a certain amount of load balancing is obtained by overlapping the disks around the sinks.

Our study was limited to the case where each sensor reports to one sink. Extensions to a multi-sink configuration are possible. First, if the sinks are deployed at random, then each sensor may decide which sink to report to, and this may not necessarily be the closest one. Next, the sinks may have limited or unlimited energy resources. Our results assume that sinks have relatively unlimited energy compared to individual sensors. The next step is to extend our results for the case of sinks with limited energy, and to find optimal design of sensor networks that balance energy between sensors and sinks. This extends known results assuming fixed transmission radius to the case of adjustable transmission radii.

We have observed that transmission radii of coronas increase as we move away from the sink. Since they are limited by the maximum transmission radius t_x, there exist a maximum i such that the width of the i-th corona is still $r_i - r_{i-1} < t_x$. Beyond this point, all coronas have radius limited by t_x. Considering our results, this means that energy imbalance is unavoidable among such coronas. If one can place sinks at strategic locations rather than at random, it appears that an optimal design should consider these limitations. That is, the radius $R = r_i$ for each sink should be selected so that the energy of sensors within this radius can be balanced. However, the determination of optimal R is not straightforward since corona sizes depend on R. Fortunately, we can modify the inequality (26) to read: $r_1^\alpha - \frac{t_x^\alpha + c}{R^2} r_1^2 + c = 0$ which allows optimal R to be determined directly. Therefore we have shown that the optimal network radius around the sink, which allows that energy expenditure at all sensors to be balanced, is also merely a function of α, c and t_x. Assuming that energy at sinks is unlimited, sinks can then be placed at strategic locations, e.g. vertices of a regular hexagon of sides approximately R, so that each sensor is at distance at most R to the nearest sink. Note that hexagonal size here is approximated rather than equated with R because of difference between circular and hexagonal shapes for regions around each sink. This may require some details to determine precise value for the size of this hexagon.

In [24] we have studied the case of preserving equal transmission radii and therefore equal corona sizes, and balancing energy consumption by applying nonuniform sensor distribution. Let ρ_i be the sensor density in i-th corona. The energy consumption will be balanced if ρ_i is proportional to $k + 1 - i$, where k is the optimal number of coronas [24]. That is, the densities should be proportional to $k, k-1, \ldots, 1$, from the closest corona to the sink to the furthest one. Details will be also given in the journal version of this article.

REFERENCES

[1] F. Akyildiz, W. Su, Y. Sankarasubramanian, and E. Cayirci, Wireless sensor networks: A survey, *Computer Networks*, 38(4), 2002, 393–422.

[2] D. Culler, D. Estrin, and M. Srivastava, Overview of sensor networks, *IEEE Computer*, 37(8), 2004, 41–49.

[3] D. Culler and W. Hong, Wireless sensor networks, *Communications of the ACM*, 47(6), 2004, 30–33.

[4] R. G. Graham, D. E. Knuth, and O. Patashnik, *Concrete Mathematics*, Addison-Wesley, NY, 1989.

[5] W. Heinzelman, Application-specific protocol architectures for wireless sensor networks, Doctoral Dissertation, MIT, 2000.

[6] W. Heinzelman, A. Chandrakashan and H. Balakrishnan, An application-specific protocol architecture for wireless microsensor networks, *IEEE Transactions on Wireless Communications*, 1(4), 2002, 660–670.

[7] F. Ingelrest, D. Simplot-Ryl, I. Stojmenovic, Target transmission radius over LMST for energy-efficient broadcast protocol in ad hoc networks, *Proc. IEEE International Conference on Communications*, ICC'04, Paris, June 2004.

[8] R. Ishak, S. Olariu, and S. Salleh, Multiple training in massively-deployed sensor networks, in preparation.

[9] J. J. Lee, B. Krishnamachari, C. C. Jay Kuo, Impact of heterogeneous deployment on lifetime sensing coverage in sensor networks, *Proc. IEEE SECON*, 2004.

[10] J. Li and P. Mohapatra, Mitigating the energy hole problem in many to one sensor networks, a manuscript, 2005.

[11] J. Lian, K. Naik and G. B. Agnew, Data capacity improvement of wireless sensor networks using non-uniform sensor distribution, *Intern. Journal of Distr. Sensor Networks*, to appear, 2005.

[12] J. Lian, L. Chen, K. Naik, M T. Ozsu, and G. B. Agnew, BBS: An energy-efficient localized routing scheme for query processing in wireless sensor networks, *International Journal of Distributed Sensor Networks*, to appear, 2006.

[13] J. Lian, K. Naik and G. B. Agnew, Modeling and enhancing data capacity in wireless sensor networks, *IEEE Monograph on Sensor Network Operations*, IEEE Press, 2004.

[14] J. Lian, L. Chen, K. Naik, T. Otzu, G. Agnew, Localized routing trees for query processing in sensor networks, *Proc. ACM Knowledge and Information Management*, October 2005, 259-260.

[15] K. Martinez, J. K. Hart and R. Ong, Sensor network applications, *IEEE Computer*, 37(8), 2004, 50–56.

[16] V. Mhatre, C. Rosenberg, Design guidelines for wireless sensor networks: communication, clustering and aggregation, *Ad Hoc Networks*, 2, 2004, 45–63.

[17] V. Mhatre, C. Rosenberg, D. Kofman, R. Mazumdar, and N. Shroff, A minimum cost heterogeneous sensor network with a lifetime constraint, *IEEE Transactions on Mobile Computing*, 4(1), 2005, 4–15.

[18] S. Olariu, A. Wadaa, L. Wilson, and M. Eltoweissy, Wireless sensor networks: leveraging the virtual infrastructure, *IEEE Network*, 18(4), (2004), 51–56.

[19] S. Olariu, M. Eltoweissy and M. Younis, ANSWER: Autonomous, wireless sensor networks, *Proc.* 1_{st} *ACM Intern. Workshop on Quality of Service and Security in Wireless and Mobile Networks*, Montreal, October 2005.

[20] M. Perillo, Z. Cheng, and W. Heinzelman, On the problem of unbalanced load distribution in wireless sensor networks, *Proc. IEEE GLOBECOM*, 2004.

[21] T. Rappaport, *Wireless Communications: Principles and Practice*, Prentice-Hall, 1996.

[22] M. Sheldon, D. Chen, M. Nixon, A. K. Mok, A practical approach to deploy large scale wireless sensor networks *Proc. IEEE MASS*, Washington, Nov. 2005.

[23] S. Soro, W.B. Heinzelman, Prolonging the lifetime of wireless sensor networks via unequal clustering, *Proc. IPDPS*, April 2005, Denver, CA, USA 2005.

[24] I. Stojmenovic and S. Olariu, Data-centric protocols for wireless sensor networks, in: *Handbook of Sensor Networks: Algorithms and Architectures* (I. Stojmenovic, ed.), Wiley, 2005, 417-456.

[25] N. Vlajic and D. Xia, Wireless sensor networks: To cluster or not to cluster, University of York, November 2005.

[26] A. Wadaa, S. Olariu, L. Wilson, K. Jones, and M. Eltoweissy, Training a sensor networks, *MONET*, January 2005.