Activity-based interfaces in the context of Cabri Geometry II: Exploiting the results of an experimental field study design

Maria Kordaki
Dept of Computer Engineering and Informatics
Patras University, 26500, Rion Patras, Greece
e-mail: kordaki@cti.gr

Abstract: This study focuses on the design of activity-based interfaces suitable for the learning of geometrical concepts in the context of Cabri Geometry II (Laborde, 1990). This design has emerged from a field study with real students aiming at the formation of appropriate learning settings, including learning materials and activities concerning Eucleidian geometry. Three different types of learning settings were tested: firstly, a combined setting consisting of Cabri-tools and paper and pencil, secondly, a hypermedia setting consisting of Word documents hyperlinked with Cabri constructions and thirdly a learning setting that was totally performed in the context of Cabri, by integrating the presentation, the performance and the evaluation of the activities proposed into specific 'acivity-oriented' interfaces. The analysis of the data presented better *student* performance in the last learning setting due to the fact that the 'activity-based' interfaces proposed helped students to avoid wasting their energy on unnecessary keystrokes and navigation and concentrate on solving the tasks at hand.

Introduction

The role of both appropriately-designed learning activities and tools to perform these activities is acknowledged by many researchers as being crucial to the formation of constructivist learning settings (von Glasersfeld, 1987; Vygotsky, 1978; Noss & Hoyles, 1996; Nardi, 1996). Constructivist computer learning environments can act as catalysts and distinctive participants in the whole learning context and can play a significant, crucial, and unique role in student learning (Hillel, 1993; Dorfler, 1993; Laborde, 1993; Noss & Hoyles, 1996). Appropriately-designed computer learning environments can act as scaffolding factors for the development of student mathematical activity, while also helping students to reorganize their thinking (Hoyles & Noss, 1989; Hillel, 1993) and to make connections between informal and typical mathematics (Noss, 1988) as well as to experiment and make generalizations exploiting their experience (Hoyles & Noss, 1989).

Among the wide range of educational software designed to support constructivist mathematical learning settings, Cabri Geometry II (Laborde, 1990) holds a remarkable position. Cabri provides the students with potential opportunities in terms of:

- *means of construction*, providing a rich set of tools for a variety of concepts concerning Euclidean Geometry which can be exploited by the students to perform a number of different geometrical constructions and to face a variety of geometrical problems
- *Dynamic, direct manipulation of geometrical constructions* by using the 'drag mode' operation that gives learners the possibility to experiment with geometrical constructions and to form dynamic views of the concepts in focus (Mariotti, 2001). More specifically, students have the possibility of handling, in a physical sense, the theoretical objects which appear as diagrams on the computer screen (Laborde and Laborde, 1995). In these Cabri-constructions, their geometrical properties are retained under dragging, while their visual output is different. The 'drag mode' can be used in three modes, as an 'exploratory' mode, a 'verification' mode and an 'adjustment' mode (Kordaki and Balomenou, 2006)
- *The possibility of collecting large amounts of numerical data*; the 'drag mode' could be used in combination with automatic measurements of specific elements of the geometrical constructions under study. These measurements can be automatically tabulated providing learners with opportunities to reflect on them and form and verify conjectures about specific geometrical concepts and relationships

- *Tools to construct a variety of representations, both numerical and visual*, such as geometrical figures, tables, equations, graphs and calculations. These representations are of different cognitive transparency; consequently, students can select the most appropriate relative tools to express their knowledge. In this way, students have the possibility of expressing both inter-individual and intra- individual differences. The representation systems used also affect the kind of knowledge that students construct (Kordaki, 2003; Mariotti, 1995)
- *Linking representations*, by exploiting its capability for continuous transformations
- *Interactivity and feedback*; intrinsic visual feedback and extrinsic numerical feedback, providing learners with opportunities to form and verify conjectures as well as to self-correct their constructions. This is important because learner actions are closely connected with their consequences, contrary to the static and silent paper and pencil environment where there is no possibility to provide immediate response to those actions (Kaput, 1994),
- *Presenting information to the students in text form*
- *Capturing the history of student actions* to provide teachers and researchers with a valuable amount of information for further studies
- *Extension*, Cabri could be extended by adding certain operations through forming specific macros and manipulating them by using appropriate buttons on its interface.

As regards the kind of learning activities, Cabri has strong capabilities for the design of such activities that encourage learners to: take an investigative perspective, express their inter-individual and intra- individual learning differences, make self-corrections, formulate and verify conjectures and exploit the advantages from the negotiation of their knowledge with the knowledge of their classmates in cooperative settings (Sutherland, 1995; Straesser, 2001; Kordaki & Balomenou, in press). In addition, authentic meaningful real life learning activities can be integrated within the context of Cabri, activities that can develop strong learner motivation.
As regards the learning settings reported for student learning with the use of Cabri tools, these usually consisted of mixed materials. In fact, paper and pencil materials were used for the presentation and evaluation of learning activities while Cabri tools were usually used to support the performance of these activities. In this paper, the design of specific activity-based interfaces integrating presentation, implementation and performance of learning activities is presented. This design was the result of a 'teaching experiment' methodology (Cobb, Wood & Yakel, 1990) that took place in the field, aiming to exploit student feedback in the design of appropriate learning materials for student learning of geometrical concepts using Cabri II tools. Such activity-based interface design has not yet been reported. In the next section of the paper, the context of this study is presented, followed by the results of the three phases of the teaching experiment realized in the field. Subsequently, these results are discussed and conclusions drawn.

The context of the study

This study presents the design of activity-based interfaces aiming to assist student learning regarding geometrical concepts. This design emerged from a teaching experiment that took place in the field with real students. According to this methodology, the teacher acts as a researcher taking into account the feedback given by his/her students in order to modify their decisions appropriately regarding the appropriateness of all the factors comprising the specific learning setting, such as the activities used, the assessment materials, the tools used etc.

The reported experiment happened in three phases. In the first phase, a combined setting was designed and tested with 30 first grade middle school students. This setting consisted of a combination of Cabri-tools and paper and pencil. The learning activities were presented and evaluated through paper and pencil while Cabri was used for the performance of the activities presented. The analysis of the data presented student difficulties in using this combination of materials and prompted the second phase where a new learning setting was designed in a hypermedia context consisting of Word documents hyperlinked with Cabri constructions. Learning activities were presented and evaluated within the context of Word, while appropriate hyperlinks were designed to lead students to the specific learning activities to be realized in Cabri. This hypermedia setting was tested with 28 first grade middle school students. As student difficulties seemed to persist, the third phase was designed, emphasizing a learning setting to be performed wholly within the context of Cabri, by integrating the presentation, performance and evaluation of the activities proposed into specific activity-oriented interfaces. These integrated learning settings were also tested in the field with 28 first grade middle school students. The data from the first phase of this experiment included the students' paper sheets, from the second phase, their Word documents and, from the third phase, the specific electronically-saved files automatically created by Cabri, including all student answers to the questions posed and their actions. The duration of each phase followed student needs, the first phase being about 3.5 hours, the second phase about 2 hours and the third phase about 1 hour. To obtain a clear picture of the advantages and disantvantages of each learning setting, each procedure was repeated five times with the use of different learning activities.

The step by step learning materials design-experiment

The design of learning materials provided to the students and the data emerging from each of the three phases of the aforementioned learning experiment are presented in the following section.

First phase

Design. The learning activities were presented to students on paper sheets, then performed by students in Cabri , followed by students returning to the paper and pencil environment to answer questions regarding the learing concepts in focus. The structure of the paper sheets was: a) description of the learning aim of the activity at hand, b) description of the keystrokes needed to realize the geometrical construction under study, using specific Cabri-tools, c) open-ended questions and d) focus on specific topic questions. Each paper work-sheet presented students with a number of activities.

Six types of learning activities have been designed to be performed by the students in the context of Cabri-Geometry II, namely (Laborde, 2001, Kordaki and Balomenou, in press, Kordaki, 2005): a) *Forming/verifying conjectures by focusing on the alteration of a geometrical construction using the drag-mode operation.* For example, when a student draws a triangle and its heights they can conjecture that 'the heights of a triangle intersect internally when the triangle is acute, but externally when the triangle is obtuse, and on the vertex of the right angle when the triangle is right-angled. In this way, students can also verify conjectures formed in some way during their experience. b) *Forming/verifying conjectures by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation.* A case in point is when a student draws a triangle and then automatically measures its area and perimeter while at the same time dragging the vertices of this triangle and tabulates the data automatically produced by the aforementioned measurement operations. By focusing on these numerical data, this student can conjecture that 'the area and the perimeter of a triangle are different entities'. In this way, students also can verify their conjectures formed in some way (eg. using their visual perception) during their experience. c) *Verifying a formula by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation.* For instance, students can verify the truth of the formula S=[sin(x)^2+[cos(x)]^2=1 in right-angled triangles by: i) drawing a right-angled triangle ABC, ii) measuring the size of the angle B, iii) calculating the S=[sin(x)^2+[cos(x)]^2, iv) tabulating the size of angle B and the result S while dragging the vertices of ABC, and v) focusing on the numerical data automatically collected. d) *Black-box activities.* Students can participate in activities where they have to explore geometrical constructions with some of their properties hidden, which they then have to discover. To illustrate this, students can be asked to justify why a class of triangles with common bases and the opposite vertex sliding on a parallel line have equal areas, as displayed by their automatic measurement. e) *Multiple solution activities.* For instance, in problems where students are asked to construct pairs of triangles with equal areas in as many ways as possible, using any tool provided. f) *Constructions simulating real life problems.* Such real life problems can help students to develop a strong motivation in their learning and approach mathematics as a human activity (Bishop, 1988) as well as put mathematical concepts into an interdisciplinary context (Clements, 1989).

Feedback. The data analysis shows that: a) students expressed enjoyment in using the provided tools easily and that the Cabri interface was friendly to them, b) students also expressed enjoyment over designing geometrical shapes using the specific Cabri-tools. Students were of the opinion that this design was easier and faster to construct than the design in the paper and pencil environment, c) a lot of time was spent by students arriving at the keystrokes needed for the geometrical constructions under study. Because they spent so much time, students focused on the technical points of the geometrical construction in focus and lost sight of the aims and the mathematical meaning of the activity at hand. d) Students were confused alternating between paper and pencil and Cabri and were unable to transfer knowledge from Cabri to paper and pencil. Taking this feedback into account, it was decided to design another type of material, avoiding the description of the keystrokes needed for a specific geometrical construction, by providing them ready for study and exploration. In addition, to avoid students becoming confused with the different media, it was decided to integrate the whole learning activity into a hypermedia setting, presented in the next section.

Second phase

Design. In this phase of the experiment, the learning setting was integrated into a hypermedia framework. The learning activities were presented to the students through a word document, then students were asked to click on the hyperlinks provided to visit the specific Cabri geometrical constructions; next students had to return to the word document to answer the related assessment questions. The structure of these hyperlinked frameworks was: a) description of the learning aim of the activities in focus, b) availability of hyperlinks providing access to the geometrical constructions appropriate for the learning of the concepts in focus, c) open-ended questions and d) focused questions on specific topics. A number of activities were presented to the students in each hyperlinked work-sheet. The types of learning activities designed were those mentioned in the previous section.

Feedback. Data analysis shows that these hypermedia frameworks were more convenient to the students than the combined frameworks reported in the first phase of this experiment. In fact, the cognitive load from the large amount of keystrokes was omitted and the time spent on the design of specific geometrical constructions was reduced. Despite this, the majority of students seemed to be confused when changing between the Word document and the Cabri environment. In addition, students seemed unable to exploit the knowledge they acquired experimenting with one geometrical construction to successfully solve another relative problem. Taking this feedback into account, it was agreed to integrate all sub-geometrical constructions under study and all necessary text-based information, including presentation of the activity and assessment in a specific learning activity-based interface. To do this, the capabilities of Cabri for macros and buttons were exploited. The design of these interfaces is presented below.

Third phase

Design. In this phase of the experiment, students were presented with an interface specifically designed for the learning of the concepts in focus through a number of activities related to study and exploration of specific interactive geometrical constructions. The aim of these activities is presented in the form of text within a text box that appears after entering the said Cabri file. A number of buttons have been designed and positioned on the aforesaid interface for the manipulation of all resources integrated into it. These buttons provide the capability of hiding and illuminating certain types of data. For example, one button has been designed to give students technical information such as the necessary actions needed to manipulate a specific interactive geometrical construction and how the buttons existent on this interface could be used. Other buttons have been designed to help students hide or illuminate specific parts of the interactive geometrical construction under study. In addition, other buttons have been designed to provide students with essential questions to answer. The general architecture of these activity-based interfaces consists of specific spaces dedicated for presentation of: a) a network of appropriate interactive geometrical constructions in focus, dedicated to student experimentation, reflection and formation/verification of hypotheses, b) interactive tables providing opportunities for automatic tabulation of specific numerical data; providing students with the opportunity to reflect and form/verify conjectures and formulas, c) individual buttons, each designed to hide/illuminate the appropriate part of the interactive geometrical construction needed for the understanding of the corresponding aspect of the learning concept in focus, so that students can integrate different aspects of the aforementioned concepts, d) individual buttons, each designed to hide/illuminate the appropriate questions corresponding to the specific aspect studied and e) a specific button designed to hide/illuminate the appropriate help for the technical skills needed for the manipulation of all the actions available to the student, as presented in this specific activity-based interface. It is worth noting that it is also possible to form specific interfaces for the learning of a network of concepts included in a chapter. Figure 1 illustrates an example of the organization of the aforementioned 'activity-based interfaces'. This interface was designed to encourage student learning regarding the concept of medians of a triangle. The basic parts of this interface are described below:

a) *Space dedicated for Experimentation with interactive geometrical Constructions (SEC).* The centre left part of the computer screen is dedicated to students' active experimentation with the specific interactive geometrical constructions constructed for the learning of the concepts in focus, e.g. (a triangle and its medians).
b) *Interactive table for tabulation of automatic measurements of the geometric elements of interest (IT).* The lower left part of the computer screen is dedicated to an interactive table whose columns are interlinked with elements of interest of the interactive geometrical construction under study. This table can be updated with the automatic measurements of these elements during the modification of the geometrical construction using the drag mode operation. In Figure 1, there is a table consisting of 4 columns where the automatic measurements of the distances of the point of intersection of the medians of a triangle from its vertices for each specific instance of the geometrical construction could be stored correspondingly.
c) *The navigation bar.* On the right of the computer screen, there is a column including all the appropriate buttons designed to help students to manipulate all resources provided for the learning of the specific learning concepts. The navigation bar is divided into parts such as the basic aspects of the learning concept in focus. In each part,

there are two buttons: one used to hide-illuminate the appropriate part of the geometrical construction related to each specific aspect and another button to hide-illuminate appropriate questions to help students think about the geometrical aspects in focus and give a sense of progression to their learning. In the navigation bar, there is also an extra part including a button designed to provide information about the technical skills needed for the manipulation of the geometrical constructions and how the buttons provided operate. In Figure 1, the buttons related to the concepts of middle of a segment, median of a triangle and the relationship between the medians of a triangle are presented. By enabling these buttons, the corresponding interactive geometrical constructions are presented in SEC. Appropriate questions formed for the learning of each of these aspects could be also illuminated by the use of specific buttons in the lower-right of the computer screen. Figure 1 also features the interactive Table for the tabulation of sizes of the appropriate segments and ratios.

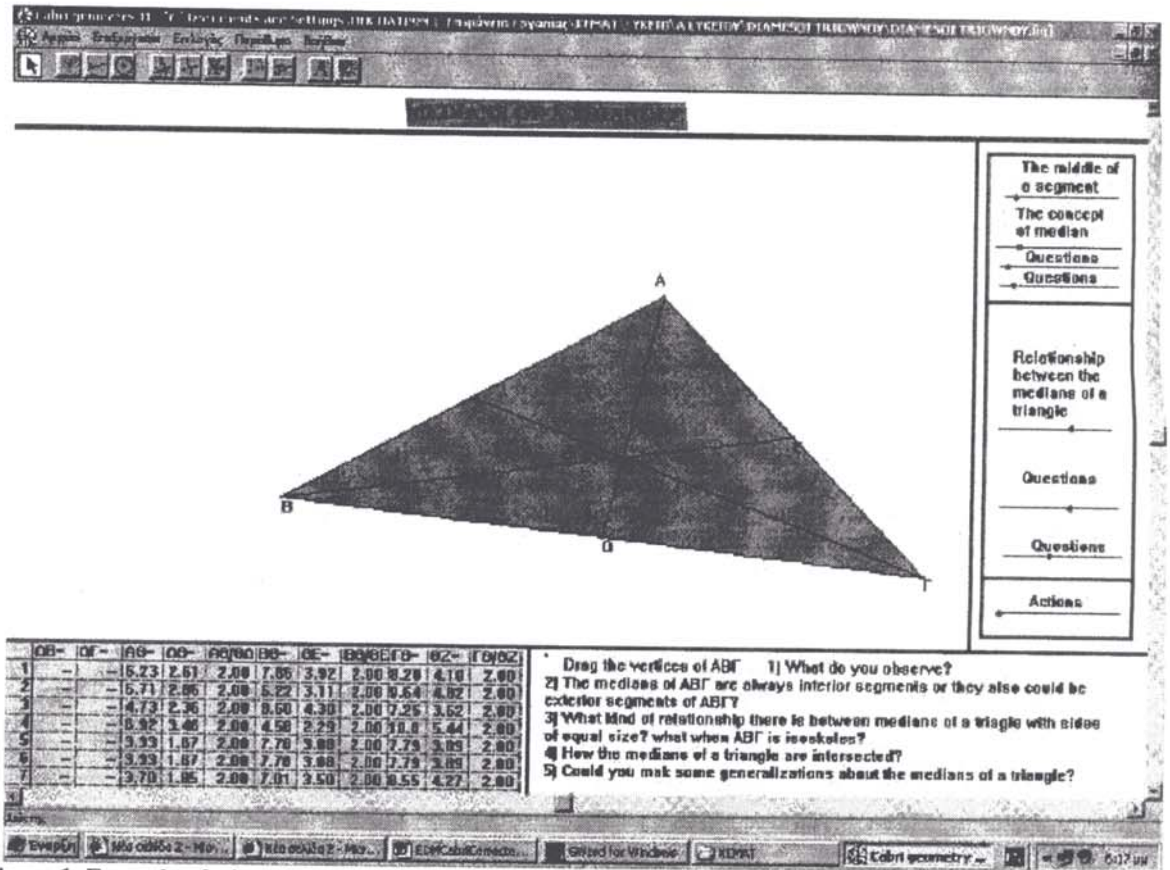

Figure 1. Example of a learning setting in the form of a specific 'activity-based interface' integrated into the context of Cabri Geometry II

Feedback. The analysis of the data shows that students: a) easily and quickly familiarized themselves with the possibilities provided through the proposed 'activity-based interfaces', b) navigated through the different sub-geometrical contructions and subtasks very well, as these were presented in the same interface and students did not have to change learning materials, c) successfully managed the whole geometrical construction, as it was designed in an holistic way, consisting of specific parts all of which could be presented in the same interface. In particular, students studied an initial sub-geometrical construction and then had the opportunity to illuminate some sub-constructions in order to construct gradually the full version of the interactive geometrical construction in focus.

Discussion

The design of 'activity based' interfaces in the context of the well known educational software Cabri-Geometry II is presented in this paper. This design emerged from field studies where different learning settings consisting of a combination of learning materials were tested with real students. In particular, students were presented with the same activities in three learning settings, namely: i) a learning setting consisting of combined learning materials such as Cabri-tools to assist students to create, study and explore appropriate interactive geometrical constructions for the learning of concepts in focus in combination with paper and pencil materials used for the presentation of the whole activity, including the description of the geometrical construction in focus and appropriate questions for student assessment. The structure of these paper sheets was: a) description of the learning aim of the activity at hand, b) description of the keystrokes needed to realize the geometrical construction under study using specific Cabri-tools, c) open-ended questions and d) focus on specific topic questions. The following types of activities were designed to be performed in Cabri: a) Forming/verifying conjectures by focusing on the alteration of a geometrical construction using the drag-mode operation. b) Forming/verifying conjectures by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation. c) Verifying a formula by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation. d) Black-box activities. e) Multiple solution activities. f) Constructions simulating real life problems. ii) a learning setting performed in a hypermedia context including Word documents for the presentation of the learning activities and questions for learner assessment. The structure of these hyperlinked frameworks was: a) description of the learning aim of the activities in focus, b) availability of hyperlinks providing access to geometrical constructions appropriate for the learning of the concepts in focus, c) open-ended questions and d) focused questions on specific topics. iii) a learning setting integrating text based resources and interactive geometrical constructions in 'activity based interfaces'. The general architecture of these interfaces consisted of specific spaces dedicated to the presentation of: a) a network of appropriate interactive geometrical constructions in focus, dedicated to student experimentation, reflection and formation/verification of hypotheses, b) interactive tables providing opportunities for automatic tabulation of specific numerical data; providing students with the opportunity to reflect and form/verify conjectures and formulas, c) individual buttons, each of which was designed to hide/illuminate the appropriate part of the interactive geometrical construction needed for the understanding of the corresponding aspect of the learning concept in focus; consequently, students can integrate different aspects of the aforesaid concepts, d) individual buttons, each of which was designed to hide/illuminate the appropriate questions corresponding to the specific aspect studied and e) a specific button designed to hide/illuminate the appropriate help for the technical skills needed for the manipulation of the whole range of actions available to the student, as presented in this specific activity-based interface. The three learning settings were tested in the field using real students. The hypermedia learning setting was designed to minimize student cognitive load to move from the paper and pencil medium to Cabri and vice versa as well as to help students to focus on the mathematical meanings of the geometrical constructions under study and not on the keystrokes needed to perform them. The 'activity-based interfaces' were designed to liberate students from the confusion emerging from navigating in different computer based resources as well as to help them to integrate different aspects of the concepts in focus by providing them with capabilities to add step by step to a primitive geometrical construction the appropriate parts corresponding to specific aspects needed to build the holistic construction appropriate for successful understanding of the learning concept in focus. The results from the field studies showed that students navigated easily and fast through the operations integrated into these interfaces. These data also revealed that students faced the tasks at hand better in the context of these specifically designed interfaces than in that of the hypermedia or combined paper and pencil and Cabri learning settings.

Conclusions

This study presented a comparative learning experiment aiming at the formation of appropriate settings for the learning of geometrical concepts in the context of Cabri-Geometry II. Three different learning settings were tested in the field using real students. The data analysis showed that students can perform better in learning settings managed through operations posed in specifically designed 'activity-based interfaces' than when these settings are managed through different computer programs or performed in combined static and computer based media. In fact, combined Cabri and paper and pencil learning settings and hypermedia settings place an extra navigational burden on learners. In addition, requiring students to form the interactive constructions under study and explore by themselves takes their attention away from the learning aims of the task at hand and focuses it on the technical part of the procedure. Contrariwise, the data analysis showed that the 'activity-based' interfaces proposed in this paper helped students to avoid wasting their energy on unnecessary keystrokes and navigation and concentrate on solving the tasks at hand.

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