Transmission properties of pair cables

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Overview

• Part I: Physical design of a pair cable
• Part II: Electrical properties of a single pair
• Part III: Interference between pairs, crosstalk
• Part IV: Estimates of channel capacity of pair cables. How to exploit the existing cable plant in an optimum way
Part I
Basic design of pair cables

- A single twisted pair
- Binder groups
- The cross-stranding principle
- Building binder groups and cables of different sizes
- A complete cable

Cross section of a single pair

Most common conductor diameters: 0.4 mm, 0.6 mm (0.5 mm, 0.9 mm)
A twisted pair

Typical pair cables of the Norwegian access network

- 0.4 and 0.6 mm conductor diameter
- Polyethylene insulation (expanded)
- Twisting periods in the interval 50 - 150 mm
- 10 pair cross-stranded binder groups
- 10 - 2000 pairs in a cable
Cross-stranding [13]:
The positions of all the pairs are alternated randomly along the cable. Interference is thus randomised, and all pairs will be almost uniform.

Cross-stranding technique:
Each pair runs through a die in the cross-stranding matrix. The positions of the dies are set by a random generator.
Design of binder groups up to 100 pairs

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<th>Kabelområde</th>
<th>Minimall</th>
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<td>100</td>
<td>3+7</td>
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<td>Hovedgruppe 100 paires</td>
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Design of binder groups up to 1000 pairs

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<td>1000</td>
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Typical cable design ("Kombikabel")

This cable may be used as:
1) overhead cable
2) buried cable
3) in water (fresh water)

Part II
Properties of a single pair

- Equivalent model of a single pair
- Capacitance
- Inductance
- Skin effect
- Resistance
- Per unit length model of a pair
- The telegraph equation - solution
- Propagation constant
- Characteristic impedance
- Reflection coefficient, terminations
A single pair in uniform insulation

A coarse estimate of the primary parameters $R$, $L$, $C$, $G$ may be found assuming $a \gg r$, uniform insulation, and no surrounding conductors.

Model of a single pair in a cable

The electrical influence of the surrounding pairs in the cable may be modelled as an equivalent shield. This model will give accurate estimates of $R$, $L$, $C$ and $G$ [12].
Capacitance of a single pair

The capacitance per unit length of a single pair is given by:

\[ C = \frac{\pi \varepsilon_0 \varepsilon_r}{2a} \ln \frac{2a}{r} \]

The capacitance of cables in the access network is 45 nF/km

Inductance of a single pair

The inductance per unit length of a single pair is given by:

\[ L = \frac{\mu_0}{\pi} \ln \frac{2a}{r} \]
Skin effect

At high frequencies the current will flow in the outer part of the conductors, and the skin depth is given by [12]:

$$\delta = \frac{1}{\sqrt{\pi f \mu \mu_0 \sigma}}$$

$\sigma$ is the conductance of the conductors

For copper the skin depth is given by:

$$\delta = \frac{2.11}{\sqrt{F_{kHz}}} \text{ mm}$$

$\delta = 2.11 \text{ mm at } 1 \text{ kHz}$

$\delta = 0.067 \text{ mm at } 1 \text{ MHz}$

Resistance of a conductor

The resistance per unit length of a conductor is for $r << a$ given by:

$$R_c = \begin{cases} 
\frac{1}{\sigma \pi r^2} & \text{for } \delta \gg r \quad \text{(low frequencies)} \\
\frac{1}{2 \sigma \pi r \delta} & \text{for } \delta \ll r \quad \text{(high frequencies)} 
\end{cases}$$
Resistance of a pair

The resistance per unit length of a pair will be the sum of the resistances of the two conductors and is given by:

\[
R = 2 \cdot R_C = \begin{cases} 
\frac{2}{\sigma \pi r^2} & \text{for } \delta >> r \quad \text{(low frequencies)} \\
\frac{1}{\sigma \pi r \delta} & \text{for } \delta << r \quad \text{(high frequencies)} 
\end{cases}
\]

Conductance of a pair

The conductance per unit length of a pair is given by:

\[
G = \delta_l \cdot \omega C
\]

\(\delta_l\) is the dielectric loss factor

A typical value of the loss factor is \(\delta_l = 0.0003\). This means that the conductance is usually negligible for pair cables.
Per unit length model of a pair

$$U \rightarrow I \rightarrow U + \Delta U$$

$$C \Delta x$$

$$R \Delta x$$

$$G \Delta x$$

$$\Delta x$$

Model of a pair of length $l$

$$I(x)$$

$$U(x)$$

$R, L, C, G$

$x=0$ to $x=l$
The telegraph equation

From the circuit diagram:

\[
\frac{d}{dx} U(x) = -(R + j\omega L) I(x) = -Z \cdot I(x)
\]

\[
\frac{d}{dx} I(x) = -(G + j\omega C) U(x) = -Y \cdot U(x)
\]

Combining the equations:

\[
\frac{d^2}{dx^2} U(x) = Z \cdot Y \cdot U(x)
\]

Solution of the telegraph equation

\[
U(x) = c_1 \cdot e^{\gamma x} + c_2 \cdot e^{-\gamma x}
\]

\[
I(x) = -\frac{c_1}{Z_0} \cdot e^{\gamma x} + \frac{c_2}{Z_0} \cdot e^{-\gamma x}
\]

c_1 and c_2 are constants
\(\gamma\) is propagation constant
\(Z_0\) is characteristic impedance
Propagation constant

\[ \gamma = \sqrt{Z \cdot Y} = \sqrt{(R + j\omega L) \cdot (G + j\omega C)} = \sqrt{(R + j\omega L) \cdot j\omega C} \]

\[ \gamma = \alpha + j\beta \]

\( \alpha \) is the attenuation constant in Neper/km
\( \beta \) is the phase constant in rad/km

Neper to dB:

\[ \alpha_{\text{dB}} = \frac{20}{\ln(10)} \alpha = 8.69\alpha \]

Characteristic impedance

\[ Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} = \sqrt{\frac{(R + j\omega L)}{j\omega C}} \]

At high frequencies \( R << j\omega L \):

\[ Z_0 = \sqrt{\frac{L}{C}} \]

The characteristic impedance is approximately 120 ohms at high frequencies for pair cables in the access network.
Attenuation constant

At high frequencies \((f > 100 \text{ kHz})\) \(R \ll \omega L\).
By series expansion of \(\gamma\):
\[
\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}} = \frac{R}{2} \sqrt{\frac{C}{L}} = k_1 \cdot \sqrt{f}
\]

At low frequencies \((f < 10 \text{ kHz})\) \(R \gg \omega L\). Hence:
\[
\gamma = \sqrt{j \omega C \cdot R} = (1 + j) \sqrt{\frac{\omega \cdot R \cdot C}{2}}
\]
\[
\alpha = \beta = \sqrt{\frac{\omega \cdot R \cdot C}{2}} = k_2 \sqrt{f}
\]
Phase constant

At high frequencies ($f > 100$ kHz):

$$\beta = \omega \sqrt{L \cdot C} = k_3 \cdot f$$

Phase velocity:

$$v = \frac{\Delta x}{\Delta t} = \frac{2\pi}{2\pi/\beta} = \frac{\omega}{\beta} = \frac{1}{\sqrt{L \cdot C}} = \frac{c}{\sqrt{\varepsilon_r}}$$

Phase velocity is 200 000 km/s for pair cables at high frequencies ($\varepsilon_r = 2.3$ for polyethylene)

Termination of a pair

$$x=0$$

$$x=l$$

$$R, L, C, G$$

$$I(l)$$

$$U(l)$$

$$Z_T$$
Reflection coefficient

\[ U(x) = V_e \cdot e^{\gamma(x-l)} + V_r \cdot e^{-\gamma(x-l)} \]

\[ I(x) = \frac{V_e}{Z_0} \cdot e^{\gamma(x-l)} - \frac{V_r}{Z_0} \cdot e^{-\gamma(x-l)} \]

\[ Z_T = \frac{U(\ell)}{I(\ell)} = Z_0 \cdot \frac{V_e + V_r}{V_e - V_r} \]

Solution of telegraph equation including termination imp. \( Z_T \)

\( V_e \) is voltage of wave in positive direction at \( x=l \)

\( V_r \) is voltage of reflected wave in at \( x=l \)

Reflection coefficient:

\[ \rho = \frac{V_r}{V_e} = Z_0 \cdot \frac{Z_T - Z_0}{Z_T + Z_0} \]

No reflections (\( \rho = 0 \)) for \( Z_T = Z_0 \). Ideal terminations are assumed in later crosstalk calculations.

Part III

Crosstalk in pair cables

- Basic coupling mechanisms
- Crosstalk coupling per unit length
- Near end crosstalk, NEXT
- Far end crosstalk, FEXT
- Statistical crosstalk coupling
- Average NEXT and FEXT
- Crosstalk power sum - crosstalk from many pairs
- Statistical distributions of crosstalk
Crosstalk mechanisms

Main contributions:
Capacitive coupling
Inductive coupling

Ideal and real twisting of a pair

Ideal twisting

Actual twisting of a real pair

The crosstalk level observed in real cables is caused mainly by deviations from ideal twisting
Crosstalk coupling per unit length

Normalised NEXT coupling coefficient [11]:

\[
\kappa_{Ni,j}(x) = \frac{1}{j\beta_0} \cdot \frac{dU_{2N}}{U_1 \cdot dx} = \frac{1}{2} \left( \frac{C_{i,j}(x)}{C} + \frac{L_{i,j}(x)}{L} \right)
\]

Normalised FEXT coupling coefficient [11]:

\[
\kappa_{Fi,j}(x) = \frac{1}{j\beta_0} \cdot \frac{dU_{2F}}{U_1 \cdot dx} = \frac{1}{2} \left( \frac{C_{i,j}(x)}{C} - \frac{L_{i,j}(x)}{L} \right)
\]

\(C_{i,j}(x)\) is the mutual capacitance per unit length between pair \(i\) and \(j\)
\(L_{i,j}(x)\) is the mutual inductance per unit length between pair \(i\) and \(j\)
\(\beta_0\) is the lossless phase constant, \(\beta_0 = \omega \sqrt{L \cdot C}\)
Near end crosstalk, NEXT

Assuming weak coupling, the near end voltage transfer function is given by [11]:

\[ H_{NE}(f) = \frac{V_{20}}{V_{10}} = j\beta_0 \int_{0}^{\ell} \kappa_N(x) e^{-2\alpha x - 2j\beta x} \, dx \]
NEXT between two pairs

![NEXT graph](image)

Stochastic model of crosstalk couplings

Crosstalk coupling factors are white Gaussian stochastic processes:

NEXT autocorrelation function [6]:

$$R_N(\tau) = E[\kappa_N(x) \cdot \kappa_N(x + \tau)] = k_N \cdot \delta(\tau)$$

FEXT autocorrelation function [6]:

$$R_F(\tau) = E[\kappa_F(x) \cdot \kappa_F(x + \tau)] = k_F \cdot \delta(\tau)$$

$k_N$ and $k_F$ are constants
Average NEXT

Average NEXT power transfer function between two pairs [6]:

\[ p(f) = E[H_{NE}(f)]^2 = E \left[ \frac{V_{20}}{V_{10}} \right]^2 = \]

\[ \beta_0^2 k_N \int_0^\ell e^{-4\alpha x} dx = \frac{\beta_0^2 \cdot k_N}{4\alpha} (1 - e^{-4\ell}) = \frac{\beta_0^2 \cdot k_N}{4\alpha} = k_{N2} \cdot f^{1.5} \]

NEXT increases 15 dB/decade with frequency

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Far end crosstalk, FEXT
Far end crosstalk, FEXT

Assuming weak coupling, the far end voltage transfer function is given by [11]:

\[ H_{FE}(f) = \frac{V_{2\ell}}{V_{1\ell}} = j\beta_0 \int_0^\ell \kappa_F(x) dx \]

Average FEXT

Average FEXT power transfer function between two pairs [6]:

\[ q(f) = E\left[ |H_{FE}(f)|^2 \right] = E\left[ \left| \frac{V_{2\ell}}{V_{1\ell}} \right|^2 \right] = \]

\[ \beta_0^2 \int_0^\ell k_F dx = k_F \cdot \beta_0^2 \cdot \ell = k_{F2} \cdot f^2 \cdot \ell \]

FEXT increases 20 dB/decade with frequency
FEXT increases 10 dB/decade with cable length
Crosstalk from \( N \) different pairs
crosstalk power sum

- Only crosstalk between identical systems is considered (self NEXT and self FEXT)
- Crosstalk from different pairs add on a power basis
- Effective crosstalk is given by the sum of crosstalk power transfer functions, which is denoted **crosstalk power sum**

\[
\text{NEXT crosstalk power sum for pair no } i: \\
\left| H_{NE, ps}(f) \right|^2 = \sum_{j=1 \atop j \neq i}^N \left| H_{NE, i,j}(f) \right|^2
\]

\[
\text{FEXT crosstalk power sum for pair no } i: \\
\left| H_{FE, ps}(f) \right|^2 = \sum_{j=1 \atop j \neq i}^N \left| H_{FE, i,j}(f) \right|^2
\]
Probability distributions of crosstalk

Crosstalk power transfer function for a single pair combination at a given frequency is gamma-distributed with probability density [6]:

\[
p_z(z) = \frac{1}{\Gamma(\nu)} \cdot \left( \frac{\nu}{a} \right)^\nu \cdot z^{\nu-1} \cdot e^{-\frac{\nu z}{a}}
\]

\( \nu = 1.0 \) for NEXT
\( \nu = 0.5 \) for FEXT
\( a \) is average crosstalk power
Probability density of NEXT and FEXT for a single pair combination

![Probability density graph](image)

**Probability of crosstalk for an arbitrary pair combination**

Different pair combinations will have different levels of crosstalk coupling (different $k_N$ and $k_F$).

It can be shown that:

The crosstalk power transfer function for a random pair combination is approximately gamma-distributed

The number of degrees of freedom, $\nu$ must be found empirically. $\nu < 1.0$ for NEXT and $\nu < 0.5$ for FEXT
Probability of crosstalk power sum

Crosstalk from different pairs add on a power basis. Hence, crosstalk power sum is approximately gamma-distributed with probability distribution:

\[ p_{ps}(z) = \frac{1}{\Gamma(\nu_{ps})} \left( \frac{\nu_{ps}}{a_{ps}} \right)^{\nu_{ps}} \cdot z^{\nu_{ps}-1} \cdot e^{-\frac{\nu_{ps}z}{a_{ps}}} \]

\( a_{ps} = Na \), where \( a \) is the average crosstalk of one pair combination
\( \nu_{ps} = N\nu \), where \( \nu \) is the number of degrees of freedom for a random pair combination
\( N \) is the number of disturbing pairs

Probability density of crosstalk power sum
Worst case crosstalk

Crosstalk dimensioning is usually based upon the 99% point of NEXT and FEXT power sum, which is given by (1% of the pairs will have crosstalk that exceeds this limit):

\[ p_{ps99}(f) = N \cdot E[k_{N2}] \cdot f^{1.5} \cdot c_{99}(\nu_{ps}) \quad \text{for \ NEXT} \]
\[ q_{ps99}(f) = N \cdot E[k_{F2}] \cdot f^2 \cdot \ell \cdot c_{99}(\nu_{ps}) \quad \text{for \ FEXT} \]

\[ c_{99}(\nu) \] is the ratio between the 99% point and the average power sum in the gamma distribution.

The expectations \( E[k_{N2}] \) and \( E[k_{F2}] \) are taken over all pair combinations.

Empirical worst case NEXT model

International model of 99% point of NEXT power sum based upon 50 pair binder groups:

\[ p_{ps99}(F) = 10^{-4} \cdot \left( \frac{N}{49} \right)^{0.6} \cdot F^{1.5} \]

\( N \) is the number of disturbing pairs in the cable
\( F \) is the frequency in MHz

This model fits well with 100% filled Norwegian cables with 10 pair binder groups.
Empirical worst case FEXT model

International model of 99% point of FEXT power sum based upon 50 pair binder groups:

\[ q_{ps,99}(F) = 3 \cdot 10^{-4} \cdot \left( \frac{N}{49} \right)^{0.6} \cdot F^2 \cdot L \]

* \( N \) is the number of disturbing pairs in the cable
* \( F \) is the frequency in MHz
* \( L \) is the cable length in km

This model fits well with 100% filled Norwegian cables with 10 pair binder groups

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Part IV
Channel capacity estimates [1,2,8]

- Shannon’s channel capacity formula
- Signal and noise models
- Realistic estimates of channel capacity
- Channel capacity per bandwidth unit
- One-way transmission
- Two-way transmission
- Crosstalk between different types of systems, alien NEXT, alien FEXT
Shannon’s theoretical channel capacity

Maximum theoretical channel capacity in the frequency band \([f_l, f_h]\):

\[
C_{Sh} = \int_{f_l}^{f_h} \log_2 \left( 1 + \frac{S(f)}{N(f)} \right) df \quad \text{bit/s}
\]

\(S(f)\): signal power density  
\(N(f)\): noise power density

Capacity per bandwidth unit

A realistic estimate of bandwidth efficiency [2]:

\[
\eta(f) = \frac{\Delta C}{\Delta f} = k_{eff} \cdot \log_2 \left( 1 + \lambda \cdot \frac{S(f)}{N(f)} \right) \quad \text{bit/s/Hz}
\]

\(S(f)\): signal power density  
\(N(f)\): noise power density  
\(\lambda \leq 1\): factor for margin (safety margin + margin for mod.meth.)  
\(k_{eff} \leq 1\): factor for overhead (sync bits, RS-code, cyclic prefix)
Signal transmission

- Attenuation proportional to $\sqrt{f}$ due to skin effect ($f > 100$ kHz)
- Signal transfer function:

$$H(f) = 10^{\frac{\alpha_{db}\ell}{20}} = \exp\left(-k\ell\sqrt{f}\right)$$

$\alpha_{db}$: attenuation constant in dB

Signal and noise models

- Signal:

$$S(F) = e^{-2\alpha L}$$

- Noise models:

$$N(F) = \begin{cases} N_{NEXT} = 10^{-4} \cdot F^{1.5} & NEXT \\ N_{FEXT} = 3 \cdot 10^{-4} \cdot F^2 \cdot L \cdot e^{-2\alpha L} & FEXT \\ N_{AWGN} = 10^{-8} & AWGN \end{cases}$$
Channel capacity vs. frequency

L=1 km

Channel capacity vs frequency II

L=1 km  L=3 km
Total channel capacity

One-way transmission:
\[ R_{\text{one-way}} = k_{\text{eff}} \int_{f_l}^{f_h} \log_2(1 + \lambda \cdot \frac{S(F)}{N_{\text{FEXT}} + N_{\text{AWGN}}}) \, df \]

Two-way transmission:
\[ R_{\text{two-way}} = k_{\text{eff}} \int_{f_l}^{f_h} \log_2(1 + \lambda \cdot \frac{S(F)}{N_{\text{NEXT}} + N_{\text{FEXT}} + N_{\text{AWGN}}}) \, df \]

The channel capacity is somewhat greater than this expression for two-way transmission due to uncorrelated NEXT in different frequency bands [9,10]

Assumptions for estimation of bitrates

- For one-way transmission, the total bitrate found in the calculations must be divided by downstream and upstream transmission
- Identical systems in all pairs of the cable (only self NEXT and self FEXT)
- All pairs are used, the cable is 100% filled
- Net bitrate is 90% of total bitrate \( k_{\text{eff}} = 0.90 \)
- Frequency band: \( f \geq 100 \, \text{kHz} \), upper limit 11 MHz
Assumptions II

• Multicarrier modulation [7]
• Adaptive modulation in each sub-band
• M-TCM modulation in each sub-band
  \[ 4 \leq M \leq 16384, \quad 1 - 13 \text{ bit/s/Hz} \]
• Distance to Shannon (\(\lambda\)): 9 dB
  (6 dB margin + 3 dB for modulation)
• White noise: 80 dB below output signal
• Cable: 0.4 mm, 22.5 dB/km at 1 MHz

Potential range for .4 mm cable
Digital Subscriber Line systems, xDSL

- ADSL; Asymmetric Digital Subscriber Lines [5]
  - Asymmetric data rates, 256 kbit/s - 8 Mbit/s downstream, range up to 4 - 5 km
- SHDSL; Symmetric High-speed Digital Subscriber Lines [3]
  - Symmetric data rates, 192 kbit/s - 2.3 Mbit/s, range up to 6 - 7 km
- VDSL; Very high-speed Digital Subscriber Lines
  - Asymmetric or symmetric data rates (still under standardisation), up to 52 Mbit/s downstream, range typically ≤ 1 km [4]
Conclusions

- Frequency planning in pair cables is very important
- New systems should be introduced with great care in order to preserve the potential transmission capacity of the cable
- Full rate SHDSL systems overlaps with ADSL
References I


References II

[9] N. Holte, “A new method for calculating the channel capacity in pair cables with respect to near end crosstalk”, DSLcon Europe, Munich, November 2001
References III


