Transmission properties of pair cables

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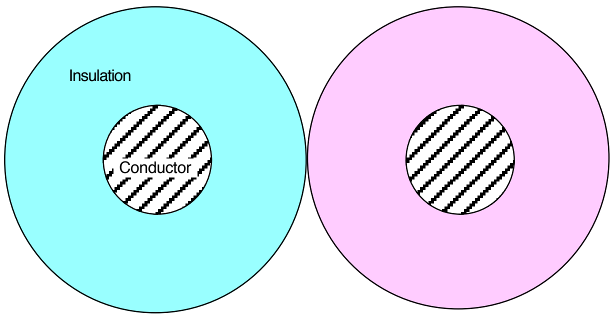
Overview

- Part I: Physical design of a pair cable
- Part II: Electrical properties of a single pair
- Part III: Interference between pairs, crosstalk
- Part IV: Estimates of channel capacity of pair cables. How to exploit the existing cable plant in an optimum way

Part I
Basic design of pair cables

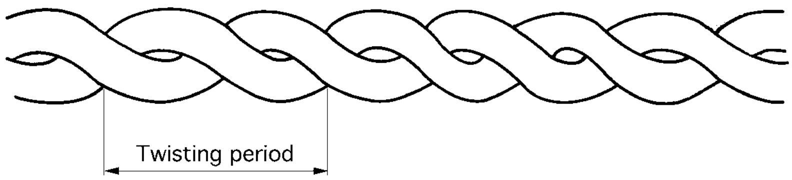
- A single twisted pair
- Binder groups
- The cross-stranding principle
- Building binder groups and cables of different sizes
- A complete cable

Cross section of a single pair



Most common conductor diameters: 0.4 mm, 0.6 mm
(0.5 mm, 0.9 mm)

A twisted pair



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Typical pair cables of the Norwegian access network

- 0.4 and 0.6 mm conductor diameter
- Polyethylene insulation (expanded)
- Twisting periods in the interval 50 - 150 mm
- 10 pair cross-stranded binder groups
- 10 - 2000 pairs in a cable

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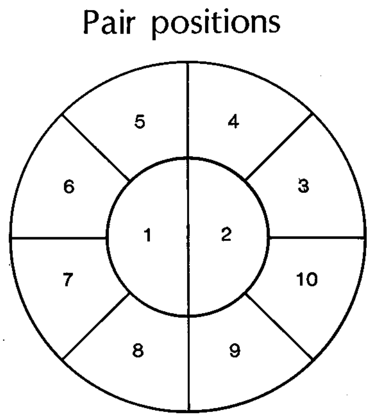
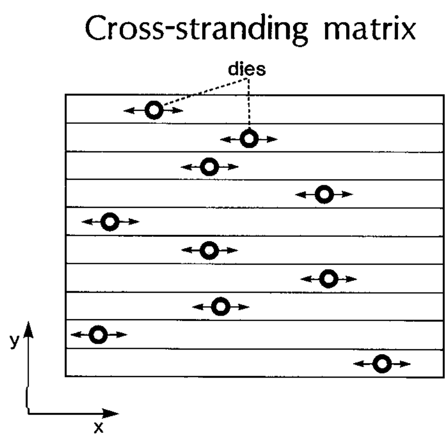
Pair positions in a 10 pair binder group



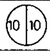
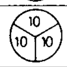
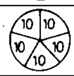
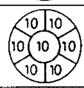
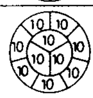
Cross-stranding [13]:
The positions of all the pairs are alternated randomly along the cable. Interference is thus randomised, and all pairs will be almost uniform

Cross-stranding

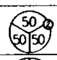
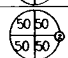
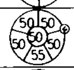
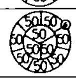
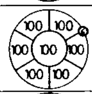
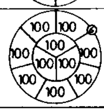


Cross-stranding technique:
Each pair runs through a die in the cross-stranding matrix.
The positions of the dies are set by a random generator

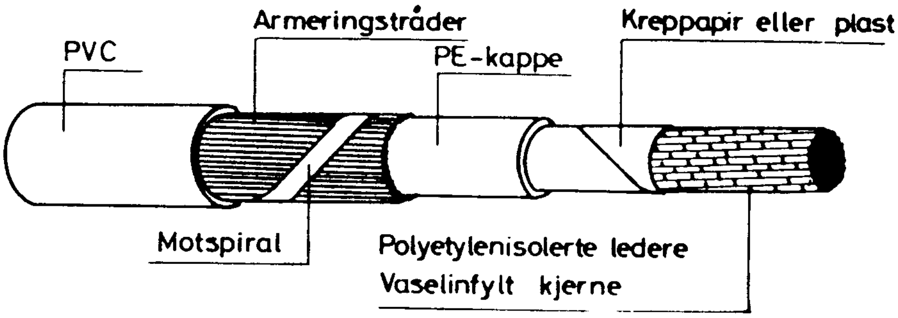
Design of binder groups up to 100 pairs

Partall	Grunn-grupper	Kabelsnitt	Merknad
10	1		Grunngruppe
20	2		
30	3		
50	5		Hovedgruppe 50 pars
70	1+6		
100	3+7		Hovedgruppe 100 pars

Design of binder groups up to 1000 pairs

Partall	Grupper	Kabelsnitt	Reservepar
150	3		2
200	4		2
300	1+5		4
500	3+7		6
700	1+6		6
1000	3+7		6

Typical cable design (”Kombikabel”)

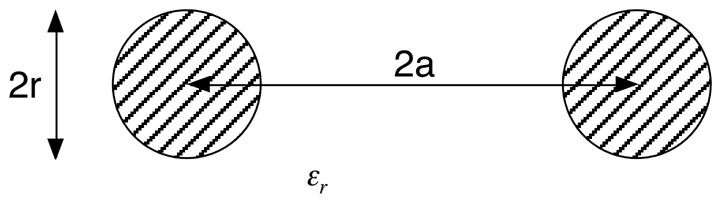


This cable may be used as:
1) overhead cable
2) buried cable
3) in water (fresh water)

Part II
Properties of a single pair

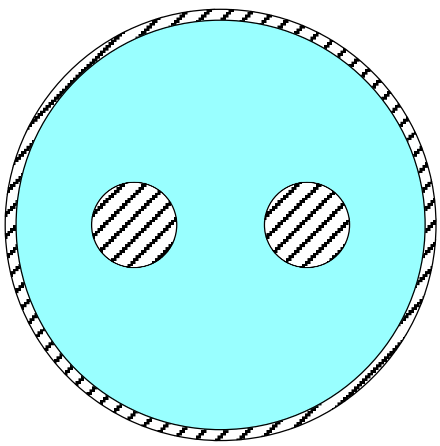
- Equivalent model of a single pair
- Capacitance
- Inductance
- Skin effect
- Resistance
- Per unit length model of a pair
- The telegraph equation - solution
- Propagation constant
- Characteristic impedance
- Reflection coefficient, terminations

A single pair in uniform insulation



A coarse estimate of the primary parameters R, L, C, G may be found assuming a >> r, uniform insulation, and no surrounding conductors

Model of a single pair in a cable



The electrical influence of the surrounding pairs in the cable may be modelled as an equivalent shield. This model will give accurate estimates of R, L, C and G [12]

Capacitance of a single pair

The capacitance per unit length of a single pair is given by:

$$C = \frac{\pi \varepsilon_0 \varepsilon_r}{\ln \frac{2a}{r}}$$

The capacitance of cables in the access network is 45 nF/km

Inductance of a single pair

The inductance per unit length of a single pair is given by:

$$L = \frac{\mu_0}{\pi} \ln \frac{2a}{r}$$

Skin effect

At high frequencies the current will flow in the outer part of the conductors, and the skin depth is given by [12]:

$$\delta = \frac{1}{\sqrt{\pi f \mu_r \mu_0 \sigma}}$$

σ is the conductance of the conductors

For copper the skin depth is given by:

$$\delta = \frac{2{,}11}{\sqrt{F_{\text{kHz}}}} \quad \text{mm}$$

δ = 2.11 mm at 1 kHz

δ = 0.067 mm at 1 MHz

Resistance of a conductor

The resistance per unit length of a conductor is for r << a given by:

$$R_C = \begin{cases} \dfrac{1}{\sigma \pi r^2} & \text{for} \quad \delta >> r \quad \text{(low frequencies)} \\ \dfrac{1}{2\sigma \pi r \delta} & \text{for} \quad \delta << r \quad \text{(high frequencies)} \end{cases}$$

Resistance of a pair

The resistance per unit length of a pair will be the sum of the resitances of the two conductors and is given by:

$$R = 2 \cdot R_C = \begin{cases} \dfrac{2}{\sigma \pi r^2} & \text{for} \quad \delta >> r \quad \text{(low frequencies)} \\ \dfrac{1}{\sigma \pi r \delta} & \text{for} \quad \delta << r \quad \text{(high frequencies)} \end{cases}$$

Conductance of a pair

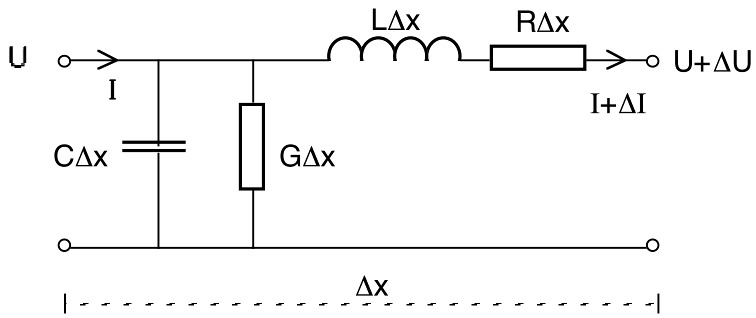
The conductance per unit length of a pair is given by:

$$G = \delta_l \cdot \omega C$$

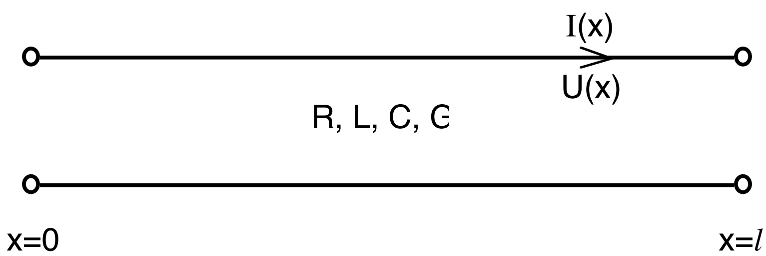
δ_l is the dielectric loss factor

A typical value of the loss factor is $\delta_l = 0.0003$. This means that the conductance is usually negligible for pair cables.

Per unit length model of a pair



Model of a pair of length *l*



The telegraph equation

From the circuit diagram:

$$\frac{d}{dx}U(x) = -(R + j\omega L)I(x) = -Z \cdot I(x)$$

$$\frac{d}{dx}I(x) = -(G + j\omega C)U(x) = -Y \cdot U(x)$$

Combining the equations:

$$\frac{d^2}{dx}U(x) = Z \cdot Y \cdot U(x)$$

Solution of the telegraph equation

$$U(x) = c_1 \cdot e^{\gamma \cdot x} + c_2 \cdot e^{-\gamma \cdot x}$$

$$I(x) = -\frac{c_1}{Z_0} \cdot e^{\gamma \cdot x} + \frac{c_2}{Z_0} \cdot e^{-\gamma \cdot x}$$

c_1 and c_2 are constants
γ is propagation constant
Z_0 is characteristic impedance

Propagation constant

$$\gamma = \sqrt{Z \cdot Y} = \sqrt{(R + j\omega L) \cdot (G + j\omega C)} =$$

$$\sqrt{(R + j\omega L) \cdot j\omega C}$$

$$\gamma = \alpha + j\beta$$

α is the attenuation constant in Neper/km
β is the phase constant in rad/km

Neper to dB:

$$\alpha_{dB} = \frac{20}{\ln(10)} \alpha = 8.69\alpha$$

Characteristic impedance

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} = \sqrt{\frac{(R + j\omega L)}{j\omega C}}$$

At high frequencies $R << j\omega L$:

$$Z_0 = \sqrt{\frac{L}{C}}$$

The characteristic impedance is approximately 120 ohms at high frequencies for pair cables in the access network

Attenuation constant

At high frequencies (f > 100 kHz) $R \ll \omega L$.
By series expansion of γ :

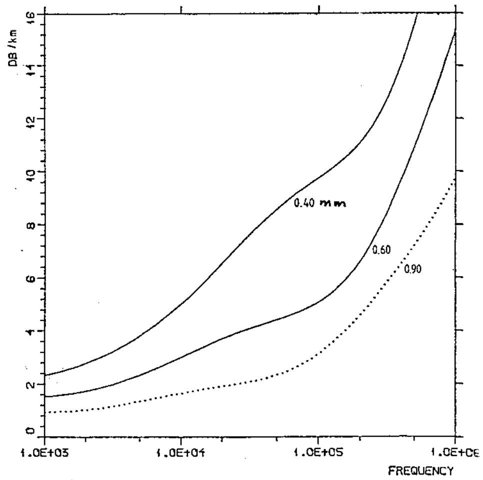
$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} = \frac{R}{2}\sqrt{\frac{C}{L}} = k_1 \cdot \sqrt{f}$$

At low frequencies (f < 10 kHz) $R \gg \omega L$. Hence:

$$\gamma = \sqrt{j\omega C \cdot R} = (1+j)\sqrt{\frac{\omega \cdot R \cdot C}{2}}$$

$$\alpha = \beta = \sqrt{\frac{\omega \cdot R \cdot C}{2}} = k_2\sqrt{f}$$

Attenuation constant of pair cables



Phase constant

At high frequencies (f > 100 kHz):

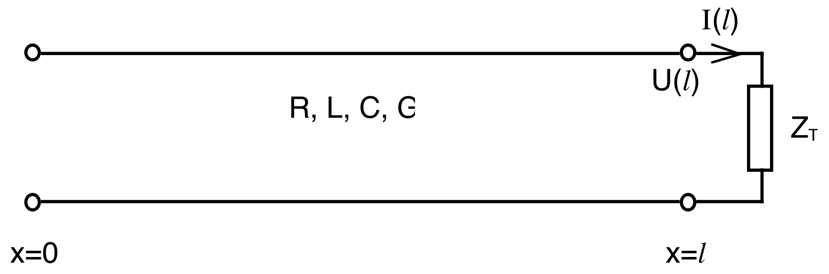
$$\beta = \omega\sqrt{L \cdot C} = k_3 \cdot f$$

Phase velocity:

$$v = \frac{\Delta x}{\Delta t} = \frac{\text{wavelength}}{\text{cycle}} = \frac{2\pi/\beta}{2\pi/\omega} = \frac{\omega}{\beta} = \frac{1}{\sqrt{L \cdot C}} = \frac{c}{\sqrt{\varepsilon_r}}$$

Phase velocity is 200 000 km/s for pair cables at high frequencies (ε_r = 2.3 for polyethylene)

Termination of a pair



Reflection coefficient

$$U(x) = V_+ \cdot e^{\gamma \cdot (\ell - x)} + V_- \cdot e^{-\gamma \cdot (\ell - x)}$$

$$I(x) = \frac{V_+}{Z_0} \cdot e^{\gamma \cdot (\ell - x)} - \frac{V_-}{Z_0} \cdot e^{-\gamma \cdot (\ell - x)}$$

$$Z_T = \frac{U(\ell)}{I(\ell)} = Z_0 \frac{V_+ + V_-}{V_+ - V_-}$$

Solution of telegraph equation including termination imp. Z_T
V_+ is voltage of wave in positive direction at $x=l$
V_- is voltage of reflected wave in at $x=l$

Reflection coefficient:

$$\rho = \frac{V_-}{V_+} = Z_0 \frac{Z_T - Z_0}{Z_T + Z_0}$$

No reflections ($\rho = 0$) for $Z_T = Z_0$. Ideal terminations are assumed in later crosstalk calculations

Part III
Crosstalk in pair cables

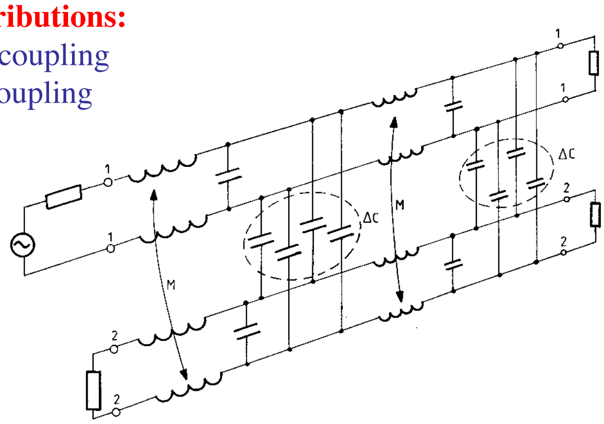
- Basic coupling mechanisms
- Crosstalk coupling per unit length
- Near end crosstalk, NEXT
- Far end crosstalk, FEXT
- Statistical crosstalk coupling
- Average NEXT and FEXT
- Crosstalk power sum - crosstalk from many pairs
- Statistical distributions of crosstalk

Crosstalk mechanisms

Main contributions:

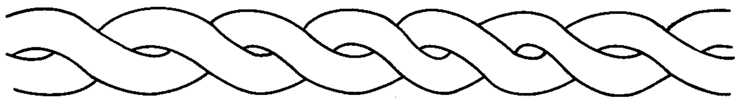
Capacitive coupling

Inductive coupling



Ideal and real twisting of a pair

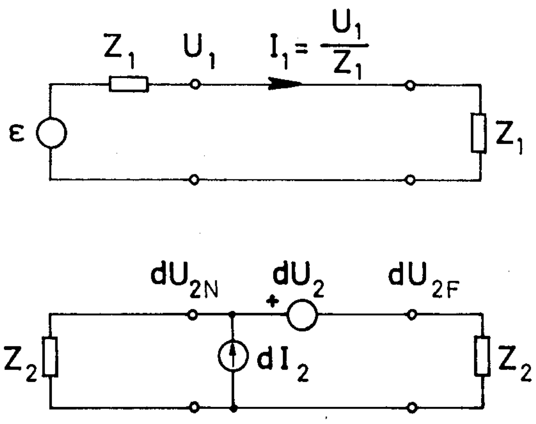
Ideal twisting



Actual twisting of a real pair

The crosstalk level observed in real cables
is caused mainly by deviations from ideal twisting

Crosstalk coupling per unit length



Crosstalk coupling per unit length

Normalised NEXT coupling coefficient [11]:

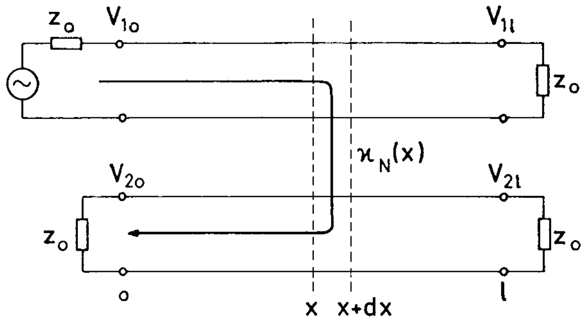
$$\kappa_{\mathrm{N}i,j}(x) = \frac{1}{j\beta_0} \cdot \frac{dU_{2\mathrm{N}}}{U_1 \cdot dx} = \frac{1}{2} \cdot \left(\frac{C_{i,j}(x)}{C} + \frac{L_{i,j}(x)}{L} \right)$$

Normalised FEXT coupling coefficient [11]:

$$\kappa_{\mathrm{F}i,j}(x) = \frac{1}{j\beta_0} \cdot \frac{dU_{2\mathrm{F}}}{U_1 \cdot dx} = \frac{1}{2} \cdot \left(\frac{C_{i,j}(x)}{C} - \frac{L_{i,j}(x)}{L} \right)$$

$C_{i,j}(x)$ is the mutual capacitance per unit length between pair i and j
$L_{i,j}(x)$ is the mutual inductance per unit length between pair i and j
β_0 is the lossless phase constant, $\beta_0 = \omega \sqrt{L \cdot C}$

Near end crosstalk, NEXT

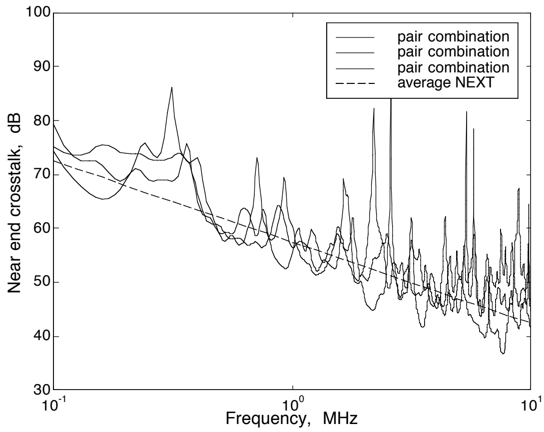


Near end crosstalk, NEXT

Assuming weak coupling, the near end voltage transfer function is given by [11]:

$$H_{NE}(f) = \frac{V_{20}}{V_{10}} = j\beta_0 \int_0^{\ell} \kappa_N(x) e^{-2\alpha x - 2j\beta x} dx$$

NEXT between two pairs



Stochastic model of crosstalk couplings

Crosstalk coupling factors are white Gaussian stocastic processes:

NEXT autocorrelation function [6]:

$$R_N(\tau) = E\left[\kappa_N(x)\cdot\kappa_N(x+\tau)\right] = k_N \cdot \delta(\tau)$$

FEXT autocorrelation function [6]:

$$R_F(\tau) = E\left[\kappa_F(x)\cdot\kappa_F(x+\tau)\right] = k_F \cdot \delta(\tau)$$

k_N and k_F are constants

Average NEXT

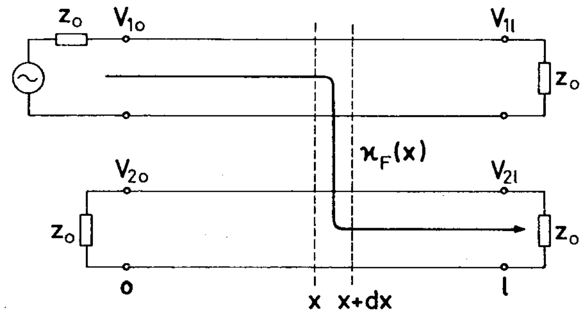
Average NEXT power transfer function between two pairs [6]:

$$p(f) = E\left[\left|H_{NE}(f)\right|^2\right] = E\left[\left|\frac{V_{20}}{V_{10}}\right|^2\right] =$$

$$\beta_0^2 k_N \int_0^{\ell} e^{-4\alpha x} dx = \frac{\beta_0^2 \cdot k_N}{4\alpha}\left(1 - e^{-4\ell}\right) \approx \frac{\beta_0^2 \cdot k_N}{4\alpha} = k_{\mathrm{N}2} \cdot f^{1.5}$$

NEXT increases 15 dB/decade with frequency

Far end crosstalk, FEXT



Far end crosstalk, FEXT

Assuming weak coupling, the far end voltage transfer function is given by [11]:

$$H_{FE}(f) = \frac{V_{2\ell}}{V_{1\ell}} = j\beta_0 \int_0^{\ell} \kappa_F(x)\, dx$$

Average FEXT

Average FEXT power transfer function between two pairs [6]:

$$q(f) = E\left[\left|H_{FE}(f)\right|^2\right] = E\left[\left|\frac{V_{2\ell}}{V_{1\ell}}\right|^2\right] =$$

$$\beta_0^2 \int_0^{\ell} k_F\, dx = k_F \cdot \beta_0^2 \cdot \ell = k_{F2} \cdot f^2 \cdot \ell$$

FEXT increases 20 dB/decade with frequency
FEXT increases 10 dB/decade with cable length

Crosstalk from *N* different pairs crosstalk power sum

- Only crosstalk between identical systems is considered (self NEXT and self FEXT)
- Crosstalk from different pairs add on a power basis
- Effective crosstalk is given by the sum of crosstalk power transfer functions, which is denoted **crosstalk power sum**

Crosstalk from N different pairs crosstalk power sum

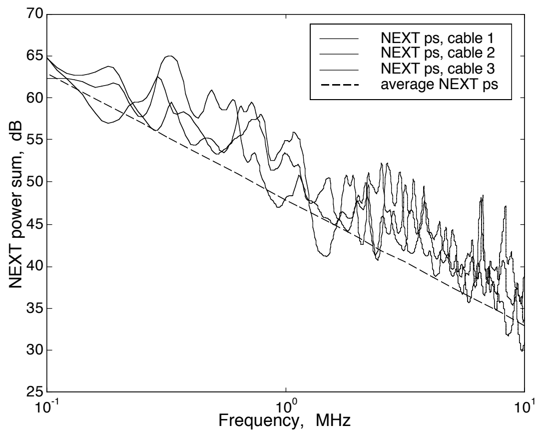
NEXT crosstalk power sum for pair no *i*:

$$\left|H_{NE\,ps}(f)\right|^2 = \sum_{\substack{j=1 \\ j \neq i}}^{N} \left[\left|H_{NE\,i,j}(f)\right|^2\right]$$

FEXT crosstalk power sum for pair no *i*:

$$\left|H_{FE\,ps}(f)\right|^2 = \sum_{\substack{j=1 \\ j \neq i}}^{N} \left[\left|H_{FE\,i,j}(f)\right|^2\right]$$

NEXT power sum



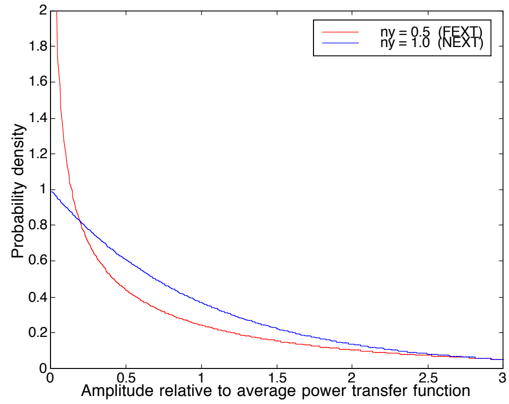
Probability distributions of crosstalk

Crosstalk power transfer function for a single pair combination at a given frequency is gamma-distributed with probability density [6]:

$$p_z(z) = \frac{1}{\Gamma(\nu)} \cdot \left(\frac{\nu}{a}\right)^{\nu} \cdot z^{\nu-1} \cdot e^{-\frac{\nu z}{a}}$$

$\nu = 1.0$ for NEXT
$\nu = 0.5$ for FEXT
a is average crosstalk power

Probability density of NEXT and FEXT for a single pair combination



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Probability of crosstalk for an arbitrary pair combination

Different pair combinations will have different levels of crosstalk coupling (different k_N and k_F).

It can be shown that:

The crosstalk power transfer function for a random pair combination is approximately gamma-distributed

The number of degrees of freedom, ν must be found empirically. $\nu < 1.0$ for NEXT and $\nu < 0.5$ for FEXT

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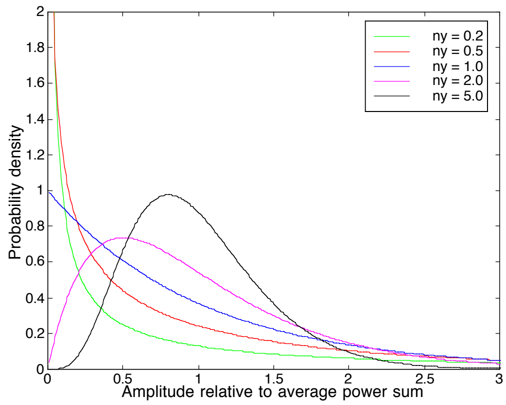
Probability of crosstalk power sum

Crosstalk from different pairs add on a power basis. Hence, crosstalk power sum is approximately gamma-distributed with probability distribution:

$$p_{ps}(z) = \frac{1}{\Gamma(\nu_{ps})} \cdot \left(\frac{\nu_{ps}}{a_{ps}} \right)^{\nu_{ps}} \cdot z^{\nu_{ps}-1} \cdot e^{-\frac{\nu_{ps} z}{a_{ps}}}$$

$a_{ps}=Na$, where a is the average crosstalk of one pair combination
$\nu_{ps}=N\nu$, where ν is the number of degrees of freedom for a random pair combination
N is the number of disturbing pairs

Probability density of crosstalk power sum



Worst case crosstalk

Crosstalk dimesioning is usually based upon the 99% point of NEXT and FEXT power sum, which is given by (1% of the pairs will have crosstalk that exceeds this limit):

$$p_{ps99}(f) = N \cdot E\left[k_{\mathrm{N2}}\right] \cdot f^{1.5} \cdot c_{99}(v_{ps}) \quad \text{for} \quad \text{NEXT}$$

$$q_{ps99}(f) = N \cdot E\left[k_{\mathrm{F2}}\right] \cdot f^{2} \cdot \ell \cdot c_{99}(v_{ps}) \quad \text{for} \quad \text{FEXT}$$

$c_{99}(v_{ps})$ is the ratio between the 99% point and the average power sum in the gamma distribution

The expectations $E[k_{\mathrm{N2}}]$ and $E[k_{\mathrm{F2}}]$ are taken over all pair combinations

Empirical worst case NEXT model

International model of 99% point of NEXT power sum based upon 50 pair binder groups:

$$p_{ps\,99}(F) = 10^{-4} \cdot \left(\frac{N}{49}\right)^{0.6} \cdot F^{1.5}$$

N is the number of disturbing pairs in the cable
F is the frequency in MHz

This model fits well with 100% filled Nowegian cables with 10 pair binder groups

Empirical worst case FEXT model

International model of 99% point of FEXT power sum based upon 50 pair binder groups:

$$q_{ps99}(F) = 3 \cdot 10^{-4} \cdot \left(\frac{N}{49}\right)^{0.6} \cdot F^2 \cdot L$$

N is the number of disturbing pairs in the cable
F is the frequency in MHz
L is the cable length in km

This model fits well with 100% filled Nowegian cables with 10 pair binder groups

Part IV
Channel capacity estimates [1,2,8]

- Shannon's channel capacity formula
- Signal and noise models
- Realistic estimates of channel capacity
- Channel capacity per bandwidth unit
- One-way transmission
- Two-way transmission
- Crosstalk between different types of systems, alien NEXT, alien FEXT

Shannon's theoretical channel capacity

Maximum theoretical channel capacity
in the frequency band [*fl*,*fh*]:

$$C_{Sh} = \int_{fl}^{fh} \log_2\left(1 + \frac{S(f)}{N(f)}\right) df \quad \text{bit/s}$$

S(*f*): signal power density
N(*f*): noise power density

Capacity per bandwidth unit

A realistic estimate of bandwidth efficiency [2]:

$$\eta(f) = \frac{\Delta C}{\Delta f} = k_{eff} \cdot \log_2\left(1 + \lambda \cdot \frac{S(f)}{N(f)}\right) \quad \text{bit/s/Hz}$$

S(*f*): signal power density
N(*f*): noise power density
$\lambda \leq 1$: factor for margin (safety margin + margin for mod.meth.)
$k_{eff} \leq 1$: factor for overhead (sync bits, RS-code, cyclic prefix)

Signal transmission

- Attenuation proportional to $\sqrt{f}$ due to skin effect (f > 100 kHz)
- Signal transfer function:

$$H(f) = 10^{-\frac{\alpha_{dB}\ell}{20}} = \exp\left(-k\ell\sqrt{f}\right)$$

α_{dB}: attenuation constant in dB

Signal and noise models

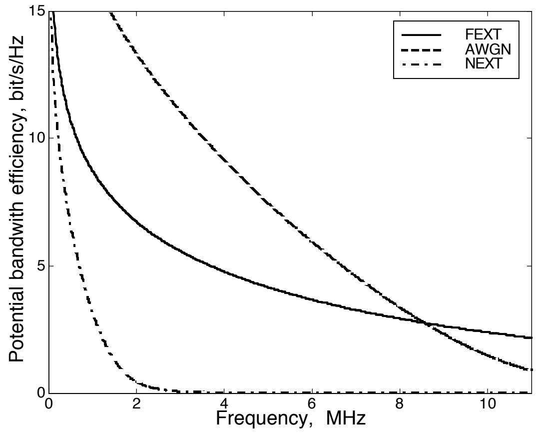
- Signal:

$$S(F) = e^{-2\alpha L}$$

- Noise models:

$$N(F) = \begin{cases} N_{NEXT} = 10^{-4} \cdot F^{1.5} & NEXT \\ N_{FEXT} = 3 \cdot 10^{-4} \cdot F^2 \cdot L \cdot e^{-2\alpha L} & FEXT \\ N_{AWGN} = 10^{-8} & AWGN \end{cases}$$

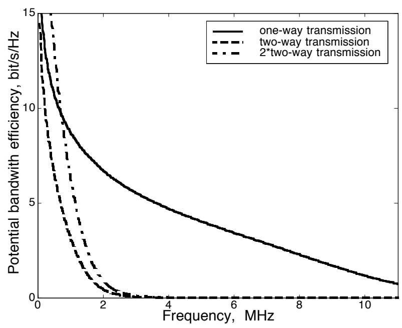
Channel capacity vs. frequency



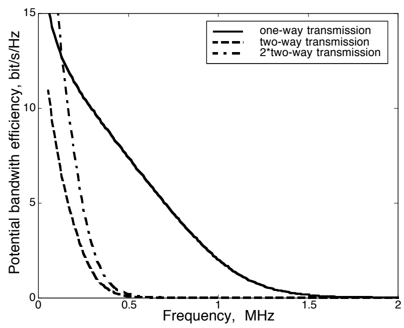
L=1 km

Channel capacity vs frequency II

L=1 km



L=3 km



Total channel capacity

One-way transmission:

$$R_{one-way} = k_{eff} \int_{fl}^{fh} \log_2 \left(1 + \lambda \cdot \frac{S(F)}{N_{FEXT} + N_{AWGN}} \right) df$$

Two-way transmission:

$$R_{two-way} = k_{eff} \int_{fl}^{fh} \log_2 \left(1 + \lambda \cdot \frac{S(F)}{N_{NEXT} + N_{FEXT} + N_{AWGN}} \right) df$$

The channel capacity is somewhat greater than this expression for two-way transmission due to uncorrelated NEXT in different frequency bands [9,10]

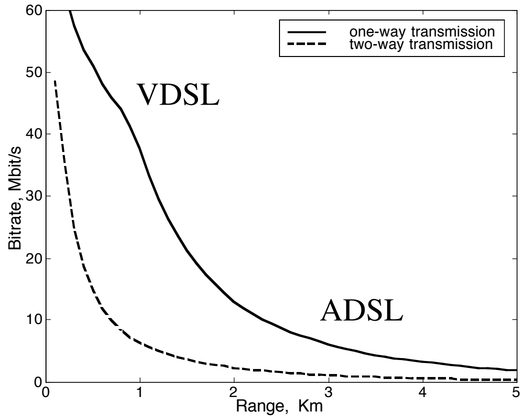
Assumptions for estimation of bitrates

- For one-way transmission, the total bitrate found in the calculations must be divided by downstream and upstream transmission
- Identical systems in all pairs of the cable (only self NEXT and self FEXT)
- All pairs are used, the cable is 100% filled
- Net bitrate is 90% of total bitrate (k_{eff}=0.90)
- Frequency band: $f \geq 100$ kHz, upper limit 11 MHz

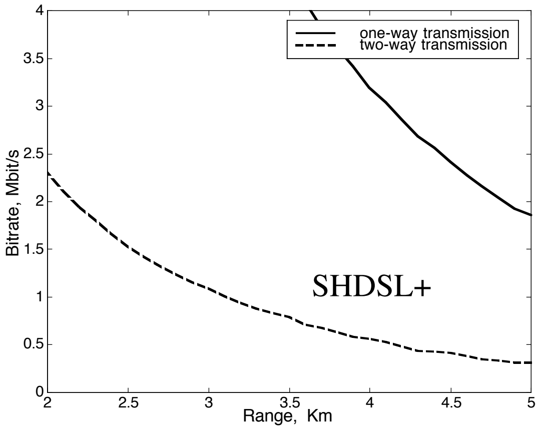
Assumptions II

- Multicarrier modulation [7]
- Adaptive modulation in each sub-band
- M-TCM modulation in each sub-band
  $4 \leq M \leq 16384$, 1 - 13 bit/s/Hz
- Distance to Shannon (λ): 9 dB
  (6 dB margin + 3 dB for modulation)
- White noise: 80 dB below output signal
- Cable: 0.4 mm, 22.5 dB/km at 1 MHz

Potential range for .4 mm cable



Potential range for .4 mm cable II

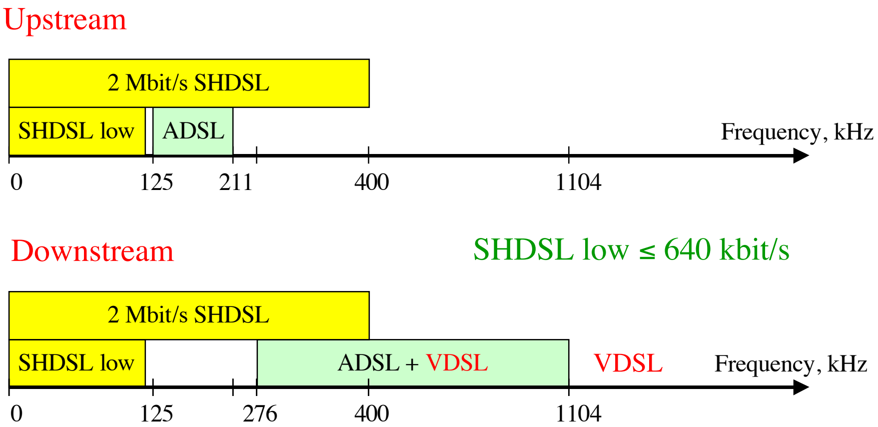


SHDSL+ means a multicarrier system using approximately the same frequency band as SHDSL

Digital Subscriber Line systems, xDSL

- ADSL; Asymmetric Digital Subscriber Lines [5]
  - Asymmetric data rates, 256 kbit/s - 8 Mbit/s downstream, range up to 4 - 5 km
- SHDSL; Symmetric High-speed Digital Subscriber Lines [3]
  - Symmetric data rates, 192 kbit/s - 2.3 Mbit/s, range up to 6 - 7 km
- VDSL; Very high-speed Digital Subscriber Lines
  - Asymmetric or symmetric data rates (still under standardisation), up to 52 Mbit/s downstream, range typically ≤ 1 km [4]

Frequency allocations in the access network



Conclusions

- Frequency planning in pair cables is very important
- New systems should be introduced with great care in order to preserve the potential transmission capacity of the cable
- Full rate SHDSL systems overlaps with ADSL

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