Adaptive coded modulation: channel prediction

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Outline

- Motivation
- System overview
- Channel model
- Antenna diversity/Maximal ratio combining
- Adaptive coded modulation
- Channel prediction
- Maximum a priori prediction
- System analysis
- Results/figures

Motivation

- Transmission in multipath fading environment
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  - High SNR: transmit with high rate code
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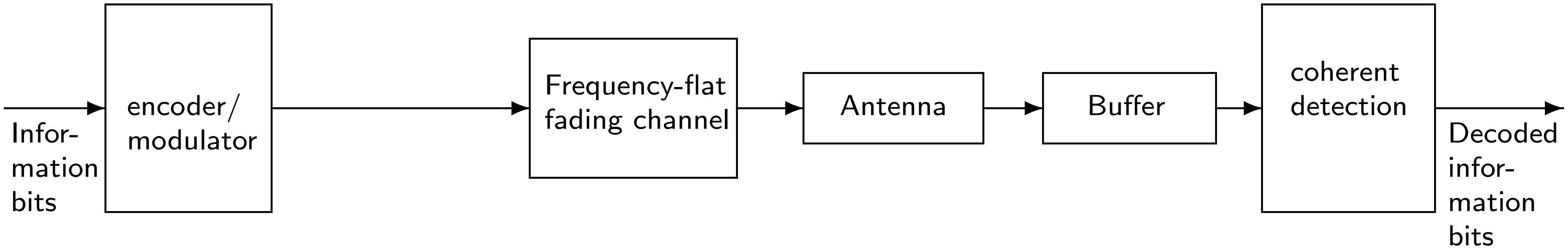
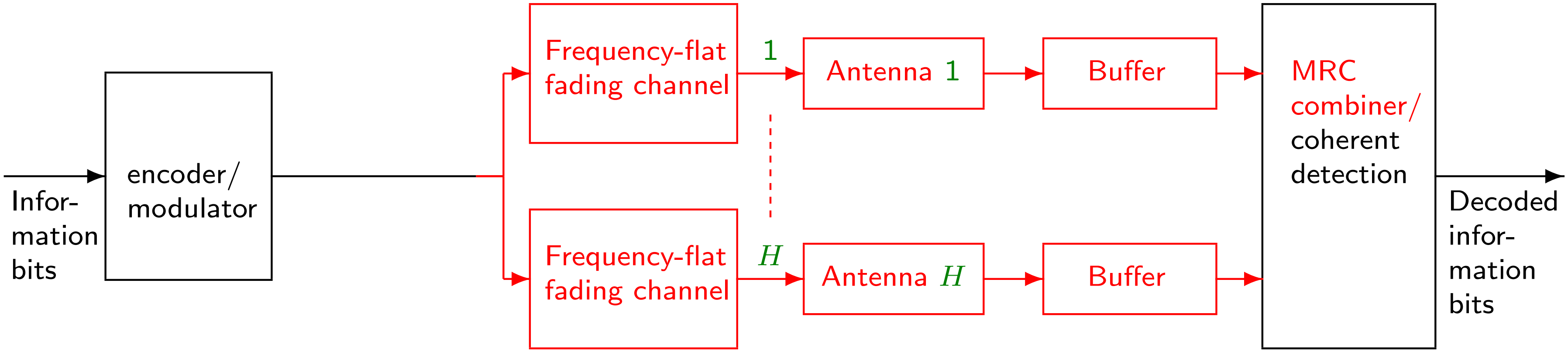
- Transmitter (Tx) needs channel state information (CSI): system relies on feedback channel
- Since delay in the feedback channel: Use prediction!

ACM system



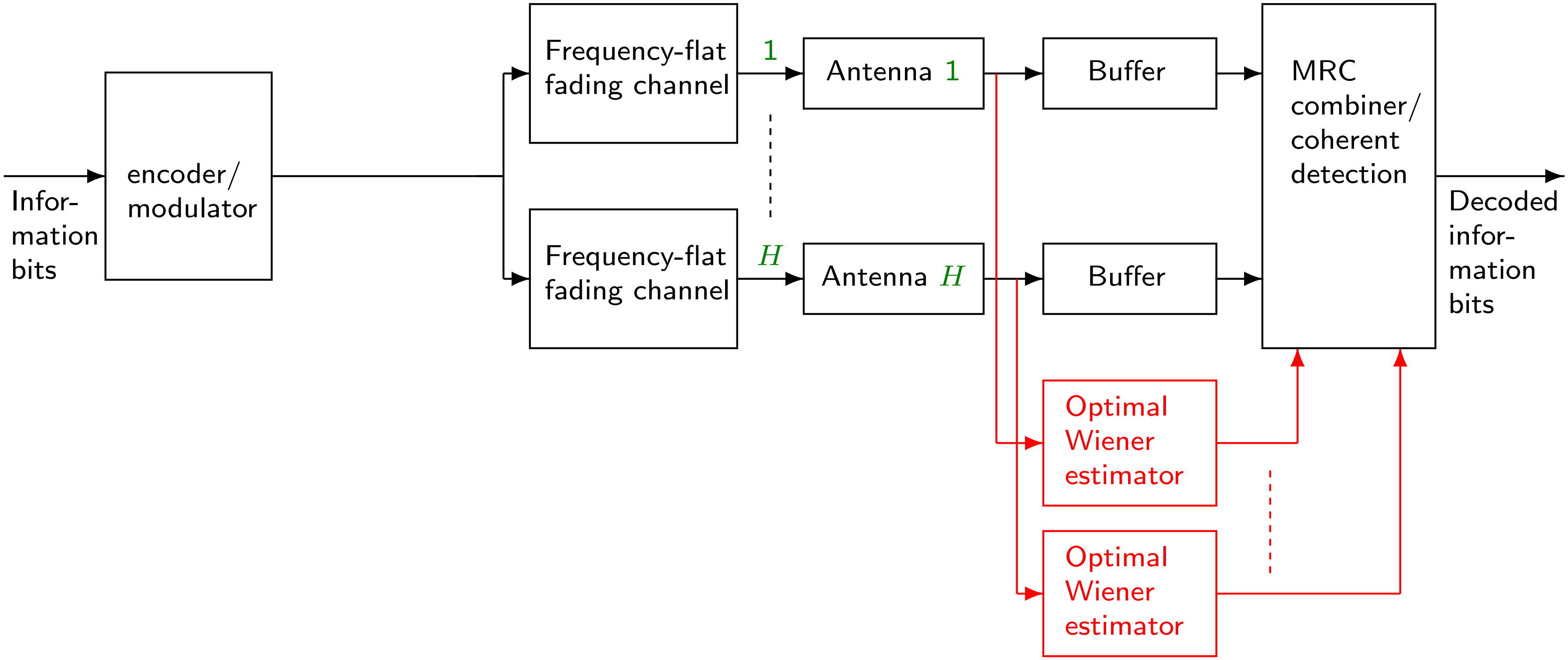
- Simple communication system

ACM system



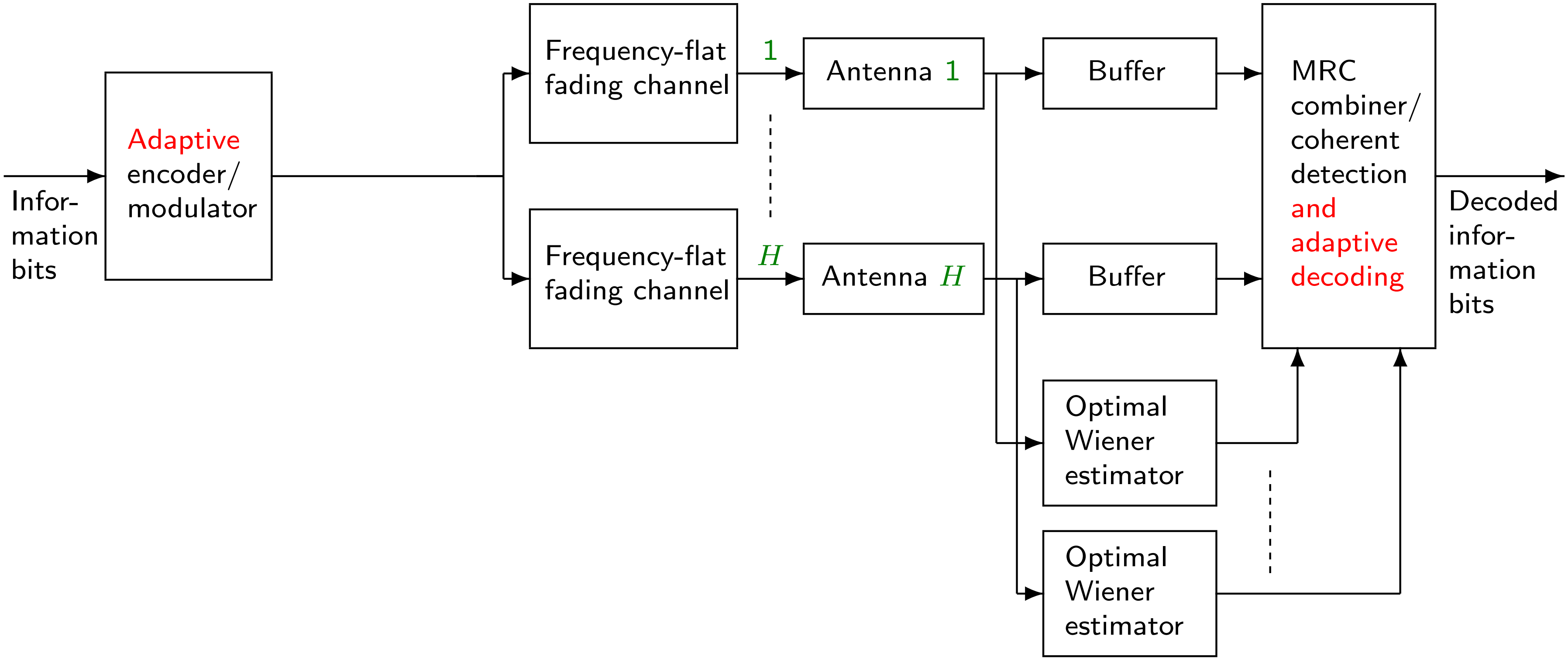
- Communication system with antenna diversity

ACM system



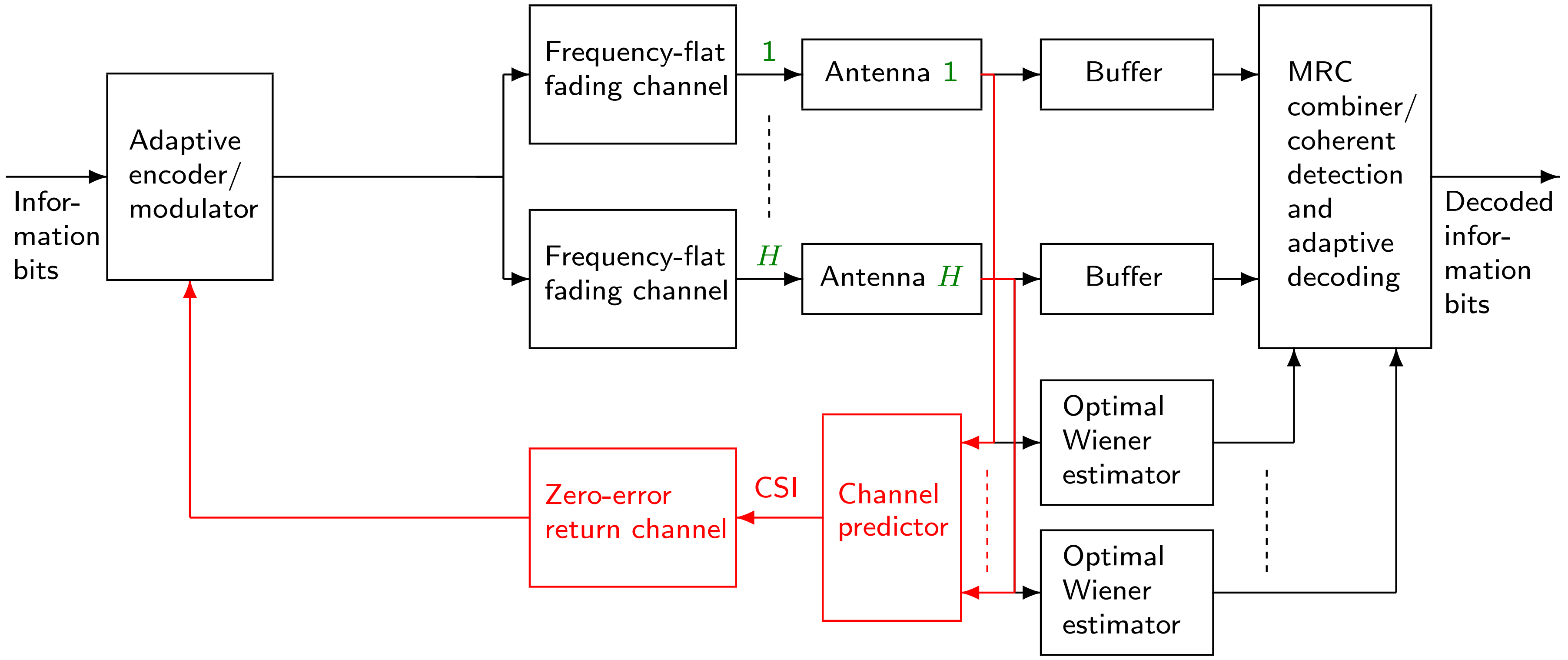
- Needs channel estimation

ACM system



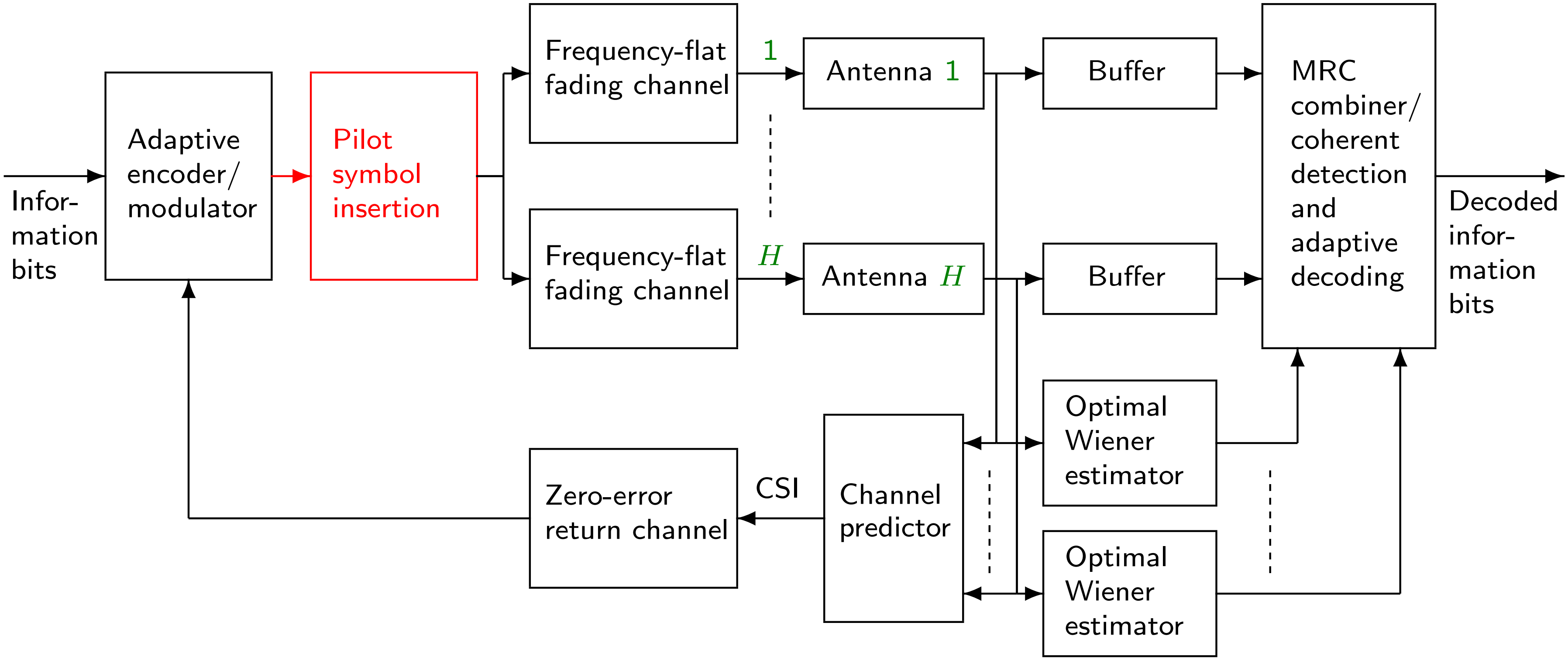
- Adaptive modulation

ACM system



- Needs feedback channel

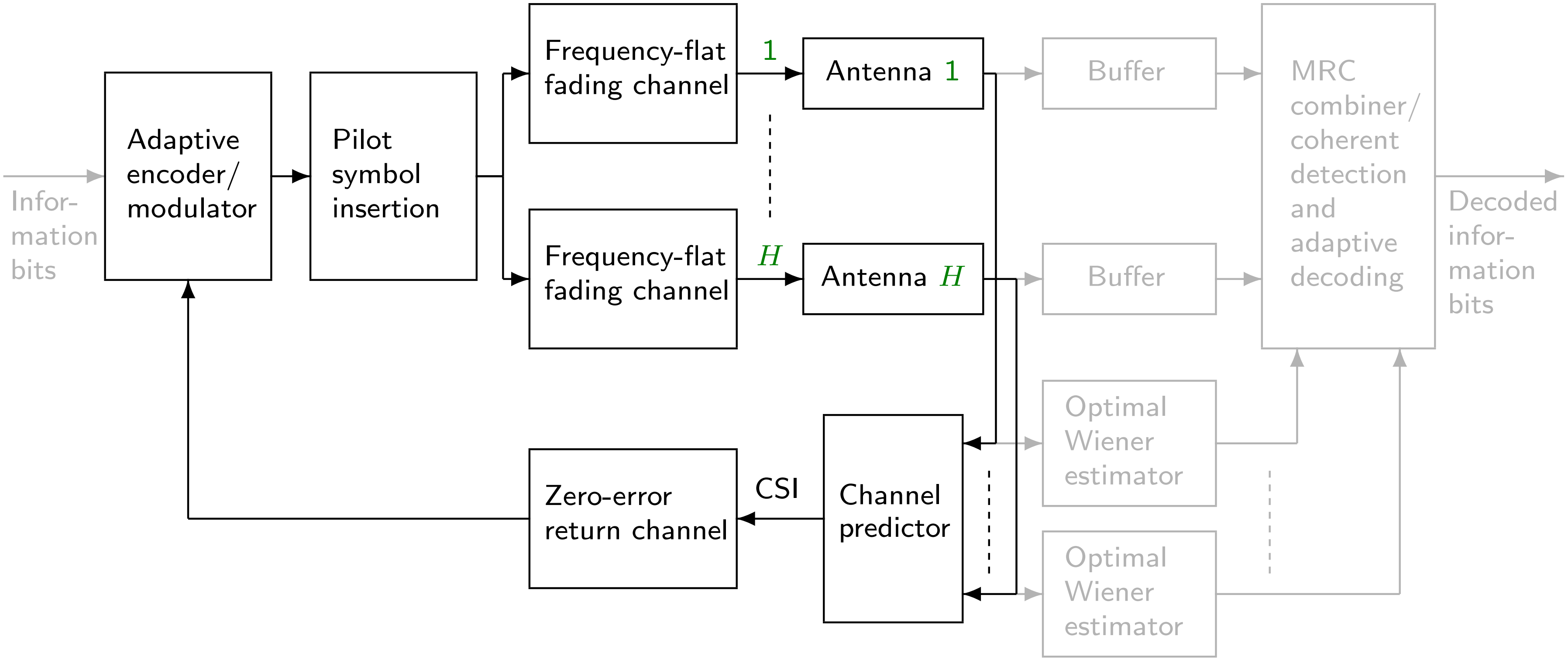
ACM system



- Pilot symbols:

  Better channel prediction (and estimation)

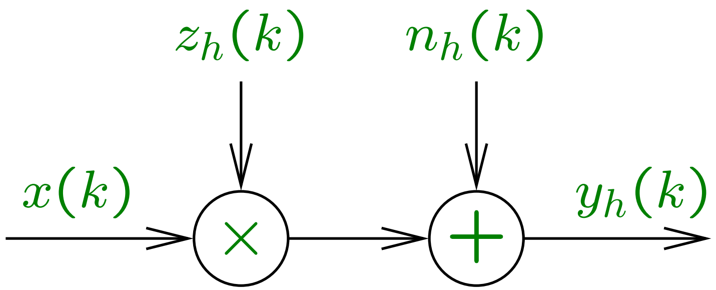
ACM system



- Will concentrate on
  - Effect of antenna diversity
  - Adaptive modulator
  - (Pilot symbol assisted) channel predictor
  - Feedback channel

Channel model

- Flat fading on subchannels:



- Received signal: $y_h(k) = z_h(k)x(k) + n_h(k)$
- $n_h(k)$ white, complex Gaussian
- $z_h(k)$ time-varying, complex Gaussian

  $\Rightarrow \alpha_h(k) = |z_h(k)|$ Rayleigh-distributed

  $\Rightarrow$ SNR $\gamma_h(k) = \frac{\alpha_h^2(k) \cdot P}{N_0 B}$ exponentially distributed

  (P: transmit power, N_0: noise spectral density, B: bandwidth)

Antenna diversity—Maximal ratio combining

- Receiver has H antennas
- Signal from each antenna is combined in an optimum manner: Maximal ratio combining (MRC)
- With MRC: $\gamma(k) = \sum_{h=1}^{H} \gamma_h(k)$
- What is the distribution of $\gamma(k)$?

Antenna diversity—MRC (cont'd)

- $\gamma_h(k)$ exponentially distributed with mean $\overline{\gamma}_h$
  $\Leftrightarrow$ $\gamma_h(k)$ gamma-distributed w/shape parameter 1
  & scale parameter $\overline{\gamma}_h$ (in short: $\gamma_h(k) \sim \mathcal{G}(1, \overline{\gamma}_h)$)

- γ_h exponentially distributed:

$$f_{\gamma_h}(\gamma_h) = \frac{e^{-\gamma_h/\overline{\gamma}_h}}{\overline{\gamma}_h}, \quad \gamma_h > 0$$

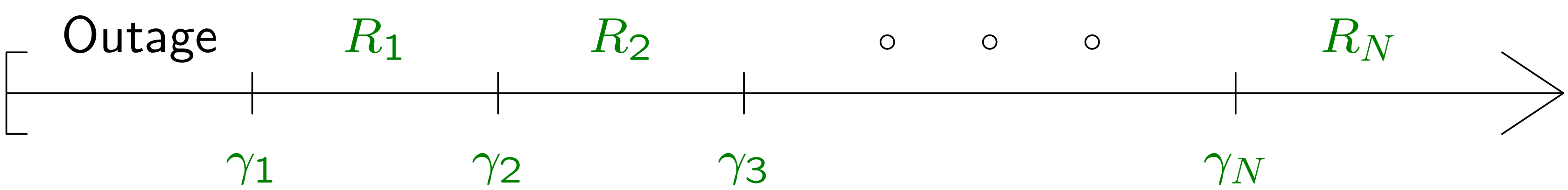
- $\gamma_h(k) \sim \mathcal{G}(1, \overline{\gamma}_h)$, $\quad \gamma(k) = \sum_{h=1}^{H} \gamma_h(k)$
  $\Rightarrow$ $\boxed{\gamma(k) \sim \mathcal{G}(H, \overline{\gamma}_h)}$ (with mean: $\overline{\gamma} = H\overline{\gamma}_h$)

- γ gamma-distributed:

$$f_\gamma(\gamma) = \frac{\gamma^{H-1} e^{-\gamma/\overline{\gamma}_h}}{\Gamma(H)\overline{\gamma}_h^H}, \quad \gamma > 0$$

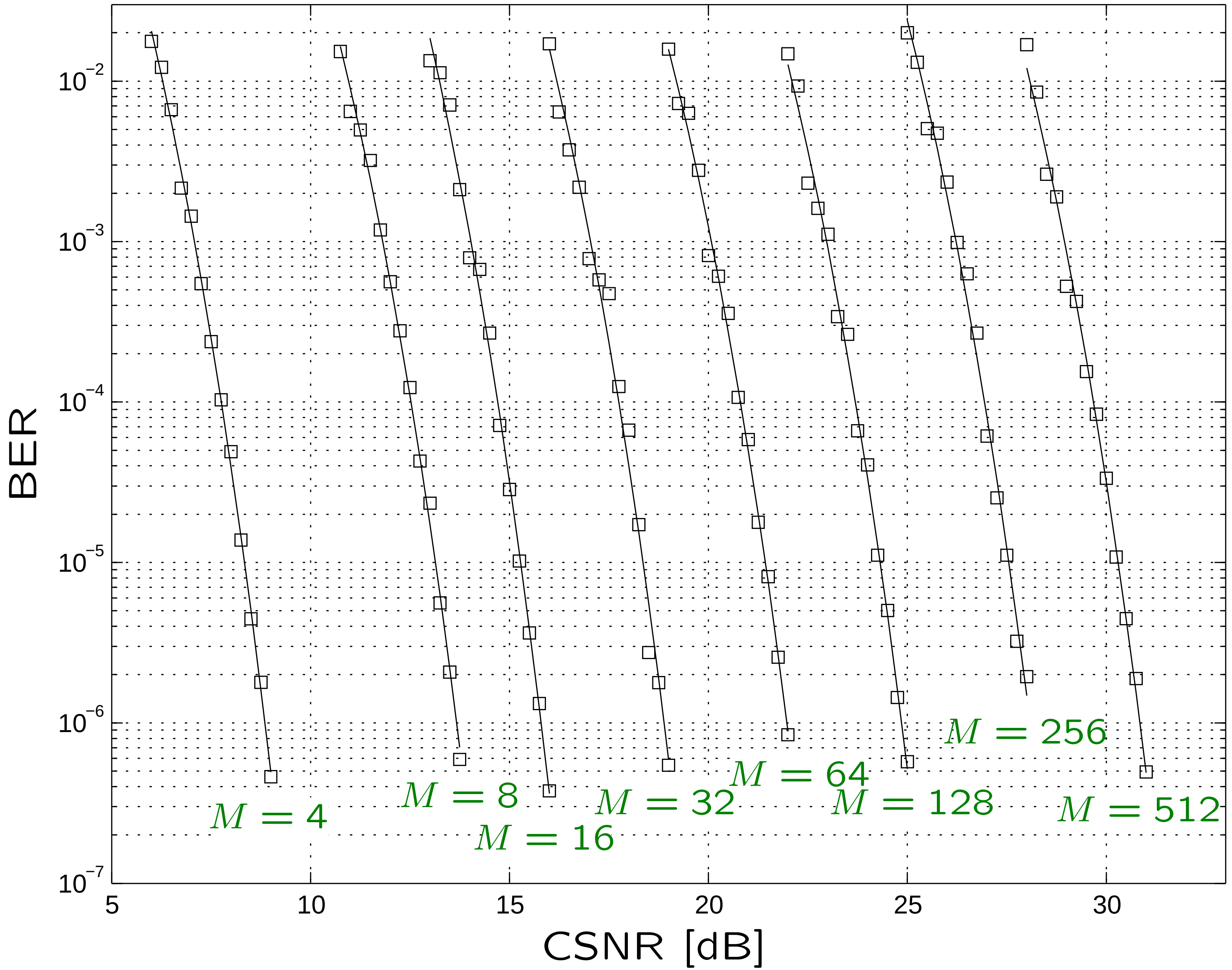
Adaptive coded modulation revisited

- Based on a set of N codes with different rate and different BER–SNR relationship
- Designer specifies a "target BER", BER_0
- Quantizing SNR range: $N + 1$ regions



- Thresholds γ_n: SNR where BER for each code equal to BER_0
- Tx attempts to use biggest code meeting the BER_0 requirement

ACM—BER simulation



Channel state information

- Tx must have information on the SNR ("CSI") (and so must the receiver (Rx))
- Ideally: Tx, Rx have perfect channel knowledge
- Instantaneous SNR estimated at Rx, then sent back to Tx
- Error sources:
  - Estimation error
  - Feedback delay
  - Errors in feedback channel

$\Rightarrow$ Tx may utilize erroneous CSI

CSI (cont'd)

- Delayed/erroneous CSI: True SNR might be in lower interval than estimated SNR
- Will affect BER
- Channel prediction will improve performance

Channel prediction

- Every L'th channel symbol: pilot with known magnitude a_p
- From noisy pilot symbols: maximum likelihood (ML) estimates (for each branch h)

$$\widetilde{z}_h(i) = z_h(i) + \frac{n_h(i)}{a_p}$$

- With collection of ML estimates

$$\widetilde{\mathbf{z}}_{h,i} = [\widetilde{z}_h(i), \widetilde{z}_h(i-L), \ldots, \widetilde{z}_h(i-(K-1)L)]^{\mathsf{T}}$$

and prediction filter $\mathbf{f}_{j,h}^{\mathsf{H}}$:

$$\boxed{\widehat{z}_h(i+j) = \mathbf{f}_{j,h}^{\mathsf{H}} \widetilde{\mathbf{z}}_{h,i}} \quad \Rightarrow \quad \widehat{\gamma}_h = |\widehat{z}_h|^2 \frac{P}{N_0 B} \quad \Rightarrow \quad \boxed{\widehat{\gamma} = \sum_{h=1}^{H} \widehat{\gamma}_h}$$

Maximum a priori (MAP) prediction

- We emphasize:
  - want to have a good estimate ($\widehat{\gamma}$) of the true SNR (γ)
  - send pilot symbols with interval L, obtain ML estimates $\widetilde{z}_h(i)$
  - do prediction via prediction filter $\mathbf{f}_{j,h}^{\mathsf{H}}$
  - $\widehat{z}_h(i+j) = \mathbf{f}_{j,h}^{\mathsf{H}} \widetilde{\mathbf{z}}_{h,i}$

⇒ To get good estimate:
need good filter coefficients $\mathbf{f}_{j,h}^{\mathsf{H}}$

MAP prediction (cont'd)

- Maximum a posteriori (MAP) criterion:

  The MAP-optimal estimate of a channel parameter (z) based on a set of observations ($\widetilde{\mathbf{z}}$) is the estimator that maximizes the PDF $f_{z|\widetilde{\mathbf{z}}}(z \mid \widetilde{\mathbf{z}})$

$$\widehat{z}_{\mathsf{MAP}} = \arg\max_{z} f_{z|\widetilde{\mathbf{z}}}(z \mid \widetilde{\mathbf{z}})$$

MAP prediction (cont'd)

- Can show that, for Gaussian RVs, $\widehat{z}_{\mathsf{MAP}}$ is

$$\widehat{z}_{\mathsf{MAP}} = \mathbf{r}_j^{\mathsf{T}} \left(\mathbf{R} + \frac{1}{\overline{\gamma}_h} \mathbf{I} \right)^{-1} \widetilde{\mathbf{z}} \quad \Rightarrow \quad \mathbf{f}_{j,\mathsf{MAP}}^{\mathsf{T}} = \mathbf{r}_j^{\mathsf{T}} \left(\mathbf{R} + \frac{1}{\overline{\gamma}_h} \mathbf{I} \right)^{-1}$$

- $\mathbf{r}_j$ is the correlation between fading at pilot symbol instants and at instant we are trying to predict:

$$\mathbf{r}_j = \frac{1}{\Omega} E[\mathbf{z}_h z_h^*(n+j)], \qquad \Omega = \mathsf{Var}(z_h)$$

- $\mathbf{R}$ is the autocorrelation between fading at pilot symbol instants:

$$\mathbf{R} = \frac{1}{\Omega} \mathsf{Cov}(\mathbf{z}_h, \mathbf{z}_h) = \frac{1}{\Omega} E[\mathbf{z}_h \mathbf{z}_h^{\mathsf{H}}]$$

PDF of $\widehat{\gamma}$

- Remember: $\gamma \sim \mathcal{G}(H, \overline{\gamma}_h)$
- What about $\widehat{\gamma}$?

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  $\Rightarrow$ $\widehat{\gamma}_h$ exponentially distributed

  $\Rightarrow$ $\widehat{\gamma}$ gamma-distributed

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- What about $\widehat{\gamma}$?
- Since $\widehat{z}_h$ is a linear combination of Gaussian random variables (RVs), it is itself a Gaussian RV

  $\Rightarrow$ $\widehat{\gamma}_h$ exponentially distributed

  $\Rightarrow$ $\widehat{\gamma}$ gamma-distributed
- Caveat: $\widehat{z}_h$ is biased estimator

$\Rightarrow$ $\widehat{\gamma} \nsim \mathcal{G}(H, \overline{\gamma}_h)$, instead: $\widehat{\gamma} \sim \mathcal{G}(H, r\overline{\gamma}_h)$

Joint PDF of γ and $\widehat{\gamma}$

- γ and $\widehat{\gamma}$ are both gamma-distributed
  - same shape factor (H)
  - different scale factors ($\overline{\gamma}_h$ and $r\overline{\gamma}_h$)
  - correlation coefficient $\rho = \dfrac{\mathrm{Cov}(\widehat{\gamma}, \gamma)}{\sqrt{\mathrm{Var}(\widehat{\gamma})\,\mathrm{Var}(\gamma)}}$
- Governed by joint PDF $f_{\gamma,\widehat{\gamma}}(\gamma, \widehat{\gamma})$

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$$f_{\gamma,\widehat{\gamma}}(\gamma,\widehat{\gamma}) = \frac{(\gamma\widehat{\gamma})^{(H-1)/2}}{\Gamma(H)(\overline{\gamma}_h r\overline{\gamma}_h)^{(H+1)/2}(1-\rho)\rho^{(H-1)/2}} \times \exp\left(-\frac{\gamma/\overline{\gamma}_h + \widehat{\gamma}/(r\overline{\gamma}_h)}{1-\rho}\right) \times I_{H-1}\left(\frac{2\sqrt{\rho}}{1-\rho}\sqrt{\frac{\gamma\widehat{\gamma}}{\overline{\gamma}_h r\overline{\gamma}_h}}\right), \quad \gamma,\widehat{\gamma} > 0$$

System analysis

- Important performance merits:
- Average spectral efficiency (ASE)
  - Expected throughput from the system?
- Bit error rate (BER)
  - "Robustness" of the system
  - In this context: target BER_0

  $\Rightarrow$ When does the system operate as required?
- Probability of no transmission (P_{out})
  - Which percentage of time will none of the codes be able to guarantee BER_0?

Average spectral efficiency (ASE)

- Analysis principle: analyze each of the codes, weight with employment probability
- Average spectral efficiency:

$$\text{ASE} = \sum_{n=1}^{N} R_n P_n$$

- R_n: Information rate of code n:

$$R_n = (\log_2(M_n) - 1/G) \cdot \frac{L-1}{L}$$

Remember:
pilot every L channel symbols

- P_n: Probability that code n is employed

$$P_n = Q\Big(H, \frac{\gamma_n}{r\bar{\gamma}_h}\Big) - Q\Big(H, \frac{\gamma_{n+1}}{r\bar{\gamma}_h}\Big)$$

  – $Q(a,z) = \frac{1}{\Gamma(a)} \int_z^{\infty} t^{a-1} e^{-t}\, dt$;
    Normalized, incomplete Gamma function

Probability of no transmission (P_{out})

- P_{out} is the probability that the system decides that the channel quality is so low that nothing should be transmitted

- Analysis principle: When the instantaneous SNR falls below the lowest threshold, γ_1, one should not transmit

- "Probability that code n is employed" formula P_n (from previous slide) can be used:

$$P_{out} = P_0 = 1 - Q\Big(H, \frac{\gamma_1}{r\overline{\gamma}_h}\Big)$$

Bit error rate (BER)

- Analysis principle: Average number of faulty bits divided by total average number of transmitted bits

$$\overline{\mathrm{BER}} = \frac{\sum_{n=1}^{N} R_n \cdot \overline{\mathrm{BER}}_n}{\sum_{n=1}^{N} R_n P_n},$$

  where $\overline{\mathrm{BER}}_n$ is the average BER experienced when code n is in use.

- R_n and P_n known from previous slides
- Must calculate $\overline{\mathrm{BER}}_n$

Average BER for code n, $\overline{\mathrm{BER}}_n$

$$\overline{\mathrm{BER}}_n = \int_{\gamma_n}^{\gamma_{n+1}} f_{\widehat{\gamma}}(\widehat{\gamma}) \int_0^{\infty} f_{\gamma|\widehat{\gamma}}(\gamma \mid \widehat{\gamma})\, \mathrm{BER}_n(\gamma \mid \widehat{\gamma})\, d\gamma\, d\widehat{\gamma}$$

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$$= \int_{\gamma_n}^{\gamma_{n+1}} \int_0^{\infty} f_{\gamma,\widehat{\gamma}}(\gamma, \widehat{\gamma}) \text{BER}_n(\gamma \mid \widehat{\gamma})\, d\gamma d\widehat{\gamma}$$

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Need expression for $\text{BER}_n(\gamma \mid \widehat{\gamma})$

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Need expression for $\text{BER}_n(\gamma \mid \widehat{\gamma})$

$$\text{BER}_n(\gamma \mid \widehat{\gamma}) = \begin{cases} a_n \cdot \exp\left(-\frac{b_n \gamma}{M_n}\right) & \text{when } \gamma \geq \gamma_n^l \\ \frac{1}{2} & \text{when } \gamma < \gamma_n^l \end{cases}$$

$\overline{\text{BER}}_n$ (cont'd)

Final expression for $\overline{\text{BER}}_n$—sum of three expressions:

$$\overline{\text{BER}}_n = \mathcal{I}1(n) - (\mathcal{I}21(n) - \mathcal{I}22(n))$$

$\mathcal{I}1$, $\mathcal{I}21$, and $\mathcal{I}22$ stem from dividing the double integral into three regions, and calculating each of these. They can be solved to provide the following expressions:

$\overline{\text{BER}}_n$ (cont'd)

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$$\mathcal{I}1(n) = a_n \left(\frac{1}{\frac{b_n \overline{\gamma}_h}{M_n} + 1} \right)^H \times \left[Q\left(H, \frac{\gamma_n}{\overline{\gamma}_h r} \cdot \frac{\frac{b_n \overline{\gamma}_h}{M_n} + 1}{(1-\rho)\frac{b_n \overline{\gamma}_h}{M_n} + 1}\right) - Q\left(H, \frac{\gamma_{n+1}}{\overline{\gamma}_h r} \cdot \frac{\frac{b_n \overline{\gamma}_h}{M_n} + 1}{(1-\rho)\frac{b_n \overline{\gamma}_h}{M_n} + 1}\right) \right]$$

$\overline{\text{BER}}_n$ (cont'd)

$$\mathcal{I}21(n) = a_n \sum_{k=0}^{\infty} \frac{\Gamma(k+H)}{\Gamma(k+1)\Gamma(H)} \left(\frac{\rho}{1-\rho}\right)^k \left(\frac{1}{\frac{b_n \overline{\gamma}_h}{M_n} + \frac{1}{1-\rho}}\right)^{k+H}$$

$$\times \left[1 - Q\left(k+H, \gamma_n^l \left(\frac{b_n}{M_n} + \frac{1}{(1-\rho)\overline{\gamma}_h}\right)\right)\right]$$

$$\times \left[Q\left(k+H, \frac{\gamma_n}{(1-\rho)\overline{\gamma}_h r}\right) - Q\left(k+H, \frac{\gamma_{n+1}}{(1-\rho)\overline{\gamma}_h r}\right)\right]$$

and

$$\mathcal{I}22(n) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{\Gamma(k+H)}{\Gamma(k+1)\Gamma(H)} \rho^k (1-\rho)^H$$

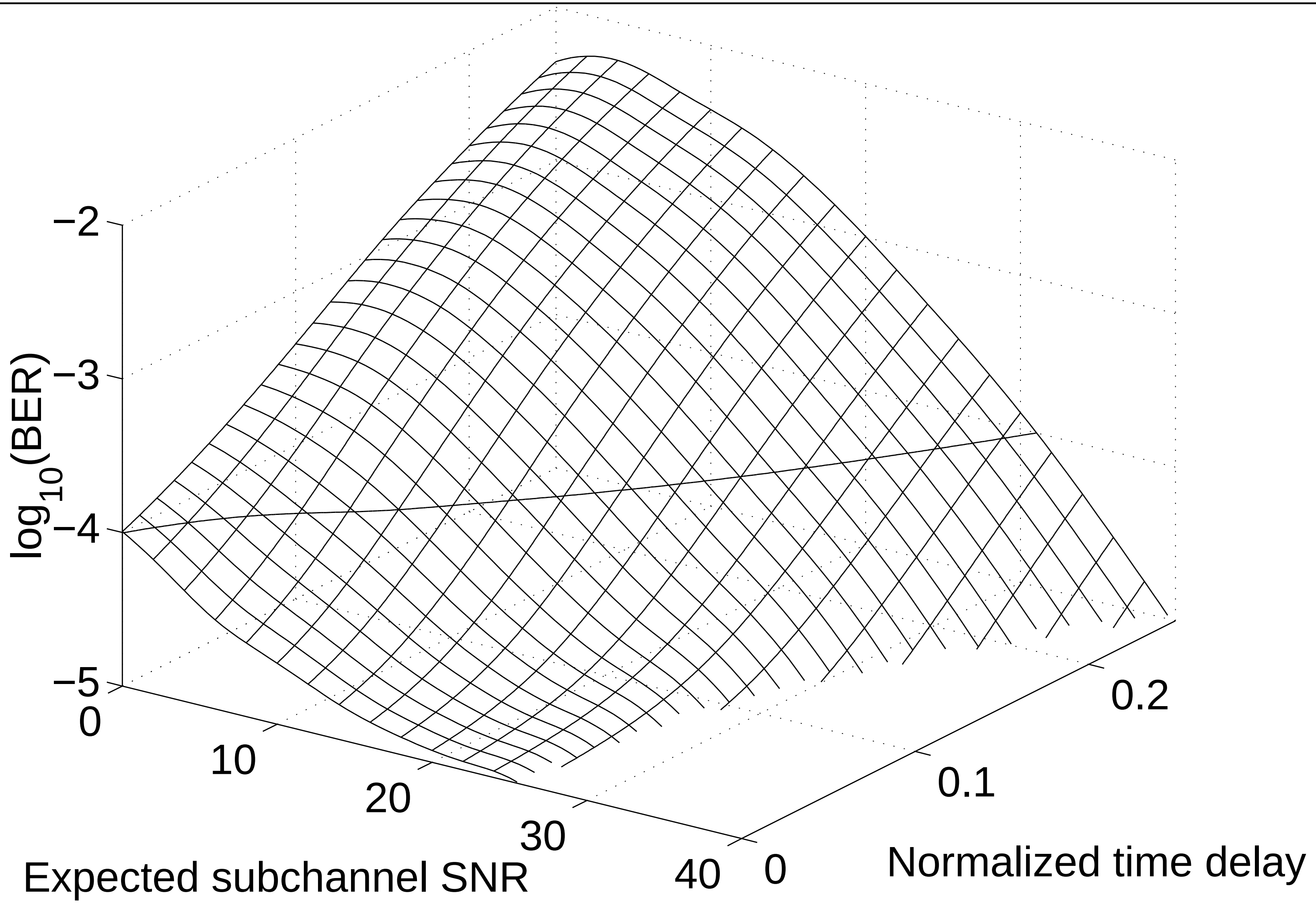
$$\times \left[1 - Q\left(k+H, \frac{\gamma_n^l}{(1-\rho)\overline{\gamma}_h}\right)\right]$$

$$\times \left[Q\left(k+H, \frac{\gamma_n}{(1-\rho)\overline{\gamma}_h r}\right) - Q\left(k+H, \frac{\gamma_{n+1}}{(1-\rho)\overline{\gamma}_h r}\right)\right]$$

Some results

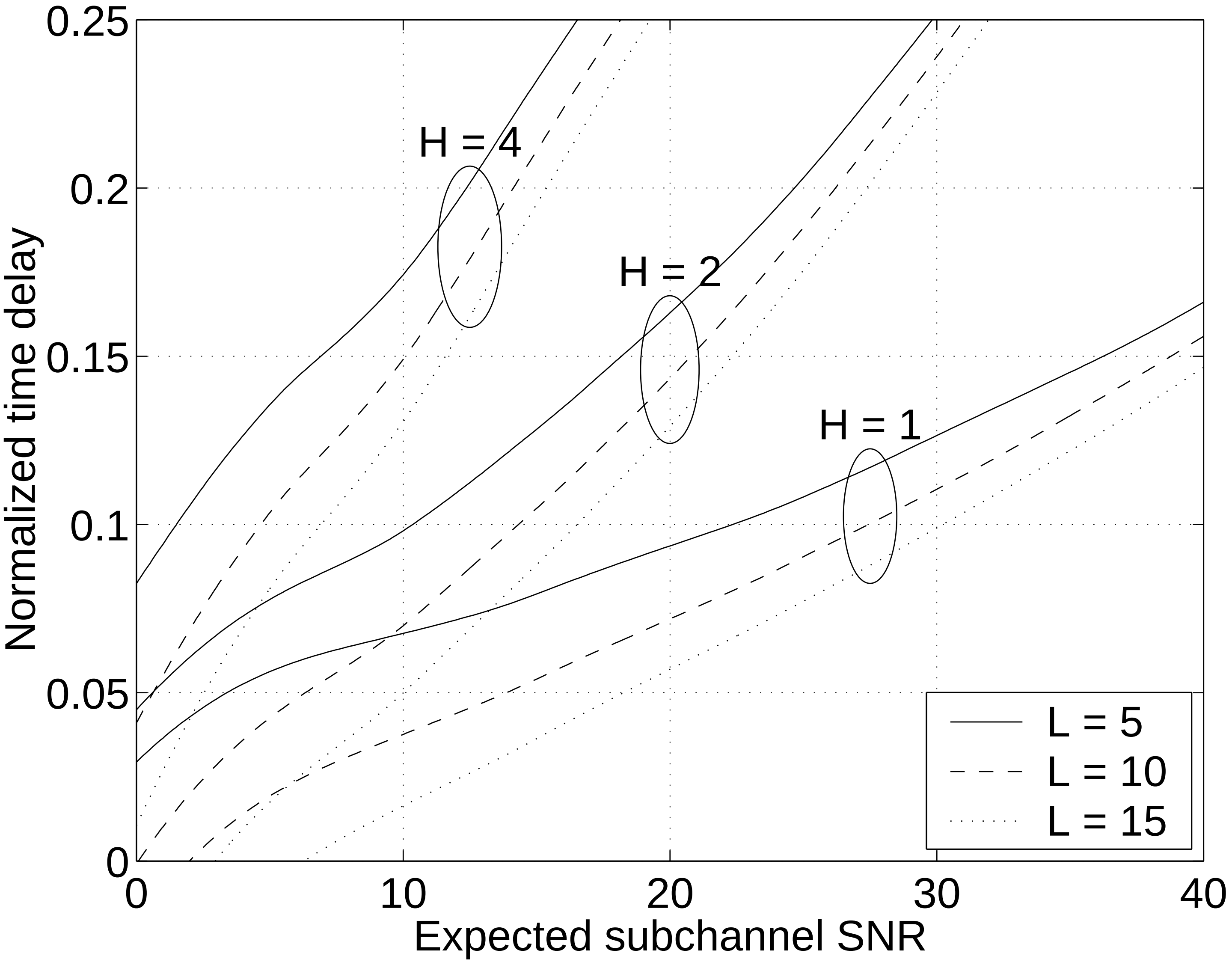
- Time for some results!
- Have plotted the expressions on the previous pages for an example system

BER I



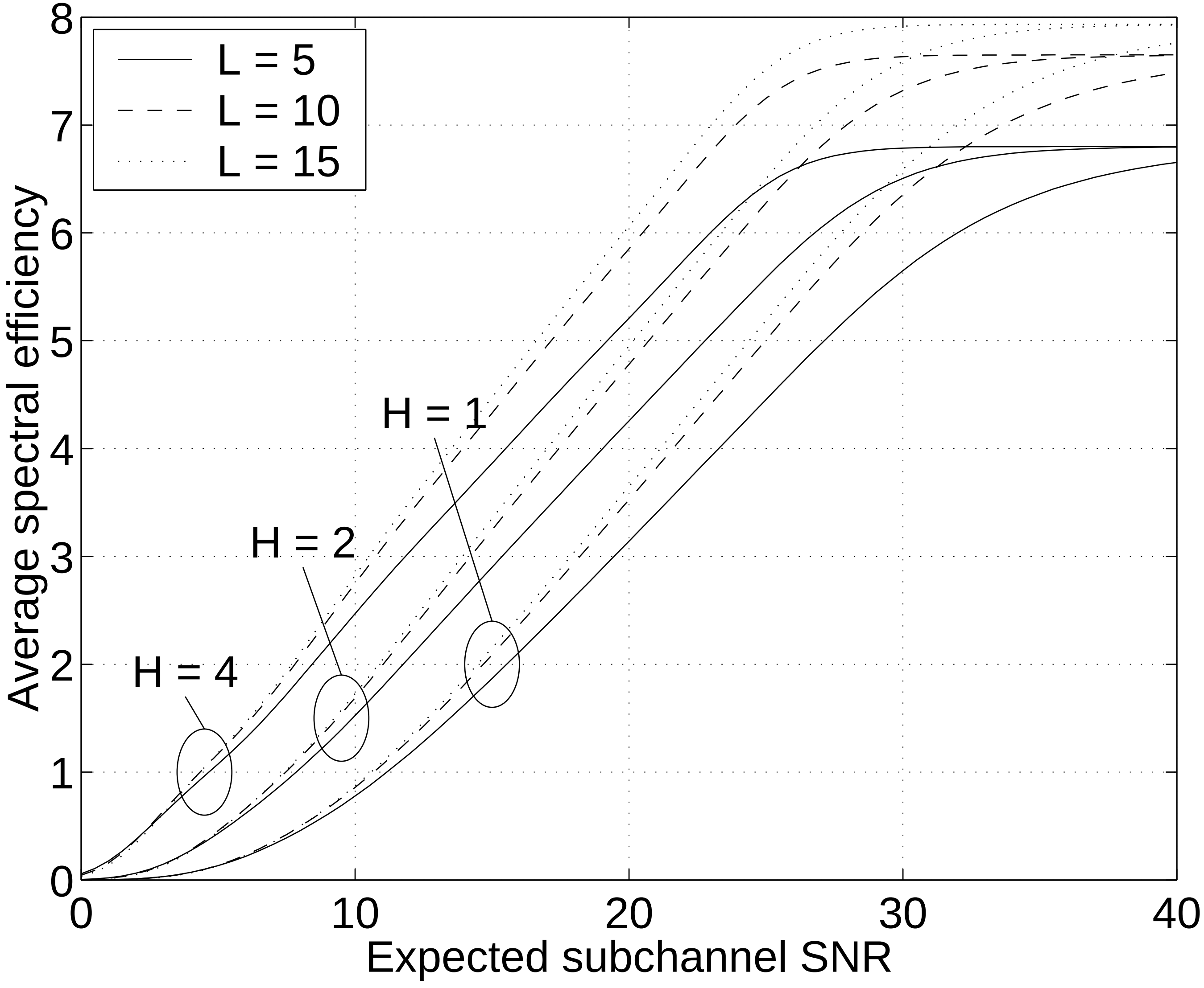
- $H = 2$ receive antennas; pilot symbol spacing $L = 10$

BER II

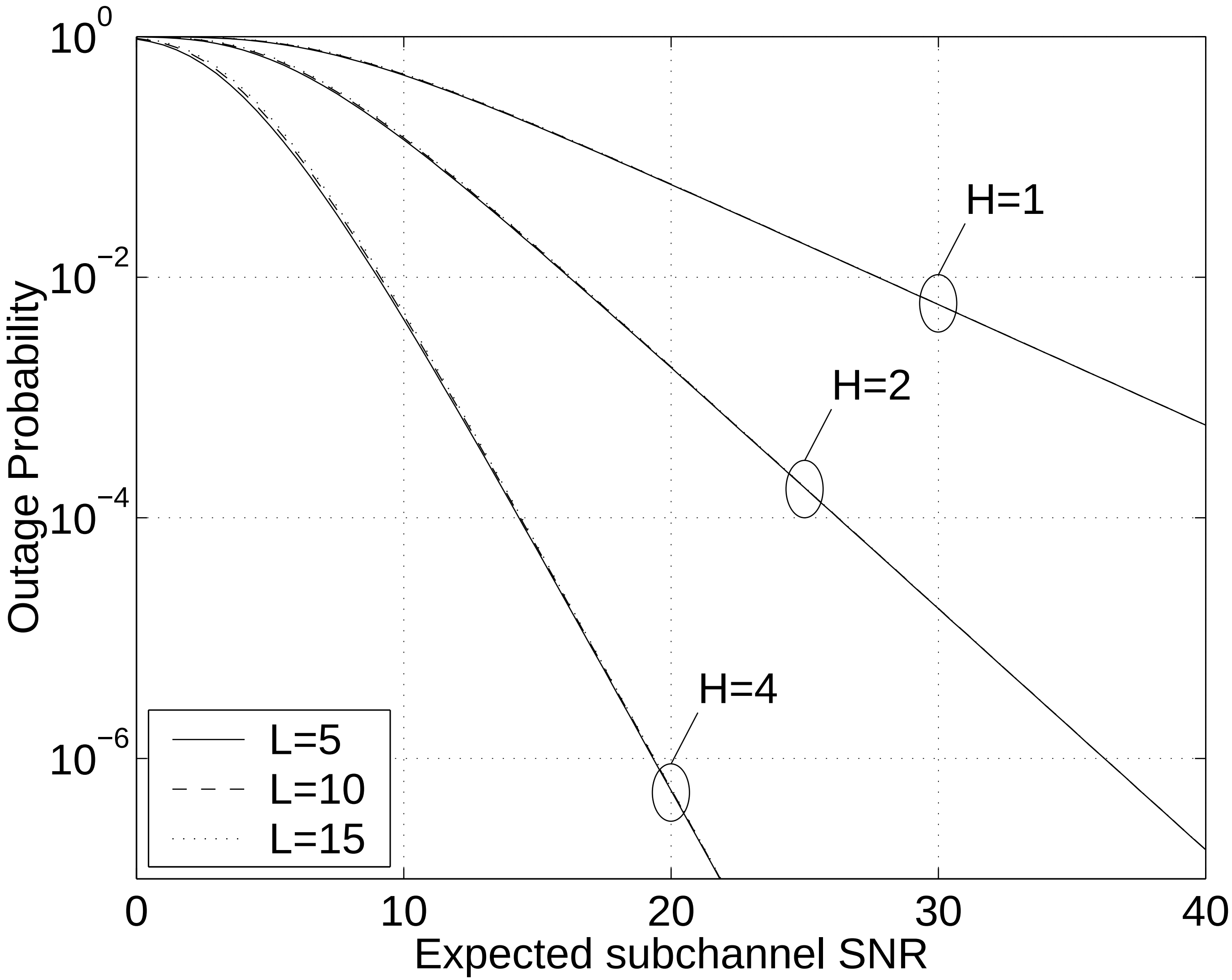


0.25
0.2
0.15
0.1
0.05
0
Normalized time delay
0
10
20
30
40
Expected subchannel SNR
H = 4
H = 2
H = 1
L = 5
L = 10
L = 15

ASE



- normalized delay 0.25



- normalized delay 0.25