General Outline

- Part 0: Background, Motivation, and Goals.
- Part I: Some Basics.
- Part II: Diversity Systems.
- **Part III: Co-Channel Interference.**
- Part IV: Multi-Hop Communication Systems.

Outline - Part III: Co-Channel Interference

1. **Co-Channel Interference (CCI) Analysis**
   - Effect of Shadowing
   - Effect of Multipath Fading
     - Single Interferer
     - Multiple Interferers
     - Minimum Desired Signal Requirement
     - Random Number of Interferers
2. CCI Mitigation
   - Diversity Combining
   - Optimum Combining/Smart Antennas
   - Optimized MIMO Systems in Presence of CCI

Effect of Shadowing

- Finding the statistics of the sum of log-normal random variables.
- No known exact closed-form available.
- Several analytical techniques have been developed over the years.
- As an example, we cover the Farley bounds on the sum of log-normal random variables.

Effect of Multipath Fading (1)

- Single Interferer
  - Let the carrier-to-interference ratio (CIR)

$$\lambda = \frac{s_d}{s_i},$$

where $s_d = \alpha_d^2$ (with average Ω_d) and $s_i = \alpha_i^2$ (with average Ω_i) are the instantaneous fading powers of desired and interfering users.

  - Outage probability

$$\begin{aligned} P_{\text{out}} &= \text{Prob}[\lambda \le \lambda_{\text{th}}] \\ &= \int_0^\infty p_{s_d}(s_d)\ \text{Prob}\left[s_i \ge \frac{s_d}{\lambda_{\text{th}}} | s_d\right]\ ds_d \\ &= \int_0^\infty p_{s_d}(s_d)\ \int_{s_d/\lambda_{\text{th}}}^\infty p_{s_i}(s_i)\ ds_i\ ds_d. \end{aligned}$$

  - Exp: Rician (Rician factor K_d)/Rayleih case

$$P_{\text{out}} = \frac{\lambda_{\text{th}}}{\lambda_{\text{th}} + b}\ \exp\left(-\frac{Kb}{\lambda_{\text{th}} + b}\right),$$

where

$$b = \frac{\Omega_d}{\Omega_i(K_d + 1)}.$$

Effect of Multipath Fading (2)

- N_I independent identically distributed interferers
  - Let the carrier-to-interference ratio
  $$\lambda = \frac{s_d}{s_I},$$
  where $s_I = \sum_{i=1}^{N_I} s_i$ and all the s_i have the same average fading power Ω_i.
  - Outage probability
  $$\begin{aligned} P_{\text{out}} &= \text{Prob}[\lambda \le \lambda_{\text{th}}] \\ &= \int_0^\infty p_{s_d}(s_d) \int_{s_d/\lambda_{\text{th}}}^\infty p_{s_I}(s_I)\, ds_I\, ds_d. \end{aligned}$$
  - Exp: Nakagami/Nakagami scenario
  $$P_{\text{out}} = I_x(m, mN_I)$$
  where
  $$x = \left(1 + \frac{\Omega_d}{\Omega_i \lambda_{\text{th}}}\right)$$
  and
  $$I_x(a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^x t^{a-1} (1-t)^{b-1}\, dt$$
  is the incomplete Beta function ratio.

Effect of Multipath Fading (3)

- N_I independent non-identically distributed interferers
  - Let the carrier-to-interference ratio
$$\lambda = \frac{s_d}{s_I},$$
    where $s_I = \sum_{i=1}^{N_I} s_i$ and all the s_i can have different average fading power Ω_i.
  - Outage probability
$$\begin{aligned} P_{\text{out}} &= \text{Prob}[\lambda \leq \lambda_{\text{th}}] \\ &= \text{Prob}\left[s_d - \lambda_{\text{th}} \sum_{i=1}^{N_I} s_i \leq 0\right]. \end{aligned}$$
  - Define $\alpha = \lambda_{\text{th}} \sum_{i=1}^{N_I} s_i - s_d$.
    * $\alpha \geq 0$ corresponds to an outage.
    * $\alpha \leq 0$ corresponds to satisfactory transmission.
  - Find characteristic function of α and then use Gil-Palaez lemma.

Gil-Palaez Lemma and Application

- Let X be a random variable with cumulative distribution function (CDF) $P_X(x) = \mathrm{Prob}[X \leq x]$ and characteristic function (CF) $\phi_X(t) = E\left[e^{jXt}\right]$ then

$$P_X(x) = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\left[\phi_X(t)e^{-jXt}\right]}{t}\, dt.$$

- Application to outage probability

$$\begin{aligned} P_{\mathrm{out}} &= \mathrm{Prob}[\alpha \geq 0] = 1 - \mathrm{Prob}[\alpha \leq 0] \\ &= 1 - P_\alpha(0) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\left[\phi_\alpha(t)\right]}{t}\, dt, \end{aligned}$$

where for the Nakagami/Nakagami fading case

$$\begin{aligned} \phi_\alpha(t) &= \phi_{s_d}(-t)\prod_{i=1}^{N_I}\phi_{s_i}(\lambda_{\mathrm{th}}t) \\ &= \left(1 + \frac{jt\Omega_d}{m_d}\right)^{-m_d}\prod_{i=1}^{N_I}\left(1 - \frac{jt\Omega_i\lambda_{\mathrm{th}}}{m_i}\right)^{-m_i}. \end{aligned}$$

Random Number of Interferers

- Depending on traffic conditions, the number of active interferers n_I is a random variable from 0 to N_I which is the maximum number of active interferers.
- Outage probability is given by

$$P_{\text{out}} = \sum_{n_I=0}^{N_I} \text{Prob}[n_I]\; P_{\text{out}}[n_I].$$

- Assume N_c available channels per cell each with activity probability p_a.
- The blocking probability

$$B = p_a^{N_c}.$$

- PDF of n_I is Binomial

$$\begin{aligned}\text{Prob}[n_I] &= \binom{N_I}{n_I} p_a^{n_I} (1-p_a)^{N_I-n_I}, \quad n_I = 0, \cdots, N_I. \\ &= \binom{N_I}{n_I} B^{n_I/N_c} \left(1 - B^{1/N_c}\right)^{N_I-n_I}.\end{aligned}$$

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Diversity Combining

- Reduce the effect via selective (or switched) antenna diversity combining techniques.
- Consider a dual-antenna diversity system with one co-channel interferer. Let α_{11} denote the fading amplitude from desired user to antenna 1, α_{12} denote the fading amplitude from desired user to antenna 2, α_{21} denote the fading amplitude from interfering user to antenna 1, and α_{22} denote the fading amplitude from interfering user to antenna 2.
- Three main decision algorithms for selective diversity.

Decision Algorithms

- CIR algorithm: picks and process the information from the antenna with the highest CIR. For the scenario describe previously:

$$\text{Max}\left[\left(\frac{\alpha_{11}}{\alpha_{21}}\right)^2, \left(\frac{\alpha_{12}}{\alpha_{22}}\right)^2\right].$$

- Desired signal algorithm: picks and process the information from the antenna with the highest desired signal, i.e.,

$$\text{Max}\left[\alpha_{11}^2, \alpha_{12}^2\right].$$

- Signal plus interference algorithm: picks and process the information from the antenna with the highest desired plus interfering signal, i.e.,

$$\text{Max}\left[\alpha_{11}^2 + \alpha_{21}^2, \alpha_{12}^2 + \alpha_{22}^2\right].$$

Interference Mitigation

- More advanced interference mitigation techniques
  - Optimum combining
  - Optimized MIMO systems

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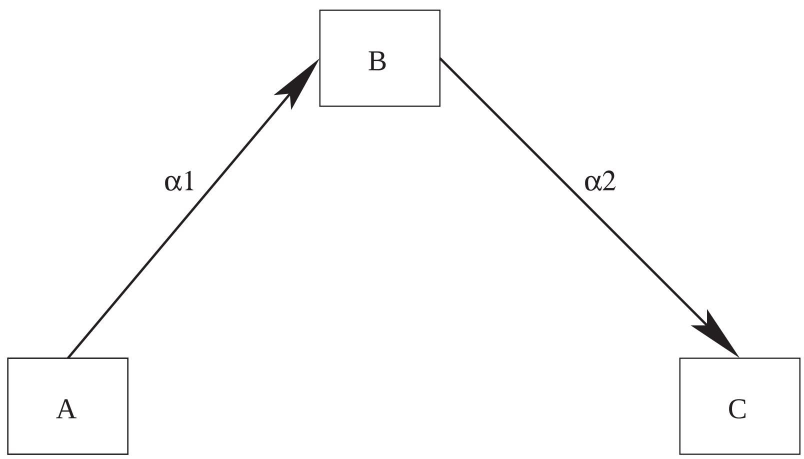
Multi-Hop Communication Systems

- Advantages of transmission with relays:
  - Broader coverage
  - Lower transmitted power (higher battery life and lower interference)
  - "Cooperative/Collaborative/Multi-user" diversity.
- "Pionnering" work on this topic:
  - Sendonaris, Erkip, and Aazhang, [ISIT'98].
  - Laneman, Wornell, and Tse [WCNC'00, Allerton'00, ISIT'01].
  - Emamian and Kaveh, [ISC'01].
- Goal:
  Develop an analytical framework for the *exact* end-to-end performance analysis of dual-hop then multi-hop relayed transmission over fading channels.

Dual-Hop Systems

- Consider the following dual-hop communication system with a relay



- Two relaying options:
  - Non-regenerative relaying (known also as analog or amplify-and-forward relaying)
  - Regenerative relaying (known also as digital or decode-and-forward relaying)

Non-Regenerative Systems

- Received signal at the relay input (B) is

$$r_b(t) = \alpha_1 s(t) + n_1(t).$$

- Received signal at the destination (C) is

$$\begin{aligned} r_c(t) &= \alpha_2 G r_b(t) + n_2(t) \\ &= \alpha_2 G(\alpha_1 s(t) + n_1(t)) + n_2(t). \end{aligned}$$

- Equivalent end-to-end SNR

$$\gamma_{\text{eq}} = \frac{\alpha_1^2 \alpha_2^2 G^2}{\alpha_2^2 G^2 N_{0_1} + N_{0_2}} = \frac{\frac{\alpha_1^2}{N_{0_1}} \frac{\alpha_2^2}{N_{0_2}}}{\frac{\alpha_2^2}{N_{0_2}} + \frac{1}{G^2 N_{0_1}}}.$$

Choice of the Relay Gain

- One possible choice of the relay gain is just channel inversion, i.e.,

$$G^2 = 1/\alpha_1^2,$$

  – Resulting equivalent end-to-end SNR

$$\gamma_{\mathrm{eq}_2} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}.$$

  – Lower bound on the performance of practical relays

  – Related to the Harmonic Mean of γ_1 and γ_2

A Second Choice of the Relay Gain

- Another possible choice of the relay gain [Laneman *et al.* '00]

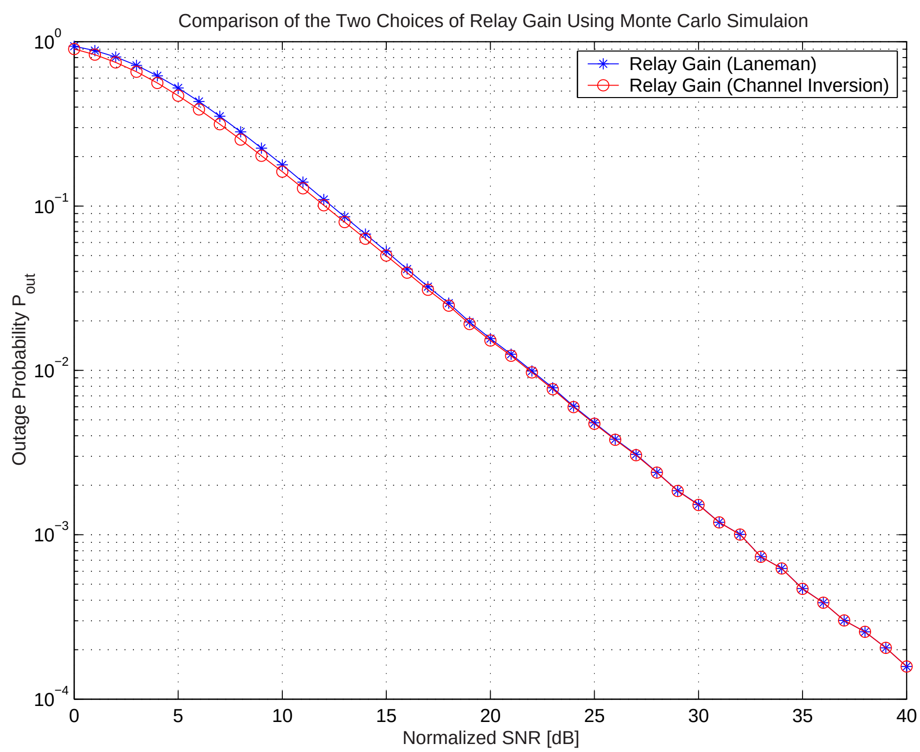
$$G^2 = \frac{1}{\alpha_1^2 + N_0}.$$

  - Limits the gain of the relay when first hop is deeply faded
  - Resulting equivalent end-to-end SNR

$$\gamma_{\mathrm{eq}_1} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1},$$

Monte Carlo Simulation

- Comparison of the outage probability for the two choices of the relay gain



Harmonic Mean

- Given two numbers $X_1,\ X_2$:
  - Arithmetic Mean
$$\mu_A(X_1, X_2) = \frac{X_1 + X_2}{2}$$
  - Geometric Mean
$$\mu_G(X_1, X_2) = \sqrt{X_1 X_2}$$
  - Harmonic Mean
$$\mu_H(X_1, X_2) = \frac{2X_1X_2}{X_1 + X_2} = \frac{2}{\frac{1}{X_1} + \frac{1}{X_2}}$$

- Relation with end-to-end SNR
$$\gamma_{\mathrm{eq}_2} = \frac{1}{2}\,\mu_H(\gamma_1, \gamma_2) \geq \gamma_{\mathrm{eq}_1},$$
where γ_1 and γ_2 are the instantaneous SNRs of hops 1 and 2, respectively.

Harmonic Mean of Exponential Variates

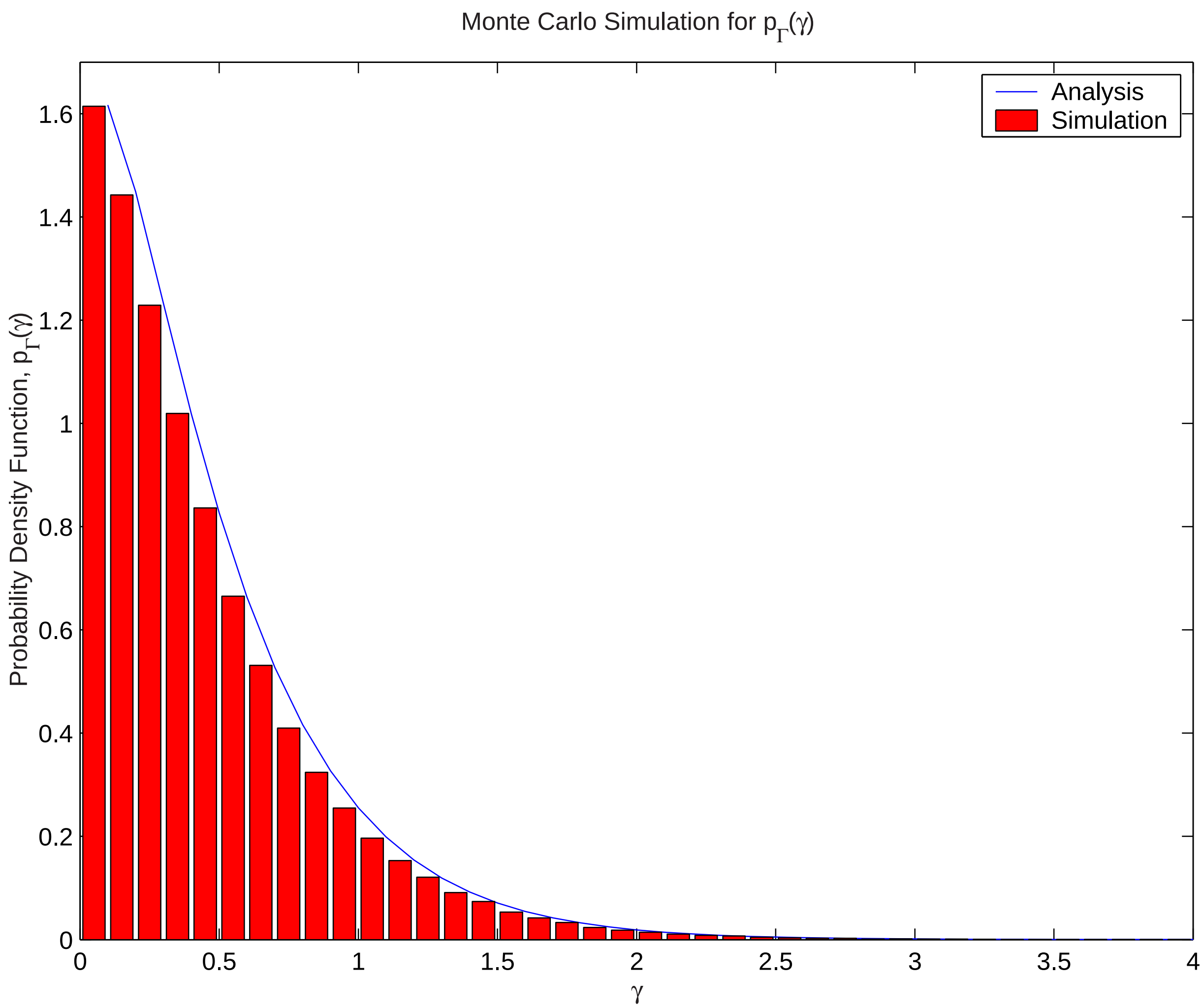
- Theorem :
  Let X_1 and X_2 be two independent exponential variates with parameters β_1 and β_2 respectively. Then, the PDF of $X = \mu_H(X_1, X_2)$, $p_X(x)$, is given by

$$p_X(x) = \frac{1}{2}\beta_1\beta_2 x e^{-\frac{x}{2}(\beta_1+\beta_2)}\left[\left(\frac{\beta_1+\beta_2}{\sqrt{\beta_1\beta_2}}\right)K_1\left(x\sqrt{\beta_1\beta_2}\right) + 2K_0\left(x\sqrt{\beta_1\beta_2}\right)\right]U(x),$$

  where $K_i(\cdot)$ is the ith order modified Bessel function of the second kind and $U(\cdot)$ is the unit step function.

- The CDF and MGF of the harmonic mean of two independent exponential variates are also available in closed-form.

Validation by Monte-Carlo Simulations



Derivation of the CDF of $X = \mu_H(X_1, X_2)$

- Let
$$Z = \frac{1}{X} = \frac{1}{2}\left(\frac{1}{X_1} + \frac{1}{X_2}\right).$$
- The CDF of X, $P_X(x)$, is given by
$$\begin{aligned} P_X(x) &= \Pr\left(X < x\right) \\ &= \Pr\left(\frac{1}{X} > \frac{1}{x}\right) = \Pr\left(Z > \frac{1}{x}\right) \\ &= 1 - \Pr\left(Z < \frac{1}{x}\right) = 1 - P_Z\left(\frac{1}{x}\right), \end{aligned}$$

  where $P_Z(\cdot)$ is the CDF of Z.

Derivation of the CDF of $X = \mu_H(X_1, X_2)$ (Continued)

- If X is an exponential random variable with parameter β then the MGF of $Y = 1/X$ can be shown to be given by

$$\mathcal{M}_Y(s) = E\left[e^{-sY}\right] = 2\sqrt{\beta s}\, K_1\left(2\sqrt{\beta s}\right).$$

- Using the differentiation property of the Laplace transform, $P_Z(z)$ can be written as

$$\begin{aligned} P_Z(z) &= \mathcal{L}^{-1}\left(\frac{\mathcal{M}_Z(s)}{s}\right) \\ &= 1 - \mathcal{L}^{-1}\Big(2\sqrt{\beta_1\beta_2}K_1\left(2\sqrt{\beta_1 s}\right) K_1\left(2\sqrt{\beta_2 s}\right)\Big)\,|_{z=\frac{1}{x}}, \end{aligned}$$

which is a tabulated inverse Laplace transform leading to

$$\begin{aligned} P_X(x) &= 1 - P_Z\left(\frac{1}{x}\right) \\ &= 1 - x\sqrt{\beta_1\beta_2}e^{-\frac{x}{2}(\beta_1+\beta_2)}K_1\left(x\sqrt{\beta_1\beta_2}\right). \end{aligned}$$

Derivation of the PDF of $X = \mu_H(X_1, X_2)$

- Taking the derivative of the CDF of X with respect to x results in

$$\frac{d}{dx}\quad (P_X(x)) = -\Big[\sqrt{\beta_1\beta_2}e^{-\frac{x}{2}(\beta_1+\beta_2)}K_1\left(x\sqrt{\beta_1\beta_2}\right)$$
$$+\ x\sqrt{\beta_1\beta_2}\left(-\frac{1}{2}(\beta_1+\beta_2)\,e^{-\frac{x}{2}(\beta_1+\beta_2)}\right.$$
$$\left.\times K_1\left(x\sqrt{\beta_1\beta_2}\right)+e^{-\frac{x}{2}(\beta_1+\beta_2)}\frac{d}{dx}\left[K_1\left(x\sqrt{\beta_1\beta_2}\right)\right]\right)\Big].$$

- Using

$$z\frac{d}{dz}K_v(z)+vK_v(z)=-zK_{v-1}(z)$$

leads to the final desired result

$$p_X(x) = \frac{1}{2}\beta_1\beta_2 x e^{-\frac{x}{2}(\beta_1+\beta_2)}\Big[\left(\frac{\beta_1+\beta_2}{\sqrt{\beta_1\beta_2}}\right)$$
$$K_1\left(x\sqrt{\beta_1\beta_2}\right)+2K_0\left(x\sqrt{\beta_1\beta_2}\right)\Big].$$

Formulas for the Outage Probability

- For non-regenerative systems, $P_{\rm out}$ is given by

$$P_{\rm out} = 1 - \frac{2\gamma_{\rm th}}{\sqrt{\overline{\gamma}_1 \overline{\gamma}_2}} K_1\left(\frac{2\gamma_{\rm th}}{\sqrt{\overline{\gamma}_1 \overline{\gamma}_2}}\right) e^{-\gamma_{\rm th}\left(\frac{1}{\overline{\gamma}_1} + \frac{1}{\overline{\gamma}_2}\right)},$$

  where $\overline{\gamma}_1$ and $\overline{\gamma}_2$ are the average SNRs of hops 1 and 2, respectively.

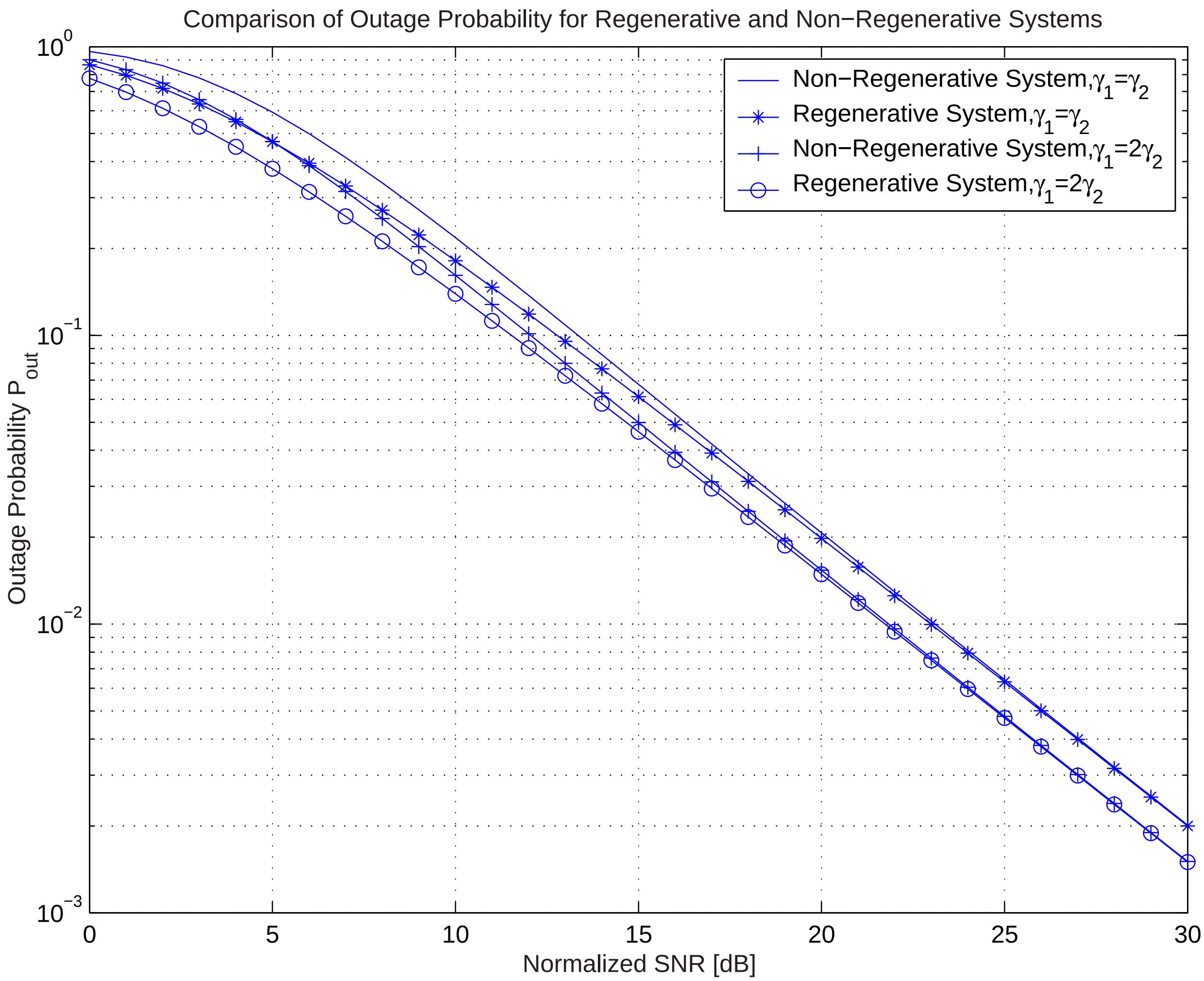
- For regenerative systems, $P_{\rm out}$ is given by

$$P_{\rm out} = 1 - e^{-\gamma_{\rm th}\left(\frac{1}{\overline{\gamma}_1} + \frac{1}{\overline{\gamma}_2}\right)}.$$

- Both formulas are equivalent at high average SNR since for small x

$$K_1(x) \simeq \frac{1}{x}.$$

Outage Probability: Numerical Example



Outage Probability of Collaborative Systems

- Consider a wireless communication system with one direct link and L collaborating paths.
- Assume direct link with average SNR $\overline{\gamma}_0$ and that the two hops in collaborating path l have the same average SNR $\overline{\gamma}_l$.
- Assume that the strongest path is selected at any given time.
- Resulting outage probability

$$\begin{aligned} P_{\text{out}} &= \left(1 - e^{-\frac{\gamma_{\text{th}}}{\overline{\gamma}_0}}\right) \\ &\times \prod_{l=1}^{L} \left(1 - \frac{2\gamma_{\text{th}}}{\overline{\gamma}_l} e^{-\frac{2\gamma_{\text{th}}}{\overline{\gamma}_l}} K_1\left(\frac{2\gamma_{\text{th}}}{\overline{\gamma}_l}\right)\right). \end{aligned}$$

Formulas for the Average BER

- The MGF of γ_{eq}, $\mathbf{E}(e^{-\gamma s})$, for identical and independent faded hops, i.e. $\overline{\gamma}_1 = \overline{\gamma}_2 = \overline{\gamma}$, is given by

$$\mathcal{M}_\Gamma(s) = \frac{\sqrt{\frac{\overline{\gamma}}{4}s\left(\frac{\overline{\gamma}}{4}s+1\right)} + \operatorname{arcsinh}\left(\sqrt{\frac{\overline{\gamma}}{4}s}\right)}{2\sqrt{\frac{\overline{\gamma}}{4}s}\left(\frac{\overline{\gamma}}{4}s+1\right)^{\frac{3}{2}}}$$

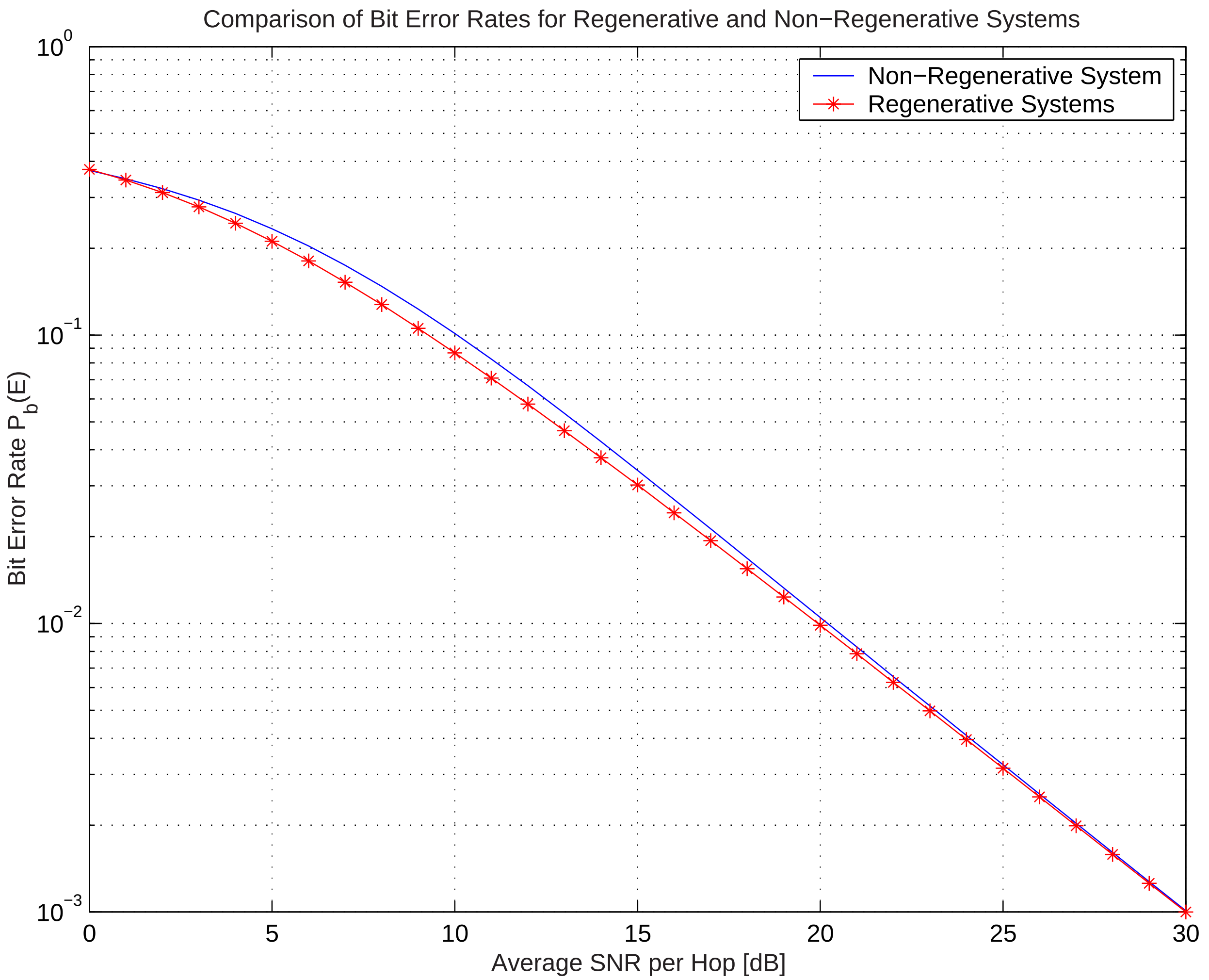
- For non-regenerative systems with DPSK the average BER is

$$P_b(E) = \frac{1}{2}\,\mathcal{M}_\gamma(1).$$

- For regenerative systems with DPSK over identical and independent faded hops

$$P_b(E) = \frac{1+2\overline{\gamma}}{2\left(1+\overline{\gamma}\right)^2}.$$

Average BER: Numerical Example



Average BER with Collaboration

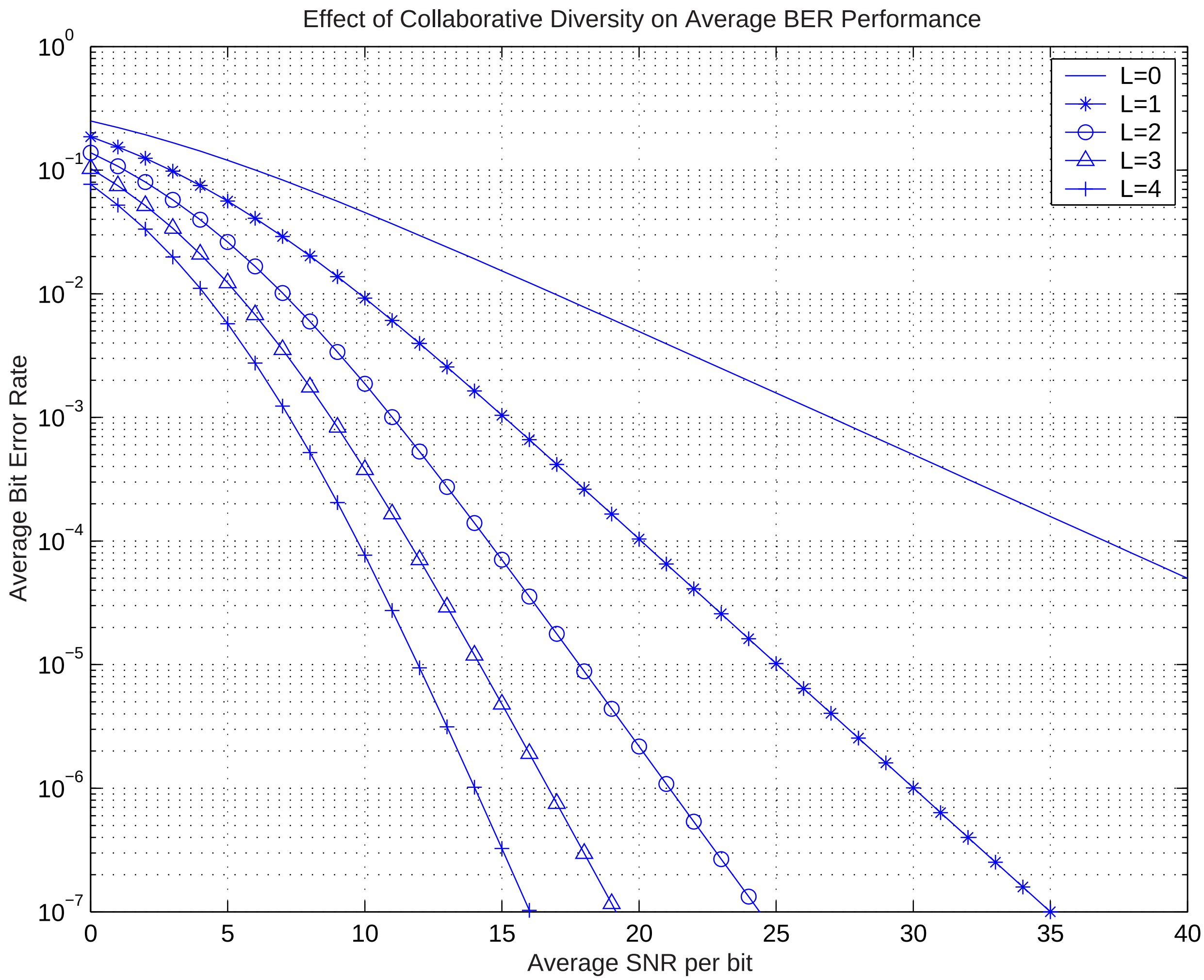
- Consider one direct link and L i.i.d. faded collaborating paths.
- Using maximal-ratio combining at the receiver, the overall SNR can be written as

$$\gamma_{\mathrm{t}} = \gamma_0 + \sum_{l=1}^{L} \gamma_l.$$

- Under these conditions the MGF of the overall combined SNR γ_{t} is given by

$$\mathcal{M}_{\gamma_{\mathrm{t}}}(s) = \mathcal{M}_{\gamma_0}(s) \prod_{l=1}^{L} \mathcal{M}_{\gamma_l}(s).$$

Diversity Gain due to Collaboration



Formulas for the Shannon Capacity

- For non-regenerative systems

$$C/W = \log_2(1+\gamma_{\text{eq}})\text{bps/Hz}$$

  - Capacity PDF is given by

$$p_C(c) = \frac{2^{c+1}\ln 2(2^c-1)e^{-(2^c-1)\left(\frac{1}{\overline{\gamma}_1}+\frac{1}{\overline{\gamma}_2}\right)}}{\overline{\gamma}_1\overline{\gamma}_2} \times\left[\left(\frac{\overline{\gamma}_1+\overline{\gamma}_2}{\sqrt{\overline{\gamma}_1\overline{\gamma}_2}}\right)K_1\left(\frac{2(2^c-1)}{\sqrt{\overline{\gamma}_1\overline{\gamma}_2}}\right)+2K_0\left(\frac{2(2^c-1)}{\sqrt{\overline{\gamma}_1\overline{\gamma}_2}}\right)\right].$$

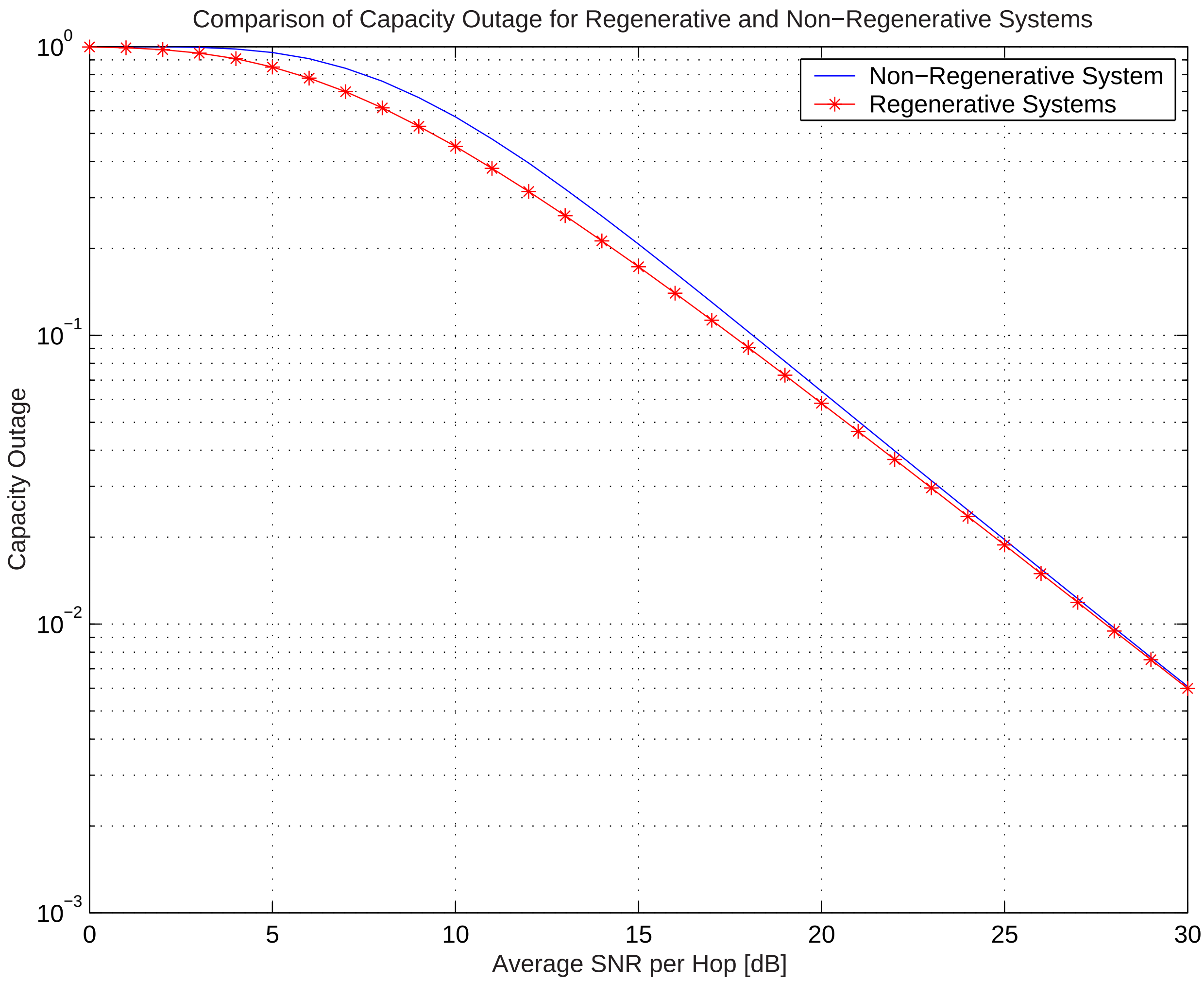
- For regenerative systems

$$C_{\text{eq}} = \min(C_1, C_2)$$

  - Capacity PDF is given by

$$p_C(c) = \ln 2\left(\frac{1}{\overline{\gamma}_1}+\frac{1}{\overline{\gamma}_2}\right)2^c e^{-(2^c-1)\left(\frac{1}{\overline{\gamma}_1}+\frac{1}{\overline{\gamma}_2}\right)}.$$

Capacity: Numerical Example



Extension to Systems with N Hops

- Analog relaying with channel inversion of the previous link

$$\gamma_{\text{eq}_2} = \left[\sum_{n=1}^{N} \frac{1}{\gamma_n} \right]^{-1}$$

  – Related to the harmonic mean of the hop's SNRs.

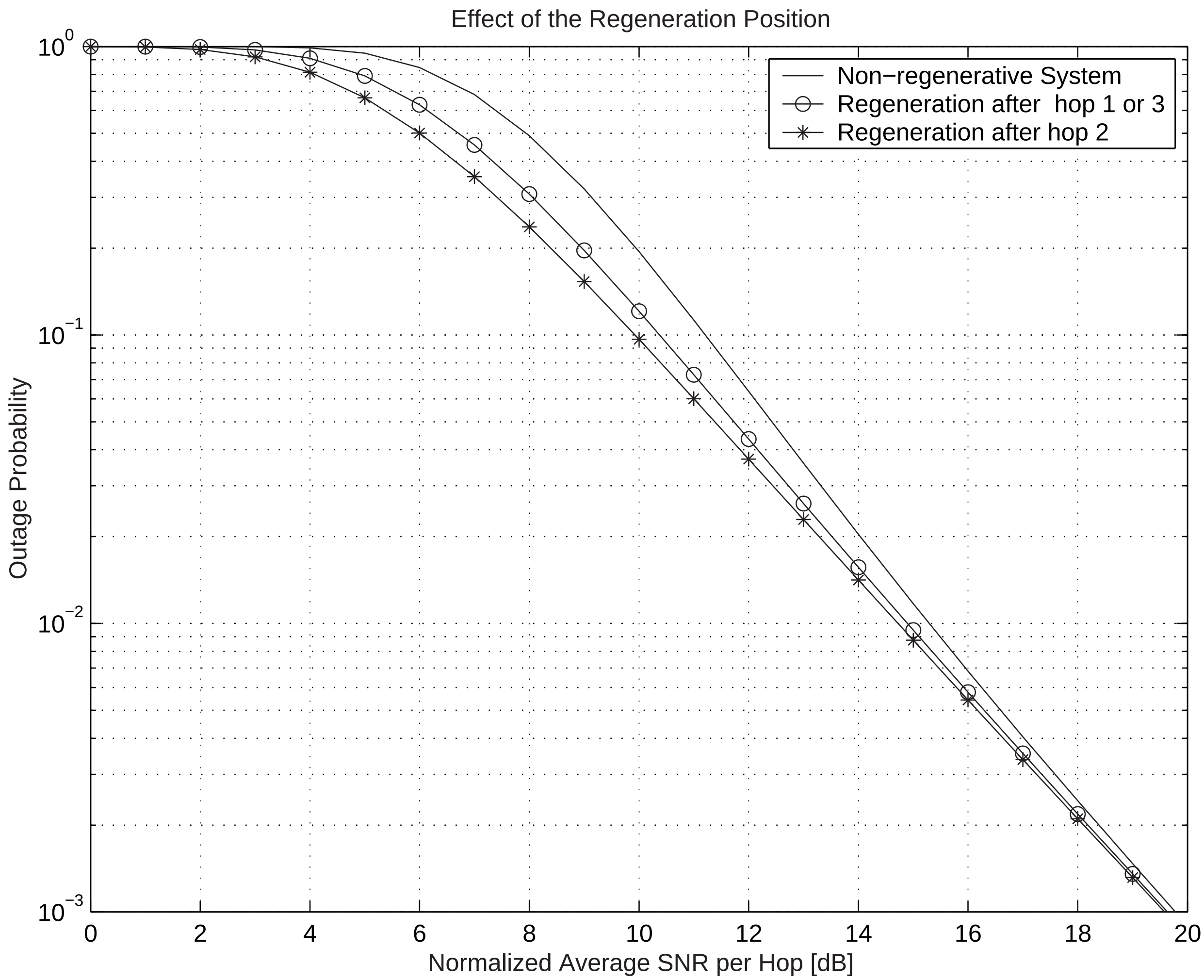
- Analog relaying with bounded relay gains

$$\gamma_{\text{eq}_1} = \left[\prod_{n=1}^{N} \left(1 + \frac{1}{\gamma_n} \right) - 1 \right]^{-1}.$$

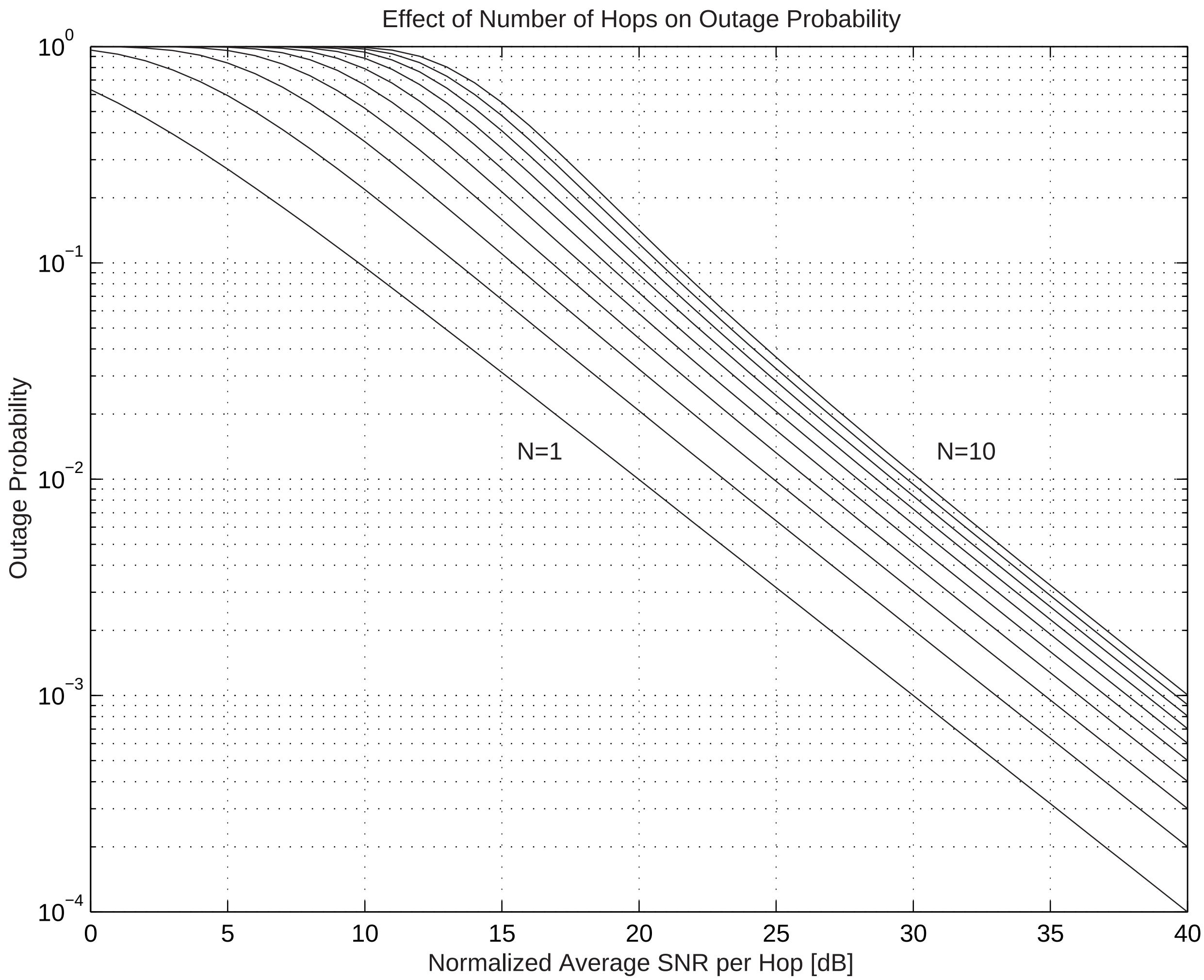
  – Example of a triple-hop system:

$$\frac{1}{\gamma_{\text{eq}_1}} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3} + \frac{1}{\gamma_1\gamma_2} + \frac{1}{\gamma_1\gamma_3} + \frac{1}{\gamma_2\gamma_3}.$$

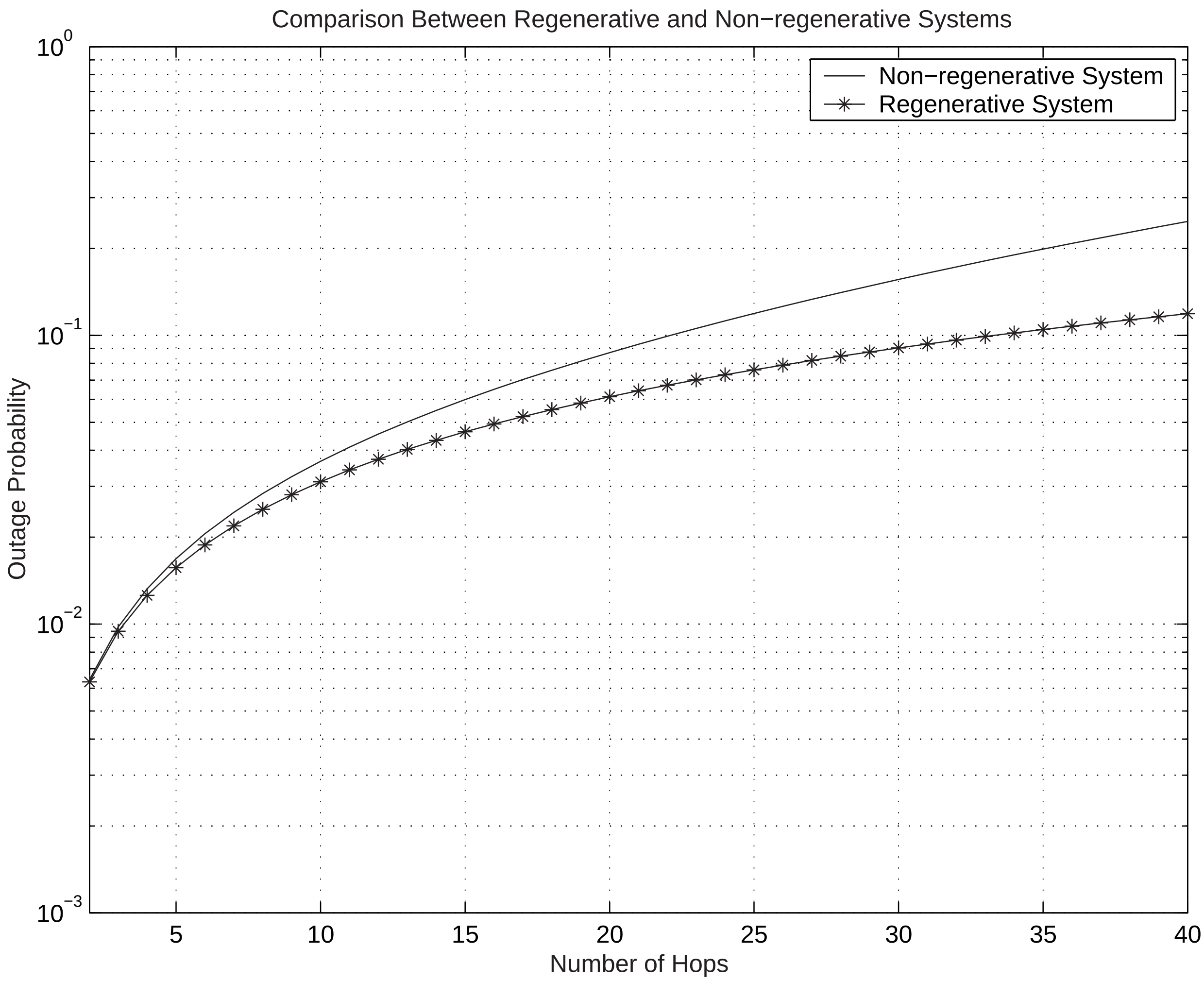
Where to Regenerate ?



Increasing the Number of Hops



Analog versus Digital Relaying



Other Topics of Interest

- Analog relaying with “fixed” relay gain.
- Variable-power and/or variable rate relays.
- Latency and delay associated with multi-hop systems.
- Global optimization versus local optimization.
- Power consumption and fairness issues.