‘MULTIPLES’: A Challenging Learning Framework for the Generation of Multiple Perspectives within e-Collaboration Settings

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Abstract. This chapter proposes a learning framework - the MULTIPLES framework (Multiple: Learning Tools, Interactions, Perspectives, Learning Experiences and Solution Strategies) - that can be used in e-collaboration settings to encourage the development of multiple perspectives for each individual student regarding the learning concepts in focus. This framework has been designed within the context of social and constructivist learning theories, acknowledging the role of asking learners, working in both groups and individually, to face appropriately designed learning tasks by using multiple learning tools and representation systems while at the same time performing various interactions in order to produce multiple solution strategies ‘*in as many ways as possible’*. To this end, a case study is reported that illuminates the role played by MULTIPLES in the enhancement of each individual student’s views by generating different solution strategies to the tasks at hand, while at the same time expressing their inter- and intra-individual differences. Specifically, 25 secondary level education students participated in a learning experiment consisting of two tasks asking for the construction of multiple solution strategies regarding the concept of area measurement by exploiting the plethora of tools and representation systems provided by the well-known educational software Cabri-Geometry II (Laborde, 1990). The analysis of the data shows that all these students truly exploited their collaborative experience during the former task and constructed various solution strategies when individually facing the second task, at the same time developing multiple perspectives of the aforementioned learning concepts.

1 Introduction

Recent advances in Internet and Web-based technologies have provided educators with great opportunities to reconceptualize learning by extending both its boundaries and pedagogies in terms of: learners’ communication capabilities, diversity of learning interactions, releasing learning from time and space, contributing towards equal learning opportunities, and providing schools with opportunities to exchange learning experiences, so they can acquire new perspectives and broaden their social horizons [13, 28, 4]. Most importantly, the Internet has been widely recognized as a medium that allows for the design of learning environments based on

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modern constructivist and social theories regarding teaching and learning [12, 17, 9]. Distributed and situated cognition views of knowledge construction can also be considered to interpret learning events within the framework of networking technologies [33, 36, 7].

Within the context of these modern theories, enhancing learner perspectives in terms of the learning concepts in focus, while at the same time acknowledging their inter- and intra-individual learning differences, is essential [27]. To motivate learners to develop such diverse learning perspectives, the role of appropriately-designed learning activities is crucial. To this end, a collaborative learning framework - the MULTIPLES framework - has been formed. This framework is situated firmly within the context of modern constructivist and social theories of learning, where the learning process is viewed as an active, subjective and constructive activity within authentic contexts rich in computer learning tools and communication interactions [40, 31, 18]. Specifically, within the MULTIPLES framework, learning is emphasized in a context supporting: a) the learning of essential aspects of each learning subject [19], b) open multiple solution-based authentic learning activities [16, 20, 38, 22], c) the expression of learners' inter- and intra-individual differences [27, 24], d) active learning using multiple computer learning tools and representation systems [21], e) the generation of multiple solution strategies to the tasks at hand by both group and individual learners, f) multiple ways of interaction and communication [13], g) appropriate intrinsic and extrinsic feedback for self-correction [32], and h) multiple ways of assessment [18]. It is worth mentioning that collaborative learning frameworks supporting the performance of multiple solution-based tasks asking to be solved '*in as many ways as possible*', while at the same time exploiting the availability of multiple learning tools and representation systems, have yet not been reported.

The role of the proposed framework was investigated through its use in the design of a learning experiment – using real students - for the learning of concepts related to conservation of area and its measurement by secondary level education students. These concepts are viewed as being essential to students' mathematical learning [14]. This chapter is organized as follows: in the next section, the background of MULTIPLES is presented and, subsequently, its architecture is demonstrated. Then, the context of the aforesaid specific learning experiment is reported while the emerging empirical results are discussed in relation to the specifications of the proposed architecture of MULTIPLES. The chapter ends with the conclusions.

2 Background

E-Learning can be defined as an open and distributed environment that provides learning tools, enabled by Internet and Web-based technologies, to facilitate knowledge building through diverse, new and meaningful learning actions and interactions such as learner-learner, learner-diverse groups, learner-various type of resources, learner-instructor, learner-expert and learner-cognitive tools. In fact, within the Web-based context, learning can be considered as a function of a plethora of interactions with others and with various tools to face appropriately-designed tasks. In this context, learning can also be reconceptualized as a social and distributed process over time and place, both synchronously and asynchronously, where the concepts of learning in groups, sharing information, meaning negotiation and co-construction of knowledge are dominant, and the concept of distance is not significant. However, the importance and necessity of linking appropriate learning theories in the design of e-learning environments has been acknowledged by many researchers, despite the aforementioned challenging teaching and learning capabilities recently facilitated in such environments [6, 1].

Understanding the coordination among individuals and artifacts in a system or community is a main principle of distributed or situated cognition in terms of knowledge construction [33, 36]. This view suggests that cognition is not an event that takes place inside one’s head in isolation, but can be looked at as a distributed phenomenon that is more global in nature—one that goes beyond the boundaries of a person to include environment, artifacts, social interactions, and culture [35]. Subsequently, it is acknowledged that individuals learn from activity and the tools supporting it to extend their cognitive capabilities [30]. The situated cognition view is consistent with the epistemological assumptions of constructivism, and social views of learning which stipulate that meaning is a function of how the individual creates meaning from his or her experiences and interactions within authentic contexts [16, 18]. In fact, promoting authentic learning activities is a central aspect of constructivist learning that is also viewed as an active, subjective and constructive process. On the whole, it is worth noting that the context or activity that frames knowledge construction is of equal importance to the learner as the knowledge itself [6].

Authentic activities engage learners in realistic and meaningful tasks that are relevant to each learner’s interests and goals, motivating him to be actively involved in their learning [41, 30]. Authentic learning tasks emphasize the encouragement of learners to explore different solution strategies while simultaneously forming various hypotheses and observing their effects [3, p. 135]. Hypothesis generation and exploration goes hand by hand with the acquisition of problem-solving and decision-making skills potentially essential in a learner’s personal and professional life [6]. Main characteristics of the problem-solving approach also involve students working collaboratively in small groups, analysing and brainstorming ideas that could lead to a solution to a problem [10]. However, appropriately-designed learning activities should also be complex enough to require the cooperation of all members in order to work toward a solution, while also being open-ended and containing the content objectives of the course.

Social views of learning also suggest that contexts rich in cognitive tools and social interactions are constantly challenging the learner’s understanding, resulting in new meanings [31]. Learners are provided with the opportunity to transform their experience of performing authentic learning activities within collective socio-cultural settings into new knowledge [34]. Various studies have also reported the positive effects of collaboration within Web-based learning settings in different fields and disciplines [8, 29]. In collaboration and social negotiation, the goal is to collaborate to face the given learning activities and to share different viewpoints and ideas [5].

In fact, communication within the context of Web-based settings can encourage learners to acquire a rich and robust knowledge base by developing multiple perspectives regarding the learning concepts in focus through the promotion, articulation and negotiation of - and reflection on - different and contrasting views. Novices can also develop their competences by exploiting their collaboration experience within the aforementioned settings. In fact, when students are encouraged to articulate their knowledge to one another, they can share multiple perspectives and make generalizations that can be applicable in a number of different contexts [3]. Promoting students' reflective thinking involves asking them to review, analyze and concentrate on their experience in order to extract the most important and useful points of view on both the learning concept in focus and the process of learning itself [3]. It is also worth noting that, when students are working with peers instead of alone, anxiety and uncertainty are reduced as they collectively find a way to face the given tasks and they find these tasks interesting and satisfying [13].

Promoting the development of multiple perspectives during collaborative learning settings is a learning approach that provides opportunities for the construction of flexible and meaningful knowledge structures [5, p. 178], at the same time encouraging the expression of learners' inter- and intra-individual differences [24]. Here, it is worth noting that student inequality in learning and achievement at school has been linked to their inter-individual learning differences [37, 39]. Most learner difficulties are found in the gap between their intuitive knowledge and the knowledge they need to express themselves in the proposed representation systems [15]. For example, prepositional, symbolic and abstract representation systems prevent some learners (usually beginners) from expressing their knowledge, the same systems being intended for use by advanced learners. Being provided with opportunities to express different perspectives on the task at hand, learners are also given opportunities to express their own knowledge, including their mistakes, as well as to master more than one learning approach. To this end, the use of multpile representation systems is acknowledged as crucial in encouraging the expression of learners' inter- and intra-individual variety regarding the subject to be learned [15, 23].

Being exposed to multiple points of view, students also have the chance to develop broad views regarding the learning concepts in focus [22]. Essentially, emphasizing multiple perspectives involves not only presenting information in various ways but encouraging learners to use multiple learning tools and representation systems to construct their own multiple solution strategies for the tasks at hand and also document their own explanations. With the promotion of multiple perspectives, learners also have the chance to become aware that there are multiple approaches to an issue, which is the case in real life situations. In addition, learners have the chance to explore each perspective to seek a meaningful resolution to the issue at hand, constructing new meaning within the context of their own experiences and knowledge [6].

3 The General Architecture of MULTIPLES

In this section, architecture of a typical representative of MULTIPLES is presented. The construction of this architecture was based on modern constructivist and social theories mentioned in the previous section, also exploiting the advantages provided by the Computer Based/Internet Based technologies. On the grounds of these learning theories, a number of specifications were formulated and used in the construction of the said architecture, which consists of four main parts: (A) learning activities, (B), learner activity space (C), learner communication and (D) learner assessment. The description of the design of these parts is presented below:

Part A. *Learning activities***.** This part of the MULTIPLES framework includes basic specifications related to the design of constructivist learning activities suitable in a computer context. In terms of design specifications, these activities have to:
a) focus on both the fundamental aspects of the learning subject and the specific learning points where the learners illuminate difficulties, b) be drawn from the context of the learners' everyday life, c) encourage problem-solving skills (that is, stimulate learners' higher mental functions in terms of analytical and synthetical thinking skills as well as critical thinking), d) promote collaboration and social negotiation, e) be solved in multiple ways using different aspects of the subject matter, as well as various representation systems, and also exploit the different kinds of knowledge that students possess, such as their previous knowledge, school-knowledge, intuitive knowledge, real life knowledge, visual knowledge, etc, f) demand each group construct solutions 'in as many ways as possible' by exploiting the provision of multiple tools and representation systems as well as the diversity of students' knowledge, g) demand each individual student perform similar tasks to those performed while working in groups, also 'in as many ways as possible' (in this way, each student would be able to exploit the advantages of his/her participation within the groupwork), h) allow students control over their learning (i.e. the activities can be solved using representation systems that provide appropriate feedback, such as intrinsic visual and/or explicit numerical), with learners being able to reflect on the feedback of their actions and then have the opportunity to correct their solution strategies), i) encourage learners to experiment by handling primary sources of data, at the same time acquiring hands-on experience, and j) not demand from learners extra complicated knowledge from other disciplines.

Part B. *Learner activity space***.** This part of MULTIPLES includes specifications for the design of a *'learner activity space'*, that is, a virtual place where the learners can actively construct their own knowledge by performing the selected group of learning activities, using various cognitive tools. These tools have to be closely related to the specific learning subject and also appropriate to help learners perform the most representative and essential activities for the learning of this subject. The main aim of the integration of various cognitive tools within the 'activity space' is to provide learners with opportunities to: a) perform different solution strategies to the selected learning activities, thereby expressing their inter- and intra-individual learning differences, b) perform the same solution strategy using different representation systems, c) solve various activities for the learning of each specific learning subject, d) overcome basic difficulties regarding each specific learning subject, e) choose from among these tools those most appropriate for the expression of their knowledge, f) express different kinds of knowledge they possess, thereby developing a broad view of the concepts in focus.

Part C. *Learner communication***.** Students have to be provided with various tools in order for them to be able to perform various communicative interactions, such as learner-learner, learner-teacher, learner-experts, learner-diverse groups, learner-whole class interactions. These interactions can be also performed synchronously and asynchronously. Learners can be encouraged to articulate their opinions and to reflect on both their experience and that presented by their colleagues. Learners can also be encouraged to share and negotiate their knowledge with their classmates. Small and whole class discussions can also be encouraged. To assist learners in performing all these communications, the learning framework can support a large number of synchronous and asynchronous conferences as well as publication of group/individual work and, of course, the use of e-mail.

Part D. *Learner assessment***.** This part of the design of MULTIPLES includes specifications in relation to the formation of assessment activities. From the constructivist perspective, assessment can become a valuable tool for learning, the emphasis being placed on both group and each individual's learning processes and not exclusively on her/his learning outcomes. Multiple solution-based activities to be performed 'in as many ways as possible' can be used as a basic structural element of learner assessment. Thus, a combination of methods can be used, such as: a) self-assessment by reflecting on intrinsic and/or extrinsic feedback appropriately provided by the electronic learning environment in use, b) peer-to-peer-assessment by explaining, articulating and negotiating the solution strategies proposed by each individual student in front of both their groups and the whole class, and c) assessment by the teacher in terms of acknowledging the variety, the kind and the number of solution strategies constructed by both the groups and each individual student. This procedure could be realized by providing learners with opportunities to add electronic portfolios, posting their work over an extended period.

4 Using MULTIPLES with Real Students: A Case Study

Aim and methodology: The aim of this experiment is to investigate, through a comparative study, the role of the MULTIPLES learning framework in the development of each individual student's knowledge concerning the mathematical notion of area. To this end, students were provided with the opportunity to interact within the rich context of tools offered by the well-known educational software Cabri-Geometry II to face 'in as many ways as possible' two similar multiple solution-based problems in both a collaborative learning setting (1st task) and a setting where the emphasis is put on the actions of each individual student (2nd task). Such a comparative study has not yet been reported. In terms of methodology, this research is a qualitative study [2] focusing on the variety of solution strategies realized by students working with Cabri tools.

Cabri Geometry II [26]**:** Within the context of this software, constructivist mathematical learning settings can be supported. In fact, Cabri is highly capable of facilitating the design of learning activities that encourage learners to take an investigative perspective, express their inter-individual and intra-individual learning differences, make self-corrections, and formulate and verify conjectures [25]. In addition, authentic, meaningful, real life learning activities can be integrated within the context of this software. In particular, Cabri provides students with potential opportunities in terms of: i) *A rich set of tools* to perform diverse geometrical constructions according to various concepts in Euclidean Geometry, *ii) Tools to construct a variety of representations, both numerical and visual*, such as geometrical figures, tables, equations, graphs and calculations. These representations are of different cognitive transparency; consequently, students can select the most appropriate to express their knowledge, iii) *Linking representations*, by exploiting the interconnection of the different representation modes provided, iv) *Dynamic, direct manipulation* of geometrical constructions, by using the ‘drag mode’ operation, enhancing their knowledge about the issue at hand by dynamically exploring the invariance of their constructions, e) *The possibility of collecting large amounts of numerical data*. These data can be used by the students to form and verify conjectures regarding the geometrical concepts in focus, f) *Interactivity and multiple types of feedback* (intrinsic visual and extrinsic numerical), providing learners with opportunities to form and verify conjectures as well as self-correct their constructions, g) *Presenting information to the students in various forms*, *h) capturing the history of student actions* to provide teachers and researchers with a valuable amount of data for further studies, and i*) Extension*. Certain operations could be added as buttons on the Cabri interface following the formation of specific macros.

*The learning experiment***:** The learning experiment took place in a typical, provincial, state secondary school in Patras, Greece [25]. Three complete classes of students participated in this experiment, consisting of: eight 1st grade students (13 years old), nine 2nd grade students (14 years old) and eight 3rd grade students (15 years old). These students were asked to perform two tasks using the Cabri II tools. The duration of tasks was commensurate with student needs; each student took about two hours to complete each task. Data resources include the electronic files of students’ visual geometrical constructions, the video recordings of all interactions performed and the field notes of the researcher.

The tasks: Two tasks were assigned, the first being: a) to ‘construct pairs of equivalent triangles, in as many ways as possible’ using Cabri tools, b) to ‘justify your solution strategy’ and c) to ‘explain what you think about the relation of the area and perimeter of these triangles’. Each student provided explanations and articulations of their solution strategies to her/his group as well as to the whole class. Whenever the students appeared to be on the point of giving up, the researcher would intervene, involving them in the task and encouraging them to continue by asking: ‘try another way of constructing another pair of triangles with equal areas. You can use other tools and the different kinds of knowledge you possess’. Students worked in groups of three to perform this task, so as to exploit the advantages of cooperation. The learning aims of this task were to enable students to: i) advance smoothly from the notion of congruent triangles to the notion of equivalent triangles, beginning with the expression of their previous knowledge of congruent triangles and then enhancing their knowledge by exploring the equivalence in triangles using the ‘drag mode’ operation, ii) distinguish the concepts of area and perimeter in triangles by studying these concepts in relation to each other, and iii) link different kinds of knowledge about the concept of area through using the diversity of the tools provided, at the same time developing a broad view of this concept.

In the second task, students were asked to 'construct a triangle and to perform any possible sequence of modifications to produce other triangles equivalent to the original'. Specifically, students were asked to: a) 'construct an original triangle, then modify it into another equivalent triangle, using the Cabri tools', b) 'justify your solution strategy' and c) 'consider the produced triangle as the original triangle and repeat (a) and (b) as many times as you can, using different ways of modification'. The researcher intervened by encouraging the students to continue, as before. Students worked individually while performing the second task. It was decided to investigate how each individual student had perceived the learning experience of the first task after participating in the aforementioned group activity. The additional learning aims of this task were to enable each individual student to: i) construct individual approaches to the concepts of area and perimeter in triangles, ii) integrate different pieces of knowledge they possessed regarding the concept of area, iii) develop a broad view of the concept of area and its invariance by constructing a class of triangles equivalent to an original triangle through a sequential process of modification.

Communication: Multiple ways of communication were encouraged, such as: learner-learner during team work, learner-whole class during whole class discussions and learner-teacher during the whole experiment. Students also interacted with the tools provided by Cabri and received feedback in multiple ways. The whole experiment took place in a face-to-face communication setting, so as to focus on the investigation of the role of collaboration using the previously mentioned type of tasks within contexts rich in cognitive tools, such as Cabri.

5 Results

Group and individual solution strategies used by the students to face both tasks were classified into ten categories. Some of these strategies lead to the construction of congruent triangles (those presented in Table 1) while some others lead to the construction of equivalent not exclusively congruent triangles (those presented in Table 2). A further description of these strategies in terms of their value in mathematics education is presented in [25].

At first glance at the results presented in both Tables indicates that students exploited the presence of the plethora of the tools provided by Cabri as well as their collaboration within both small groups and the whole class, and constructed different solution strategies to the tasks at hand, at the same time expressing different kind of knowledge as well as their inter-individual differences and forming multiple perspectives.

Table 1. Categories of group and individual solution strategies to construct congruent triangles in the context of Cabri-Geometry II

Categories of strategies performed by the students to construct congruent triangles in the context of Cabri-Geometry II
Strategies were based on…
S1: students’ visual perception and automatic control by measurement tools. *Tools used:* segment, automatic measurements of both area and perimeter.
S2: the preservation of the lengths or both; lengths and angles of the original triangle. *Tools used:* segment, automatic measurements of length, angle, area and perimeter.
S3: basic geometrical transformations such as: Translation (Strategy S3a), Reflection about an axis (Strategy S3b), Symmetry (Strategy S3c) and Rotation (Strategy S3d). *Tools used:* triangle, symmetry, reflection, rotate, translate as well as automatic measurements of both area and perimeter.
S4: the splitting of polygons: eg. Splitting: • an isosceles triangle using a perpendicular bisector (strategy S4a), • a rectangle & a square into two equivalent triangles by using one of its diagonals (strategy S4b), • a regular polygon into a number of equivalent triangles by using all its diagonals (strategy S4c), • a parallelogram into two equivalent triangles by using one of its diagonals (strategy S4d) *Tools used:* triangle, perpendicular line, median, polygon, regular polygon, parallel lines, circle as well as automatic measurements of both area and perimeter.
S5: the formation of specific geometrical constructions producing pairs of congruent triangles. *Tools used:* parallel and perpendicular lines, lines, segments, triangle, the ‘drag mode’ operation, rotation about an angle and around a point, automatic measurement of both area and perimeter, automatic tabulation of numerical data.

Specifically, students expressed intuitive approaches to area (strategy S1), spatial approaches in terms of area measurement using spatial units (strategy G4), formal approaches in terms of using the area formulae (strategies G2, G3, G5) and dynamic approaches (strategies G1, G2 and G5b) where they investigated the possibility of the existence of shapes with equal areas and different forms. Students also expressed their mistakes in terms of shapes of equal perimeters having equal areas (strategy S2). Finally, students related area to different kinds of knowledge they possessed, such as basic area transformations (strategies included in category S3), polygons (strategies fall into category S4) and specific geometrical constructions formed by using arbitrary, parallel and perpendicular lines (strategy S5). In performing all the aforementioned strategies, students also exploited the possibility of the direct manipulation of their constructions by using hands-on experience and formed/verified hypotheses and conjectures by reflecting on both the visual and numerical data automatically produced by the system. Students also exploited the multiple ways of feedback (intrinsic and numerical) provided by Cabri to verify their solution strategies at both group and individual levels. The feedback given by their teacher was also essential.

Table 2. Categories of group and individual solution strategies to construct equivalent not exclusively congruent triangles in the context of Cabri-Geometry II

Categories of strategies performed by the students to construct equivalent not exclusively congruent triangles in the context of Cabri-Geometry II
Strategies based on:
G1: the investigation of the possibility of the existence of equivalent triangles using the 'drag' mode in combination with automatic area measurement: *Tools used:* triangle, the 'drag mode' operation, as well as automatic measurements of both area and perimeter.
G2: the conservation of both; the length of the base and its distance from the opposite vertex in a triangle. *Tools used:* parallel lines, segment, point, triangle, the 'drag mode' operation, automatic measurement of both; area and perimeter, automatic tabulation of numerical data.
G3: the splitting of a triangle into two equivalent triangles using a median. *Tools used:* triangle, segment, median, automatic measurement of both area and perimeter.
G4: measuring areas using area-units: *Tools used:* triangle, square grid, segment, the 'drag mode' operation, median, automatic measurement of both area and perimeter.
G5: using area formulae. Constructing an original triangle ABC and trying to construct another triangle equivalent to the original by conserving the product of the lengths of its base and its respective altitude, eg. in right-angled triangles (Strategy G5a), in arbitrary triangles (Strategy G5b) or in an arbitrary triangle at the same time sliding the altitude on the base of the triangle. *Tools used:* triangle, perpendicular line, segment, measurement of length, calculation, , the 'drag mode' operation, median, automatic measurement of both; area and perimeter, measurement transfer, automatic measurement of both area and perimeter.

In the following section, the specific strategies constructed by both each group and each individual student are presented.

5.1 Group and Individual Multiple Solution Strategies to the Tasks at Hand Using the Tools Provided by Cabri-Geometry II

Group and individual multiple solution strategies to face both tasks are presented in Table 3.

Table 3. Group and individual multiple solution strategies performed within Cabri-Geometry II

Group and individual multiple solution strategies performed within Cabri-Geometry II						
Grades	**Group**	**Strategies performed by each group**	**Total Strat.**	**Students**	**Total Strat**	**Strategies performed by each student**
1st	***A1***	S3a, G1, G4, G2, G2, S4a, S4b, S3b, S3c, S5, G3	11	**P1**	6	S3a, G1, S3b, S2, G2
				P2	6	G1, S3a, S4a, G2
				P3	6	S3c, S1, S3d, G3, G2
	A2	S1, S4c, G2	3	**P4**	3	S1, S4c, G2
				P5	4	S1, G4, S3a, G2
	A3	G1, G2, S3b, S4c	4	**P6**	3	S3a, G1, S3b

Table 3. (*continued*)

				P7	7	S1, S3b, S4c, G1, G2 S3d
				P8	5	S3b, G4, S3d, G1, G2
2nd	**B1**	S3a,G1, G2, G4, G5a, S4d, G3	7	**P9**	6	G4, S3a, G3, G5a, G2
				P10	7	S3b, G3, S3c, G1, G5a, G2
				P11	6	G5, S4d, G2, S3
	B2	S1,G1, S3c	3	**P12**	5	G4, S3c, S4
				P13	3	S4d, S3b, G1
				P14	6	S3a, G1, S3c, G2, G5a
	B3	G5b, S2, G1, G4, S3b, S3d, G3	7	**P15**	4	G1, S3b, S2
				P16	4	S3b, G1, S3d
				P17	9	G4, S2, G2, G5b, S3b, S4c
3rd	**C1**	S1, S3d, G1, G4	4	**P18**	3	S3a, G1
				P19	3	G4, G2, S3d
				P20	5	S1, G1, S3b
	C2	S3a, G3, S3b, S3c, G1, G2	6	**P21**	4	G1, S3a
				P22	4	G1, S3b
				P23	4	S3c, G2
	C3	S1, G1	2	**P24**	7	S3a, G1, G5b ,S4b, G3, S3c
				P25	3	S3b, S1, G1

In this Table, the capital letters A, B, C are used to represent the 1st, 2nd and 3rd grades, correspondingly. These letters are used in combination with the numbers 1, 2, 3, to represent the number of groups in each respective grade that performed the specific strategy. Capital letters Pi (i=1…25) are also assigned to represent each individual student participating in this experiment. As is clearly demonstrated in this Table (3rd column), each group exploited the presence of the various tools provided by Cabri and the collaboration during the 1st task and expressed their inter-individual differences to perform multiple solution strategies for this task, while at the same time constructing multiple perspectives of the concepts in focus. It is also clearly shown (see the last column of Table 3) that each student exploited their collaborative experience during their work in small groups and within the whole class to perform multiple solution strategies to the 2nd task, at the same time expressing their intra-individual differences and forming multiple perspectives and a broad view of the concepts in focus.

As regards the ways in which students collaborated within their groups, it is worth noting that students articulated and shared their ideas with their colleagues through face-to-face communication, so as to form complete and correct solution strategies to the 1st task. As regards the role of whole class discussions, it is worth noting that, each student shared and articulated the specific solution strategy that he/she proposed in their group with the whole class. Each student also provided the whole class with detailed replies to their questions.

6 Discussion

As is clearly demonstrated by the data presented in this case study, the various tools provided by Cabri inspired each group to use them in a diversity of ways to meet the demands of the 1st task, which was for it to be solved 'in as many ways as possible'. As a result, each student expressed - within their own group - their perspectives on the concepts in focus in terms of his/her own solution strategies to the tasks at hand. With students articulating and negotiating their solution strategies within their groups and in front of the whole class, each student also developed multiple perspectives on the said concepts, which in turn helped them to construct multiple solution strategies to the 2nd task, which also required being solved 'in as many ways as possible'. Because both tasks were required to be solved 'in as many ways as possible', the students during both team and individual work were encouraged to invent various solution strategies. In this way, students developed multiple perspectives on the issues at hand at both group and personal level. The possibility of receiving multiple feedback from Cabri and from their collegues during group work and whole class discussions also gave the students confidence in the correctness of the solution strategies they constructed.

Based on the promising results to emerge from the use of MULTIPLES in the reported face-to-face communication setting, we can predict that similar encouraging results could emerge from the use of MULTIPLES within web-based settings. In fact, within such learning settings, students can be provided with fruitful collaboration support in terms of: (a) synchronous discussion capabilities, such as chat rooms - for each group and for the entire class - and videoconferencing. With the provision of such possibilities, students have the chance to discuss case issues that require real time brainstorming and sharing of information, debate controversial issues and plan their actions in a short time, (b) asynchronous discussion opportunities using bulletin boards and discussion forums. By participating in online discussions, students can articulate their understanding of the issues at hand by answering their colleagues' questions and explaining their points of view. Students also have the opportunity to hear other students' viewpoints, ask questions and revisit these discussion areas - in their own time - to reflect on their postings. On the whole, in the context of online discussions, students can collaborate and negotiate their diverse points of view on the topic at hand, while at the same time attaching multiple perspectives to it, (c) various links to multiple learning tools, search engines and learning materials, such as microworlds, simulations, and virtual reality environments, as well as online databases and information repositories presented, and by using multimedia, such as text, digital, audio and video. Students have the chance to form different perspectives on an issue by accessing these various links and learning materials, (d) publishing/sharing their work online (e.g. their solution strategies to the given problems). Group members can also work simultaneously and co-edit documents online and annotate them, if an appropriate application sharing tool (groupware) is provided. In this way, students can be engaged in both reflection and peer evaluation of each other's solution strategies. As a result, students can enhance their perspectives on the issue at hand.

7 Conclusions

In this study, a learning framework – the MULTIPLES framework – has been proposed that encourages collaboration within contexts providing multiple aids in terms of cognitive tools, representation systems and feedback, at the same time asking students to perform tasks in 'as many ways as possible'. The role of this framework has been examined through a case study using real students with promising results. In fact, students exploited the abundance of tools provided by the educational software Cabri-Geometry II as well as multiple ways of collaboration – within their groups and the whole class - and constructed a plethora of solution strategies to the tasks at hand at both group and individual level. Based on the data emerging from this experiment, it was clearly shown that, in supporting the generation of multiple perspectives of the tasks at hand, the role of engaging students in learning environments rich in learning tools, representation systems and communicating interactions is a crucial one. In addition, the kind of activities that can help learners express their inter-individual and intra-individual varieties is of major interest. To this end, multiple-solution activities to be performed in contexts providing a variety of tools are the most appropriate. Specifically, asking both groups and each individual to form solution strategies to the tasks at hand student '*in as many ways as possible'* provides them with the opportunity to exploit the richness of the aforementioned learning environments. Students can select from among various cognitive tools and representation systems those most relative to their knowledge to form their own solution strategies to the given tasks, at the same time expressing their intra- and inter-individual variety and acquiring different learning styles. Students can also use the provided communication capabilities to express their knowledge as well as to exploit the knowledge of their colleagues. The study data also clearly supported the notion that computer learning environments are ideal for integrating both cognitive tools and representation systems which can be combined by students to form multiple solution strategies to the tasks at hand. Web-based learning environments can also provide learners with rich communication capabilities so that they may exchange ideas, and articulate and negotiate opinions. On the whole, the enhancement of Web-based environments with appropriate cognitive tools, in tandem with the requirement for learners to solve tasks 'in as many ways as possible', provides students with great opportunities to develop multiple perspectives on the issue at hand.

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