Multiple representation systems and inter-individual learning differences in students

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Abstract: This study focuses on the role of Multiple Representation System (MRS) in the expression of inter-individual learning differences in students. More specifically, thirty students participated in a comparative learning experiment on the learning of the concept of area. This experiment was performed both in the traditional paper and pencil environment and in a computer microworld (The C.AR.ME. microworld; Kordaki & Potari, 1998) providing a variety of tools that support the expression of the concept of area in a variety of representation systems. Students were given the same tasks in both environments. The analysis of the data shows that students were encouraged by the MRS provided in the context of C.AR.ME to express their inter-individual learning differences in terms of the concept of area. In particular, students constructed 20 different types of solution strategies for the given tasks within the context of C.AR.ME in comparison with four types of solution strategies constructed while trying to solve these tasks in the paper and pencil environment.

Introduction

Student inequality in learning and achievement at school has been linked to their inter-individual learning differences (Snow, 1986; Schmeck, 1988). More specifically, learners seem to arrive at schools with different learning styles, such as intuitive, visual, holistic, field dependent, reflective, rational, analytic and field independent. Historically, solutions have been envisaged to resolve inequality of opportunity for success at school. The proposed solutions fall into one of three categories: a) changing the learning style of the learner so that they will better 'fit' the educational system (Corno & Snow, 1984; Bradley, 1985), b) changing the teaching and evaluation methods of the educational system so that it can match the variety of learner differences (Joyce & Weil, 1972; Bennett, 1977) and c) changing both the system and the learner so that every learner can master more than one learning style by providing more than one teaching and evaluative approach (Ramby, 1984; Lemerise, 1992). This can be achieved by engaging students in learning within such environments through the provision of: a) a variety of tools of different cognitive transparency, from which students select the most appropriate relative to their knowledge, b) possibilities to study the concepts in focus in MRS so that they can express their inter and intra-individual variety and c) rich communication capabilities for them to express their knowledge and to exploit the knowledge of their colleagues.
The role of Multpile Representation Systems (MRS) in teaching and learning is acknowledged as crucial in encouraging the expression of learners' individual variety (Dyfour-Janvier, Bednarz & Belanger, 1987; Janvier, 1987; Kordaki, 2005). Most learner difficulties are found in the gap between their intuitive knowledge and the knowledge they need to express the proposed RS for use (Janvier, 1987). For example, prepositional, symbolic and abstract representation systems prevent some learners (usually beginners) from expressing their knowledge, the same systems being intended for use by advanced learners. Contrariwise, metaphors of everyday life and visual RS are more suitable for beginners. MRS are proposed to be used to enable learners' individual variety as well as to enable each learner to acquire a broader view of the subject to be learned (Kordaki, 2003). In addition, multiple and linked RS provide learners with opportunities to study how the variation in one system can affect the other. In this way, each learner can make connections between different aspects of a learning subject (Lesh, Mehr & Post, 1987b; Janvier, 1987). The role of translation and transformation among different RS as well as within the same RS has been viewed as crucial (Lesh, Post & Mehr, 1987a; Janvier, 1987), especially in the learning of mathematical concepts.
Computers are an ideal medium providing wide opportunites for the construction of a variety of different, linked and dynamic representation systems such as; texts, images, equations, variables, tables, graphs, animations, simulations of a variety of situations, programming languages and computational objects (Kaput, 1994). Computer learning

environments providing opportunities for MRS have been also acknowledged as essential for the expression of students' inter-individual differences (Rampy, 1984; Lemerise, 1992). Well-known examples of such environments are: the Logo programming language environment, Dynamic Geometry Systems such as Cabri-Geometry II (Laborde, 1990), the Geometers' Sketchpad (Klotz, & Jakiw, 1988) etc.

In this study, the role of a multirepresentational computer microworld (the C.AR.ME. microworld; Kordaki & Potari, 1998) in the expression of students' inter-individual differences regarding the mathematical notion of area is presented. C.AR.M.E was designed for the learning of the concept of area by primary and secondary level education students and to allow learners to: a) express their problem-solving strategies in MRS, starting from intuitive representations and moving gradually to more sophisticated ones, b) focus on spatial qualitative solutions rather than on quantitative solutions mainly based on area formulae, c) be engaged in problem-solving settings including a variety of familiar and meaningful problems, and d) correct their solution strategies by receiving appropriate feedback by the system. The design methodology of C.AR.ME. was based on the construction of three models; namely: a) the learning model based on constructivist and social views of learning (von Glasersfeld, 1990; Noss & Hoyles, 1996; Crawford, 1996a; Crawford, 1996b). These theoretical considerations are translated into operational specifications of the software, b) the subject matter model, including an adequate set of necessary aspects comprising the concept of area and c) the learner model for solving essential problems regarding the concept of area, while at the same time taking into account individual learner learning differences and difficulties that have emerged from the literature.

In this paper, the role of MRS on student expression of inter-individual differences is investigated through the comparison of their strategies in solving the same problems in two learning environments: a) the traditional paper and pencil learning environment that provides opportunities for limited, static and non inter-linked representation systems and b) the C.AR.ME micro-world, providing a variety of tools for MRS of the notion of area. Such a comparative study, focusing on the role of MRS on the expression of student inter-individual differences within the context of mathematics education, has not yet been reported.

In the next section of the paper, the MRS included in the C.AR.ME microworld are briefly presented, followed by reference to the context of the comparative learning experiment realized using real students. Next, the findings of this experiment in terms of learner strateges in solving the same problems in both the paper and pencil environment and the C.AR.ME microworld are demonstrated. Subsequently, these results are discussed in terms of the role of the MRS provided and conclusions are drawn.

MRS provided by C.AR.ME microworld

Understanding area as a space inside a figure and conservation of area are prerequisite for the understanding of the concept of area measurement (Hughes & Rogers, 1979; Piaget, et al., 1981). An area may be conserved while the shape of its figure is altered. Students can primarily master conservation of area through the *cut, move* and *paste* actions by re-arranging the parts of a shape to produce a new one with equivalent area (Piaget et al., 1981). Understanding conservation of area through these processes is necessary and prerequisite for the students to understand the concept of area measurement as well as multiplication structures (Douady & Perrin, 1986). Students have difficulties in understanding conservation of area when it is represented in shapes of different forms (Carpenter et al., 1975). In addition, they are prevented from understanding the concept of conservation of area since their conclusions are based on their perceptions (Hughes & Rogers, 1979). Moreover, students confuse areas and their perimeters and use them alternatively (Hart, 1989). Difficulties in understanding the concept of conservation also arise with the concept of the unit of area (Hart, 1989). Students cannot conserve units from their parts. Most of the research literature emphasizes the study of student difficulties with standard geometrical shapes such as squares, rectangles, parallelograms and triangles (Johnson, 1986). Student difficulties regarding area measurement are related to their premature introduction to the quantitative approach to area using area formulae, while disregarding the qualitative approach which emphasizes conservation of area without the use of numbers (Johnson, 1986; Douady & Perrin, 1986). The qualitative approach acknowledges the need for students to grasp conservation of area by splitting areas into parts or units and recomposing them in order to construct new equivalent shapes (Carpenter et al., 1975; Hiebert, 1981). Moreover, student difficulties in area measurement are attributed, firstly, to their fragmented way of studying areas without dynamic relation to their perimeters and, secondly, to their inability to fill the gap between the cultural approach to area, expressed in the use of area formulae, and the qualitative approach to manipulating areas without the use of numbers (Baturo & Nason, 1996).

Taking the above into consideration, four different groups of tools were provided by C.AR.ME for the students to use, to enable them to express their knowledge in four different representation systems in order to solve problems regarding the notion of area. The tools provided in the C.AR.ME interface are presented in Figure 1. These tools are disscused below in relation to the representations of area that could be constructed by using them.

IRS: Intuitive Representation Systems. Students can use the tools demonstrated in Column 3 of Table I to express their intuitive knowledge to conserve areas in a qualitative way, without the use of numbers. A variety of different representations of equivalent areas can be created in two ways: firstly, by changing only the position of a figure while conserving its shape and, secondly, by splitting a figure into its non-overlapping parts and recomposing the parts to form a new equivalent shape. These processes can be realized by using the tools *'Select All'*, *'Select Part'*, *'Cut'*, *'Paste'*, *'Rotate'*, and *'Symmetry'*.

SURS: Spatial-Unit based Representation Systems. Students are provided with the opportunity to use the group of tools presented in Column 6 of Table I to express their visual knowledge of area measurement using spatial units. These t*ools help students to construct a variety of spatial units and grids.* Moreover, tools to perform *unit iteration* and *counting the number of units needed* to cover specific areas are also available. By using this group of tools, students have the opportunity to experience conservation of area in a more sophisticated way by splitting areas using spatial units and counting and recomposing them to construct new equivalent shapes. Students also can use these tools to measure areas in a visual way.

File	Draw	Edit	Automatic Measurements	Automatic Transformations	Measurement Tools	Help
Open	Dot Square Grid	Select Part	Areas	Square	Square unit	
Open Last	Dot Triangle Grid	Select All	Angles	Rectangles	Rectangular unit	
Save Last	Draw Polygons	Cut	Segments	Families of Rectangles	Student unit	
Save As	Draw Segments	Paste		Parallelograms	Unit iteration	
Print	End Draw Polygons	Draw an angle of Rotation		Families of Parallelograms	Counting of units	
Exit	Clear	Rotate		Triangles	Square Grid	
		Draw an axis of Symmetry		Families of Triangles	Rectangular Grid	
		Symmetry about axis		Show numerical elements	Student Grid	
		Erasers				

Table 1. The general interface of C.AR.ME.

DTARS: Dynamic Transformation of Area Representation Systems. Students can use the group of tools shown in Column 5 of Table I to study conservation of area in a dynamic way by exp loring the knowledge implied in dynamic transformations of a figure while its area is conserved. By using these tools, students can make connections between visual knowledge regarding basic properties of the figures in focus and symbolic knowledge such as area formula implied in the transformations of the area of those shapes dynamically performed by the system. More specifically, a number of different tools are provided for the students to automatically transform areas already drawn into equivalent ones. These equivalent areas are a *square*, a *rectangle with dimensions of ratio 1:2*, *a right-angled triangle with perpendicular sides of ratio 1:2*, and *classes of equivalent shapes* of the same form such as *rectangles, parallelograms and triangles* with common bases and equal heights. Students can draw the base of a representative for each one of the above classes with the tools provided by C.AR.ME. Then, students can produce a number of equivalent shapes belonging to each class by using the appropriate tools. By altering these bases, a number of different classes of equivalent shapes of the same form can be produced.

NRS: Numerical Representation Systems. Students can use the group of tools shown in Column 4 of Table I to study areas in terms of numbers, taking into account their linear elements such as perimeter, sides, heights etc. By using these tools, students have the chance to investigate the equivalence of different areas, to verify the equivalence of known equivalent ones and to clarify the diference between the notions of area and perimeter.

The context of the study

The theoretical framework and the focus of the study. This study focuses on student approaches to the concept of area used in their strategies in solving the problems of transformation and of comparison within the context of two learning environments providing different tools. The aforesaid environments were: a) the paper and pencil environment and b) the C.AR.ME. microworld. This work is part of an extensive formative evaluation of the above-mentioned microworld designed to investigate student strategies related to the concept of area (Kordaki,1999). The research on student area strategies is reported in detail in Kordaki & Potari, 2002 and Kordaki, 2003. This evaluation study was realized with students in the field and focused not only on the learning outcomes but also on their learning processes. In terms of methodology, it is a qualitative study (Cohen & Manion, 1989) as well as adopting a phenomenographic approach to evaluation (Marton, 1988). In interpreting this approach, I focus on the variety of interactions realized by the students using C.AR.ME. tools and on the different ways that these students approach the concept of conservation of area using these tools.

The Learning experiment. The learning experiment took place in a typical state secondary school in Patras, Greece. A whole class consisting of thirty 2nd grade students (14-year-old) participated in a problem-solving activity. This activity involved the conservation of area and its measurement and was not part of the students' normal classroom experience. In this experiment, students were asked to complete two tasks, initially in the paper and pencil environment and then in the context of C.AR.ME. An approximately two-hour familiarization phase in the use of the C.AR.ME tools took place before the students commenced the main study. The duration of each task was commensurate with student needs. Each student spent on average one hour per task in the paper and pencil environment and about two hours per task in the context of C.AR.ME. Students worked in a computer laboratory equipped with three computers, They worked individually in rotation except during the familiarization phase where they worked in pairs.The researcher participated in the study as an observer with minimum intervention. The data resources are the log files containing the history of student interactions with the C.AR.ME. tools, the electronic snapshots of students' drawings, the audio recordings of all verbal interactions and the field notes of the researcher.
The tasks. Students were assigned the same two tasks in both environments during this evaluation study. In the first task, students were asked to 'transform this polygon to another polygon of equal area in as many ways as possible' while in the second task they were asked to 'compare this polygon to this square in as many ways as possible'. These shapes were not easily comparable by 'eye'. In the paper and pencil environment, all the shapes to be studied were drawn on a paper sheet while in C.AR.ME these shapes were drawn on the computer screen by the researcher and the instructions were presented verbally to the students. In the paper and pencil environment, students were encouraged to ask for any tool they considered potentially useful in the problem solving process. The above tasks have been considered essential by other researchers for the students to construct, in a qualitative way, the concepts of conservation of area and its measurement (Carpenter, Coburn, Reys & Wilson 1975; Hiebert, 1981; Driscoll, 1981). By allowing and asking for a variety of solutions to the given tasks, students were encouraged to construct their own individual approaches to the concept of area and to express different kinds of knowledge they possess regarding this concept (Lemerise, 1992; Kordaki & Potari, 2002; Kordaki, 2003). The nature of these tasks allows the students to construct their own individual approaches to the relative concepts and to express their intuitive knowledge as well as to develop multiple and different solution strategies.
The process of data analysis. The various types of data were organized according to the two different learning environments. Next, these data were sorted according to the two different tasks. In each task, all individual students' multiple-solution strategies were identified, reported and then analyzed in terms of student conceptions of the concept of area and their development as they emerged during the experiment. In the next stage, the focus was on the entire group of students and strategies were classified into categories.

Results

Students constructed a variety of solution strategies to solve the given tasks both in the paper and pencil environment and in C.AR.ME. These strategies were classified in each environment with the criteria being the kind of tools used and subsequently the representation system used by the students to express their solutions. The kind and variety of categories of student strategies performed in each learning environment is an indicator of its capabilities for the expression of student inter-individual learning differences. In the following section, the various student solution strategies to the given tasks in both the paper and pencil environment and the C.AR.ME microworld are presented.

Student strategies performed in C.AR.ME. Students constructed 328 solution strategies in both tasks by exploiting the tools of C.AR.ME. These strategies were classified into 28 categories, 14 for each task, and presented in Table II. Of these categories, 20 are different. A description of these strategies in terms of student conceptualization of area is presented in Kordaki & Potari (2002) and Kordaki (2003).

Categories of student transformation (C.T) and comparison (C.C) strategies in the context of C.AR.ME.			**Number of students**
Categories performed by using...	**C.T**	**C.C**	
the 'eye'	C1	C1	1T, 1C
the perimeter of the shapes *(NRS were used)*	C2	C2	4T, 2C
the automatic transformation tools *(DTARS were used)*	**G1**	**G1**	28T, 10C
the simulation of student sensory -motor actions *(IRS were used)*	**G2**	**G2**	27T, 14C
the tools that support the operation of area measurement using spatial units *(SURS were used)*	C6	C6	19T, 25C
the simulation of student sensory -motor actions in combination with the tools for automatic transformations *(IRS and DTARS were used)*	**G3**	**G3**	8T, 16C
the tools that support the operation of area measurement using spatial units in combination with tools for automatic transformations *(SURS and DTARS were used)*	**G4**	C7	1T, 9 C
by enclosing the non-convex polygon in its minimum convex super set in combination with the simulations of student sensory -motor actions *(IRS were used)*	**G5**	-	1T
by enclosing the non-convex polygon in its minimum convex super set in combination with the simulations of student sensory -motor actions and the tools for automatic transformations *(IRS and DTARS were used)*	**G6**	-	2T
by enclosing the non-convex polygon in a minimum convex super set in combination with the operation of area measurement using spatial units *(SURS were used)*	-	**G7**	2C
by enclosing the non-convex polygon in a minimum convex super set in combination with the operation of area measurement using spatial units and the area formulae *(SURS and AF were used)*	**G8**	**G8**	2T, 2C
the area formulae *(AF were used)*	C8	-	1 T, 1C
the tool for automatic area measurement in combination with area formulae *(NRS and AF were used)*	C10	C10	1T, 1 C
the tool for automatic area measurement in combination with area formulae and the tools for automatic transformations *(NRS and AF and DTARS were used)*	**G9**	-	5T
the tool for automatic area measurement *(NR were used S)*	-	C3	23 C
the tool for automatic area measurement in combination with the simulation of student sensory -motor actions *(NRS and IRS were used)*	-	C4	4 C
the tool for automatic area measurement in combination with the tools for automatic transformations *(NRS and DTARS were used)*	-	C5	5 C
the area formulae in combination with the simulation of student sensory -motor actions *(AF and IRS were used)*	-	C9	1 C
the operation of area measurement using spatial units in combination with area formulae *(SURS and AF were used)*	C11	-	2 T
	Sum = 14 cat.	Sum = 14 cat	

Table II. Categories of student area strategies performed in the context of C.AR.ME.
(C: Task of comparison, T: Task of transformation)

As shown in Table II, C.AR.ME provided opportunities for students to express their inter-individual differences in the concept of area through 20 different solution strategies to the given tasks. These strategies were based on: a) students' visual perception (category C1) b) the perimeter of the shapes under study (category C2) c) automatic measurements of the area of the shapes in focus (category C3) d) automatic dynamic transformations of areas (category G1) e) intuitive approaches to area (category G2) f) area measurement using spatial units (category C6) g) area formulae (category C8) and h) combination approaches (categories C4, C5, C7, C9, C10, C11, G3, G4, G5, G6, G7, G8, G9).

Student strategies performed in the paper and pencil environment. Students constructed a total of 63 solution strategies in the paper and pencil environment. These strategies were classified into 6 categories, 3 for the task of transforamation and three for that of comparison. These categories are presented in Table III. A more detailed description of thes e categories, their interpretations in terms of learning the concept of area and student difficulties are presented in Kordaki (1999).

Categories of student transformation (C.T) and comparison (C.C) strategies in the paper and pencil environment			**Number of students**
Categories performed by using...	T	C	
the 'eye'		C1	1(C)
the perimeter of the shapes	C2	C2	1(T), 4(C)
Conserving the area of the original shape by changing its position and by splitting it into parts - by drawing- and changing the position of the parts to recompose another shape with equal area	G2		5(T)
the area formulae	C8	C8	24(T), 28(C)

Table III. Categories of student area strategies in the paper and pencil environment
(C : Task of comparison, T: Task of transformation)

Students expressed their inter-individual differences regarding area through four different approaches in the paper and pencil environment: a) by using their visual perception (category C1), b) by focusing on the perimeter of the shapes under study (category C2), c) by expressing their intuitive knowledge (category G2) and d) by using area formulae (category C8). The most dominant approach in both tasks focused on the use of area formulae (24 students used this approach to the task of transformation and 28 students for the task of comparison). Of note is the fact that, despite the majority of students participating in this experiment using area formula to solve the tasks at hand, sixteen students used it in wrong ways. A small number of students expressed intuitive approaches (5 students) and one student used his visual perception. Lastly, some students seemed to confuse perimeter and area (1student in the transformation task and 4 students in that of comparison).

Discussion

The role of Multiple Representation Systems in the expression of student inter-individual differences in terms of the concept of area was presented in this paper. Student inter-individual differences were expressed through the different strategies they performed in learning environments provided with different tools to construct different representations of area. The learning environments used were the traditional static paper and pencil environment and a computer environment, the C.AR.ME. micro-world - Kordaki & Potari, 1998- designed specifically for the learning of area. The analysis of the data showed that students exploited the variety of tools provided by C.AR.ME and used them independently and in combination resulting in the construction of a plethora of solution strategies, which were classified into 20 different categories. These categories of student strategies were based on: a) students' visual perception, b) the perimeter of the shapes under study, c) automatic measurements of the area of the shapes under study, d) automatic dynamic transformations of areas, e) intuitive approaches to area, f) area measurement using spatial units, g) area formulae and h) a combination of the previously mentioned approaches. It is worth mentioning that the strategies falling into all categories except C2 are correct. In my view, this means that the variety of tools provided by C.AR.ME gave students opportunities to select from among these tools those that were most appropriate to their knowledge, encouraging them to express their inter-individual differences in the concept of area. In particular, the design of such tools allowing the expression of area solution strategies in different representation systems provided students with opportunities to express their inder-individual differences to the concept of area.

In comparison to C.AR.ME, students performed a limited number of strategies (4) in the paper and pencil environment. These strategies were mainly based on recalling knowledge from school emphasizing the use of area formulae to accomplish the given tasks. It is also worth noting that only half of the students succeeded in using area formulae correctly. The fact that the number of strategies constructed in the paper and pencil environment was limited is closely related to the fact that, in this environment,students were not provided with any tools. As a result, students did not manage to fully express their inter-individual learning differences in terms of the concept of area in this environment.

Conclusions

This study showed that computer learning environments providing a variety of tools to construct solution strategies in multiple representation systems can encourage students to express their inter-individual learning differences better than in environments providing a limited number of tools, such as the traditional static paper and pencil environment. More specifically, this study showed that students exploited the variety of tools of a computer micro-world - the C.AR.ME micro-world - and constructed 20 different types of solution strategies in contrast with a mere four strategies constructed in the paper and pencil environment. In this way, students expressed the different kind of knowledge they possessed through C.AR.ME. In other words, students selected among the tools provided those most appropriate to their inter-individual variety and constructed a plethora of solution strategies to the given tasks. Also of note is the finding that all students exploited the variety of tools provided by C.AR.ME and correctly solved the given tasks in at least two ways, in contrast to the paper and pencil environment where only a limited number of students succeded in correctly solving them. On the whole, the experience of this study supports the idea that the provision of tools designed in such a way to support the expression of knowledge in different representation systems can efficiently support students to express their inter-individual learning differences in specific concepts in focus.

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