Adaptive coded modulation: channel prediction

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http://www.tele.ntnu.no/projects/beats/course.htm
Outline

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- System overview
- Channel model
- Antenna diversity/Maximal ratio combining
- Adaptive coded modulation
- Channel prediction
- Maximum a priori prediction
- System analysis
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Motivation

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• Adaptive coded modulation (ACM): promising tool
  – Low SNR: transmit with low rate code
  – High SNR: transmit with high rate code
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  - Low SNR: transmit with low rate code
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- Transmitter (Tx) needs channel state information (CSI): system relies on feedback channel
- Since delay in the feedback channel: Use prediction!
• Simple communication system
● Communication system with antenna diversity
ACM system

- Needs channel estimation
ACM system

- Adaptive modulation
ACM system

- Needs feedback channel
ACM system

- Pilot symbols:
  Better channel prediction (and estimation)
Will concentrate on

- Effect of antenna diversity
- Adaptive modulator
- (Pilot symbol assisted) channel predictor
- Feedback channel
Channel model

- Flat fading on subchannels:

\[ z_h(k) \times n_h(k) \]

\[ x(k) \rightarrow \times \rightarrow + \rightarrow y_h(k) \]

- Received signal: \( y_h(k) = z_h(k)x(k) + n_h(k) \)

- \( n_h(k) \) white, complex Gaussian

- \( z_h(k) \) time-varying, complex Gaussian

\[ \Rightarrow \alpha_h(k) = |z_h(k)| \text{ Rayleigh-distributed} \]

\[ \Rightarrow \text{SNR } \gamma_h(k) = \frac{\alpha^2_h(k) \cdot P}{N_0 B} \text{ exponentially distributed} \]

(\( P \): transmit power, \( N_0 \): noise spectral density, \( B \): bandwidth)
Antenna diversity—Maximal ratio combining

- Receiver has $H$ antennas
- Signal from each antenna is combined in an optimum manner: Maximal ratio combining (MRC)
- With MRC: $\gamma(k) = \sum_{h=1}^{H} \gamma_h(k)$
- What is the distribution of $\gamma(k)$?
Antenna diversity—MRC (cont’d)

- $\gamma_h(k)$ exponentially distributed with mean $\bar{\gamma}_h$
  $\Leftrightarrow \gamma_h(k)$ gamma-distributed w/shape parameter 1
  & scale parameter $\bar{\gamma}_h$ (in short: $\gamma_h(k) \sim \mathcal{G}(1, \bar{\gamma}_h)$)

- $\gamma_h$ exponentially distributed:
  $$f_{\gamma_h}(\gamma_h) = \frac{e^{-\gamma_h/\bar{\gamma}_h}}{\bar{\gamma}_h}, \quad \gamma_h > 0$$

- $\gamma_h(k) \sim \mathcal{G}(1, \bar{\gamma}_h), \quad \gamma(k) = \sum_{h=1}^{H} \gamma_h(k)$
  $\Rightarrow \gamma(k) \sim \mathcal{G}(H, \bar{\gamma}_h)$ (with mean: $\bar{\gamma} = H\bar{\gamma}_h$)

- $\gamma$ gamma-distributed:
  $$f_{\gamma}(\gamma) = \frac{\gamma^{H-1}e^{-\gamma/\bar{\gamma}_h}}{\Gamma(H)\bar{\gamma}_h^H}, \quad \gamma > 0$$
Adaptive coded modulation revisited

- Based on a set of $N$ codes with different rate and different BER–SNR relationship
- Designer specifies a “target BER”, $BER_0$
- Quantizing SNR range: $N + 1$ regions

\[
\begin{array}{cccccc}
\text{Outage} & R_1 & R_2 & \ddots & \ddots & R_N \\
\gamma_1 & \gamma_2 & \gamma_3 & \gamma_N
\end{array}
\]

- Thresholds $\gamma_n$: SNR where BER for each code equal to $BER_0$
- Tx attempts to use biggest code meeting the $BER_0$ requirement
ACM—BER simulation

![Graph showing BER vs. CSNR for different values of M.](image)

- 
- $M = 4$
- $M = 8$
- $M = 16$
- $M = 32$
- $M = 64$
- $M = 128$
- $M = 256$
- $M = 512$

CSNR [dB]
Channel state information

• Tx must have information on the SNR ("CSI") (and so must the receiver (Rx))

• Ideally: Tx, Rx have perfect channel knowledge

• Instantaneous SNR estimated at Rx, then sent back to Tx

• Error sources:
  – Estimation error
  – Feedback delay
  – Errors in feedback channel

⇒ Tx may utilize erroneous CSI
CSI (cont’d)

- Delayed/erroneous CSI: True SNR might be in lower interval than estimated SNR
- Will affect BER
- Channel prediction will improve performance
Channel prediction

- Every $L$’th channel symbol: pilot with known magnitude $a_p$

- From noisy pilot symbols: maximum likelihood (ML) estimates (for each branch $h$)
  \[
  \tilde{z}_h(i) = z_h(i) + \frac{n_h(i)}{a_p}
  \]

- With collection of ML estimates
  \[
  \tilde{z}_{h,i} = [\tilde{z}_h(i), \tilde{z}_h(i - L), \ldots, \tilde{z}_h(i - (K - 1)L)]^T
  \]

  and prediction filter $f_{j,h}^H$:

  \[
  \hat{z}_h(i + j) = f_{j,h}^H \tilde{z}_{h,i} \quad \Rightarrow \quad \hat{\gamma}_h = |\hat{z}_h|^2 \frac{P}{N_0 B} \quad \Rightarrow \quad \hat{\gamma} = \sum_{h=1}^{H} \hat{\gamma}_h
  \]
Maximum a priori (MAP) prediction

- We emphasize:
  - want to have a good estimate ($\hat{\gamma}$) of the true SNR ($\gamma$)
  - send pilot symbols with interval $L$, obtain ML estimates $\tilde{z}_h(i)$
  - do prediction via prediction filter $f_{j,h}^H$

\[ \hat{z}_h(i + j) = f_{j,h}^H \tilde{z}_{h,i} \]

⇒ To get good estimate:
  need good filter coefficients $f_{j,h}^H$
• Maximum a posteriori (MAP) criterion:

The MAP-optimal estimate of a channel parameter \( z \) based on a set of observations \( \tilde{z} \) is the estimator that maximizes the PDF \( f_{z|\tilde{z}}(z | \tilde{z}) \)

\[
\hat{z}_{\text{MAP}} = \arg \max_{\tilde{z}} f_{z|\tilde{z}}(z | \tilde{z})
\]
MAP prediction (cont’d)

- Can show that, for Gaussian RVs, $\hat{z}_{\text{MAP}}$ is

$$\hat{z}_{\text{MAP}} = r_j^T \left( R + \frac{1}{\gamma_h} \mathbf{I} \right)^{-1} \tilde{z} \Rightarrow f_{j,\text{MAP}} = r_j^T \left( R + \frac{1}{\gamma_h} \mathbf{I} \right)^{-1}$$

- $r_j$ is the correlation between fading at pilot symbol instants and at instant we are trying to predict:

$$r_j = \frac{1}{\Omega} E[z_h z^*_h(n + j)], \quad \Omega = \text{Var}(z_h)$$

- $R$ is the autocorrelation between fading at pilot symbol instants:

$$R = \frac{1}{\Omega} \text{Cov}(z_h, z_h) = \frac{1}{\Omega} E[z_h z_h^H]$$
PDF of $\hat{\gamma}$

- Remember: $\gamma \sim \mathcal{G}(H, \overline{\gamma}_h)$

- What about $\hat{\gamma}$?
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- Since $\hat{z}_h$ is a linear combination of Gaussian random variables (RVs), it is itself a Gaussian RV
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- What about $\hat{\gamma}$?

- Since $\hat{z}_h$ is a linear combination of Gaussian random variables (RVs), it is itself a Gaussian RV
  $\Rightarrow \hat{\gamma}_h$ exponentially distributed
  $\Rightarrow \hat{\gamma}$ gamma-distributed
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  $\Rightarrow \hat{\gamma}_h$ exponentially distributed
  
  $\Rightarrow \hat{\gamma}$ gamma-distributed

- Caveat: $\hat{z}_h$ is biased estimator
  
  $\Rightarrow \hat{\gamma} \sim G(H, \bar{\gamma}_h)$, instead: $\hat{\gamma} \sim G(H, r\bar{\gamma}_h)$
Joint PDF of $\gamma$ and $\hat{\gamma}$

- $\gamma$ and $\hat{\gamma}$ are both gamma-distributed
  - same shape factor ($H$)
  - different scale factors ($\gamma_h$ and $r\gamma_h$)
  - correlation coefficient $\rho = \frac{\text{Cov}(\hat{\gamma}, \gamma)}{\sqrt{\text{Var}(\hat{\gamma}) \text{Var}(\gamma)}}$

- Governed by joint PDF $f_{\gamma,\hat{\gamma}}(\gamma, \hat{\gamma})$
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$$f_{\gamma,\hat{\gamma}}(\gamma, \hat{\gamma}) = \frac{(\gamma \hat{\gamma})^{(H-1)/2}}{\Gamma(H)(\gamma_h r\gamma_h)^{(H+1)/2}(1 - \rho)(\rho(H-1)/2}}$$

$$\times \exp\left(-\frac{\gamma/\gamma_h + \hat{\gamma}/(r\gamma_h)}{1 - \rho}\right)$$

$$\times I_{H-1}\left(\frac{2\sqrt{\rho}}{1 - \rho} \sqrt{\frac{\gamma \hat{\gamma}}{\gamma_h r\gamma_h}}\right), \quad \gamma, \hat{\gamma} > 0$$
System analysis

- Important performance merits:
  - Average spectral efficiency (ASE)
    - Expected throughput from the system?
  - Bit error rate (BER)
    - “Robustness” of the system
    - In this context: target BER$_0$
    $\Rightarrow$ When does the system operate as required?
  - Probability of no transmission ($P_{\text{out}}$)
    - Which percentage of time will none of the codes be able to guarantee BER$_0$?
Average spectral efficiency (ASE)

- Analysis principle: analyze each of the codes, weight with employment probability

- Average spectral efficiency:

\[
ASE = \sum_{n=1}^{N} R_n P_n
\]

- \(R_n\): Information rate of code \(n\):

\[
R_n = (\log_2(M_n) - 1/G) \cdot \frac{L - 1}{L}
\]

- \(P_n\): Probability that code \(n\) is employed

\[
P_n = Q(H, \frac{\gamma_n}{r\gamma_h}) - Q(H, \frac{\gamma_{n+1}}{r\gamma_h}) - Q(a, z) = \frac{1}{\Gamma(a)} \int_{z}^{\infty} t^{a-1} e^{-t} dt;
\]

Normalized, incomplete Gamma function
**Probability of no transmission** (\(P_{out}\))

- \(P_{out}\) is the probability that the system decides that the channel quality is so low that nothing should be transmitted.

- Analysis principle: When the instantaneous SNR falls below the lowest threshold, \(\gamma_1\), one should not transmit.

- “Probability that code \(n\) is employed” formula \(P_n\) (from previous slide) can be used:

\[
P_{out} = P_0 = 1 - Q(H, \frac{\gamma_1}{r \gamma_h})
\]
Bit error rate (BER)

- Analysis principle: Average number of faulty bits divided by total average number of transmitted bits

\[ \overline{BER} = \frac{\sum_{n=1}^{N} R_n \cdot \overline{BER}_n}{\sum_{n=1}^{N} R_n P_n}, \]

where \( \overline{BER}_n \) is the average BER experienced when code \( n \) is in use.

- \( R_n \) and \( P_n \) known from previous slides

- Must calculate \( \overline{BER}_n \)
Average BER for code $n$, $\overline{\text{BER}}_n$

$$\overline{\text{BER}}_n = \int_{\gamma_n}^{\gamma_{n+1}} f_{\hat{\gamma}}(\hat{\gamma}) \int_0^\infty f_{\gamma|\hat{\gamma}}(\gamma | \hat{\gamma}) \text{BER}_n(\gamma | \hat{\gamma}) \, d\gamma \, d\hat{\gamma}$$
Average BER for code $n$, $\overline{BER}_n$

$$\overline{BER}_n = \int_{\gamma_n}^{\gamma_n+1} f_{\hat{\gamma}}(\hat{\gamma}) \int_0^{\infty} f_{\gamma|\hat{\gamma}}(\gamma | \hat{\gamma}) \, BER_n(\gamma | \hat{\gamma}) \, d\gamma \, d\hat{\gamma}$$

$$= \int_{\gamma_n}^{\gamma_n+1} \int_0^{\infty} f_{\gamma,\hat{\gamma}}(\gamma, \hat{\gamma}) \, BER_n(\gamma | \hat{\gamma}) \, d\gamma d\hat{\gamma}$$
Average BER for code $n$, $\overline{BER}_n$

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\]

Need expression for $\overline{BER}_n(\gamma | \hat{\gamma})$
Average BER for code $n$, $\overline{\text{BER}}_n$

$$\overline{\text{BER}}_n = \int_{\gamma_n}^{\gamma_{n+1}} f_{\gamma}(\gamma) \int_{0}^{\infty} f_{\gamma|\gamma}(\gamma | \gamma) \text{BER}_n(\gamma | \gamma) \, d\gamma \, d\tilde{\gamma}$$

$$= \int_{\gamma_n}^{\gamma_{n+1}} \int_{0}^{\infty} f_{\gamma,\tilde{\gamma}}(\gamma, \tilde{\gamma}) \text{BER}_n(\gamma | \tilde{\gamma}) \, d\gamma \, d\tilde{\gamma}$$

Need expression for $\text{BER}_n(\gamma | \tilde{\gamma})$

$$\text{BER}_n(\gamma | \tilde{\gamma}) = \begin{cases} a_n \cdot \exp\left(-\frac{b_n \gamma}{M_n}\right) & \text{when } \gamma \geq \gamma_n^l \\ \frac{1}{2} & \text{when } \gamma < \gamma_n^l \end{cases}$$
BER
(continue)

Final expression for BER
— sum of three expressions:

\[ BER_n = I_1(n) - (I_21(n) - I_22(n)) \]

\( I_1, I_21, \) and \( I_22 \) stem from dividing the double integral into three regions, and calculating each of these. They can be solved to provide the following expressions:
Final expression for $\text{BER}_n$—sum of three expressions:

$$\text{BER}_n = I_1(n) - (I_{21}(n) - I_{22}(n))$$

$I_1$, $I_{21}$, and $I_{22}$ stem from dividing the double integral into three regions, and calculating each of these. They can be solved to provide the following expressions:

$$I_1(n) = a_n \left( \frac{1}{b_n \gamma_h + 1} \right)^H$$

$$\times \left[ Q \left( H, \frac{\gamma_n}{\gamma_h r} \cdot \frac{b_n \gamma_h + 1}{(1 - \rho) b_n \gamma_h + 1} \right) - Q \left( H, \frac{\gamma_{n+1}}{\gamma_h r} \cdot \frac{b_n \gamma_h + 1}{(1 - \rho) b_n \gamma_h + 1} \right) \right]$$
\[ \mathcal{I}_{21}(n) = a_n \sum_{k=0}^{\infty} \frac{\Gamma(k + H)}{\Gamma(k + 1) \Gamma(H)} \left( \frac{\rho}{1 - \rho} \right)^k \left( \frac{1}{b_n \bar{\gamma}_h + \frac{1}{1 - \rho}} \right)^{k+H} \times \left[ 1 - Q \left( k + H, \gamma_n \left( \frac{b_n}{M_n} + \frac{1}{(1 - \rho)\bar{\gamma}_h} \right) \right) \right] \times \left[ Q \left( k + H, \frac{\gamma_n}{(1 - \rho)\bar{\gamma}_h r} \right) - Q \left( k + H, \frac{\gamma_n + 1}{(1 - \rho)\bar{\gamma}_h r} \right) \right] \]

and

\[ \mathcal{I}_{22}(n) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{\Gamma(k + H)}{\Gamma(k + 1) \Gamma(H)} \rho^k (1 - \rho)^H \times \left[ 1 - Q \left( k + H, \frac{\gamma_n}{(1 - \rho)\bar{\gamma}_h} \right) \right] \times \left[ Q \left( k + H, \frac{\gamma_n}{(1 - \rho)\bar{\gamma}_h r} \right) - Q \left( k + H, \frac{\gamma_n + 1}{(1 - \rho)\bar{\gamma}_h r} \right) \right] \]
Some results

- Time for some results!
- Have plotted the expressions on the previous pages for an example system
- $H = 2$ receive antennas; pilot symbol spacing $L = 10$
• normalized delay 0.25
$P_{out}$

- normalized delay 0.25