# Interaction of young children with multiple representations in an Open Environment

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## SUMMARY
This paper studies interaction of young children with a multiple representational world. Alternative tools and problem solving strategies can be applied in this world in order to investigate basic geometrical concepts like conservation of area of shapes during transformations and measurement of areas. The effect of tools usability and complexity of required cognitive processes in user alternative approaches are discussed through the description of a large scale experiment involving interaction of 30 pupils with the tools.

**KEYWORDS:** multiple representations of geometrical concepts, tools usability, problem solving strategies

## INTRODUCTION
We distinguish **internal representations,** as mental images corresponding to constructed formulations of reality [3] and **external** ones as embodiments of internal conceptualisations. If we concentrate on external representations, we observe that these play a crucial role on children thinking [16], [6], [8], since young children have the opportunity to express themselves through these external representational systems as well as to support their internal representations by external ones [17],[2]. External representations can help scientific concepts to be more interesting and attractive to the pupils [3].
Modern computer interfaces support effective integration of a variety of different representation systems. These systems can be linked and interacted so that the variations in one system can affect the others, and these alterations can be visualized [7]. Figures, drawings, geometrical shapes, text, Venn diagrams, tree diagrams, flow diagrams, graphs, tables, equations, simulations as well as computational objects are the constituent parts of different representation systems that can be integrated in a computer environment [6],[3]. More emphasis is given to graphical visual representations, since children are expressed better in visual systems than propositional ones. By interacting with different representation systems in a direct manipulation way, young children can see the results of their actions on interface objects of the representational system and after reflection they can form abstract concepts [4]. Pupils have also the opportunity to express their own individual approaches to a concept by selecting among representations the most appropriate ones for their cognitive development [3]. Different representation systems can give to the pupils the opportunity to express different kinds of knowledge, according to their cognitive development [18]. So the pupils can express their intuitive approaches as well as analytical or formal ones in different representation systems. Moreover, the same child can express his/her different approaches to a concept using different representation systems [10].
The use of multiple and linked representations has been characterized as an ideal method to learn scientific concepts[5], but translations among representations and transformations within them are also important in the understanding of these concepts [13]. By interacting within different linked representational systems, pupils have the opportunity to construct a multidimensional and dynamic view of the related concepts [3]. During this process pupils can grasp the common properties of the concepts under study and can form more abstract ones.
Representational systems can be viewed as *transparent* or *opaque* [14]. A transparent representation has no more meaning than the idea(s) or structure(s) it represents. An opaque representation would emphasize some aspects of the idea(s) or structure(s), and de-emphasize others; including properties beyond the idea(s) and structure(s) represented at the user interface.
In the following sections a specific study on the use of multiple representations of the same concept by young children and the relation of the tools usability on the developed representations are studied and discussed.

## A MICROWORLD SUPPORTING MULTIPLE REPRESENTATIONS
The 'Conservation of Area and its Measurement' C.AR.ME microworld [9] is a software environment that incorporates some of the characteristics discussed in the previous section. CARME has been designed as an interactive open problem-solving environment to support pupils' experimentation with the geometric concepts of conservation of area and its measurement. The environment has been modelled as a toolbox. A number of tools were constructed to give young children the opportunity to construct multiple views of the concepts of conservation of area and area measurement. These views are con-

sidered as qualitative, quantitative and dynamic representations of the above concepts. These tools and their corresponding representations are presented in this section.

**T1. Tools permitting direct manipulation of shapes (sensory-motoric simulation).**

A variety of different representations of **equivalent areas** can be constructed by manipulating shapes. These constructions can be realized in two ways: by changing only the position of a shape while conserving its form and by splitting the shape in its non overlapping parts and recomposing them to form new equivalent shapes, as shown in figure 1. All these representations can be viewed as approaches to the concept of conservation of area. They are 'transparent' representations, also reported by the literature, as prerequisites for young children's understanding of the concept of area measurement. These constructions can be realized by using the tools of '*Select All'*, '*Select Part'*, '*Cut*', '*Paste*', '*Rotate*', and '*Symmetry*'. These tools attempt to simulate the pupils' **sensory-motoric actions** in the context of this microworld.

In terms of usability, these tools are intuitive to use, since they simulate object manipulation activities, however the way they have been implemented in the version of the software used, through a combination of menu commands and direct manipulation operations could create confusion to the young children.

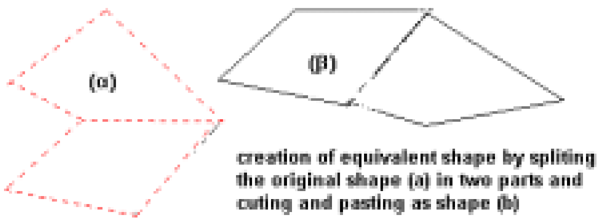


*Figure 1. Example of use of tools T1*

**T2. Tools based on use of spatial units and grids as representation systems.**

A variety of spatial units and grids are offered to the pupils as tools for active area measurement. These are *a square and a rectangular unit* and *grid* respectively. Direct manipulation of the units, which can cover the measured area by the children, is the intuitive process suggested to the pupils, see figure 2. Two additional tools are available for the pupils to create *personal units* and *grids*. Moreover tools to perform the *unit iteration* and for *counting the number of the units* needed to cover areas are also available. So the pupils have the opportunity to measure the area of given shapes using different units. The produced representations are 'transparent' and are reported by the literature as prerequisite for pupils' understanding of area formulae.

In terms of usability this approach is the most intuitive to implement, since it is based mostly on direct manipulation operations of the selected units.

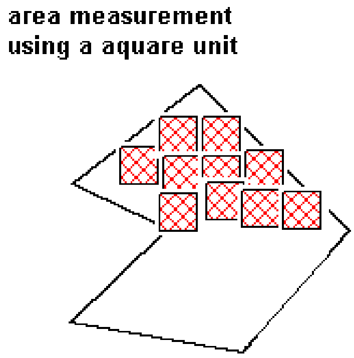


*Figure 2. Example of use of tools T2*

**T3. Tools to represent equivalent areas using dynamic representation systems.**

A number of different tools are provided to the pupils to automatically transform areas to other equivalent ones of known geometrical shapes. These equivalent shapes are a square, a rectangle with dimensions of ratio 1:2, a right-angled triangle with perpendicular sides of ratio 1:2, and classes of equivalent shapes of the same form as rectangles, parallelograms and triangles. By studying these equivalent shapes, pupils have the opportunity to explore different representations of the concept of conservation of area. These representations are 'opaque' but gave the pupils the opportunity to study the concept of conservation of area in a dynamic way, to reflect on them and to form more abstract concepts, see figure 3.

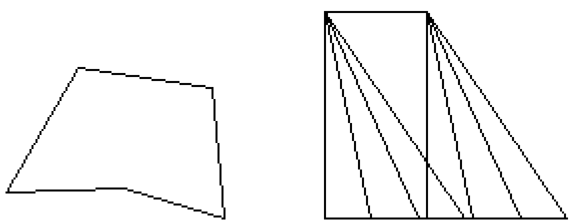

*Figure 3. Example of use of tools T3*

The role of dynamic representations in pupils' thinking is acknowledged by many researchers ([15], [1], [12]). Moreover, by studying the plethora of the shapes included in the classes described above, pupils have the opportunity to explore the equal elements (bases and heights) of these shapes. All this experimentation can help pupils to move gradually from visual to propositional representations of the concept of conservation of area. These dynamic representations are opaque and needed exploration as well as justifications by the pupils in order to be productive.

The operation of these tools was based on a long sequence of menu commands. A number of usability problems have been discovered in this area in other studies of the CARME microworld [19].

#### T4. Tools based on numerical representation.

The possibility of automatic area or perimeter measurement is provided to the pupils. In this measurement operations the use of standard units of length and area are implied. By using these tools numerical representations of area measurement can be produced. These are 'opaque' representations and in order to be productive, justifications are needed to be developed by the pupils. More specifically pupils have the chance to develop justifications of the equivalence of the variety of equivalent shapes they study, as well as to investigate the relationship between the area and the perimeter of these shapes.

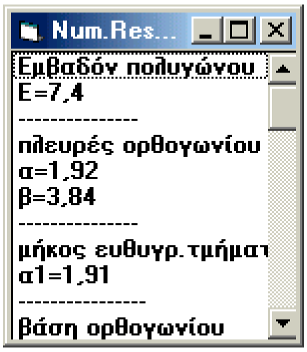


*Figure 4. Display of automatic measurements*

The simplicity of these tools makes them particularly easy to use. A limitation was that the measurements appear in a separate window, as shown in figure 4, resulting in not easy association of the displayed measurement to the measured object.

## EXPERIMENTAL USE OF THE TOOLS

In an environment like CARME, it is interesting to investigate how multiple equivalent approaches to the same problem can be followed by young children, who are asked to use the available alternative representations. The order in which the children decide to use these representations, the diversity of the approaches, the relation between usability of the tools and usage patterns are interesting subjects of investigation. A study has therefore been undertaken in order to provide some insight into these questions. The context of this study is described next.

A large-scale experiment took place in the Computer Lab of a secondary school of Patras, Greece, involving thirty (30) 14-years old pupils who participated in a problem-solving activity relating to the conservation of area and its measurement using the C.AR.ME microworld. The children worked individually, except of the familiarization phase where they worked in pairs. The aim of this phase was to familiarize the pupils with the tools of the microworld and not to get them involved in the measurement process. The need for this phase emerged from the pilot study and took place before the pupils commenced the main study. The familiarization phase was realized by asking the pupils to consecutively try all the operations available in the microworld. For instance, "draw a polygon", "draw a segment", "clean the screen", "save your work", "select the dot square grid", "rotate a shape", "select a unit", "iterate a unit" were some typical examples of the tasks posed. The familiarization phase lasted about 2 hours for each pair.

Overall, the pupils spent as much time as they needed to perform the given tasks. Each student spent on average about two hours per task. The facilitator participated with the minimum intervention. Two tasks were assigned to the pupils during this evaluation study. The task discussed here relates to *the comparison of a non-convex polygon to a square, not easily comparable by 'eye', in 'any possible way'*. The nature of the task allows the children to express their intuitive knowledge about the relative concepts as well as to develop multiple and different solution strategies [9].

The data collected during this experiment were the automatically created log files containing the history of pupils' interactions with the tools, the screenshots of pupils drawings, audio recordings of verbal interactions between the pupils and the facilitator and the field notes of researcher's observations during the study.

The data were organized in such a way that all individual pupils' solution strategies on the specific task were identified and reported. These strategies were analysed in terms of tools used and the constructed representations using these tools.

#### Analysis of problem-solving strategies

Pupils interacted within representational systems provided by this microworld to complete the given comparison task and developed a number of alternative problem-solving strategies.

During the design of the CARME environment the assumption was implicitly made that the young children would solve the task using the four alternative approaches imposed by the tools discussed in the previous section. However the experiment revealed that the users invented more complex problem solving strategies, involving often combination of tools. This mismatch between the environment designer expectations and the usage observed during experimentation, discussed in [11], suggests that such complex environments should incorporate flexible and robust tools, capable to inter-operate in many alternative ways, since it is not possible to foresee all possible uses of the tools in such open environments.

In particular the problem solving strategies used by the young pupils involved in this study fall in 12 categories, shown in Table 1. These, as shown in the table, are the

ones foreseen by the designers (C2, C5, C10, C11) and combination of them or combination with unforeseen techniques, like use of the perimeter or use of area formulas.
In total 158 different solutions were given to the problem, by the 30 pupils using the CARME environment (an average of 5.3 solutions per pupil). The min value of strategies was 2 and the max 10 (stdev=1.65).
From these solutions 113 (71%) involved use of one tool, while 45 (29%) involved combinations of the provided software tools or combinations of the tools and other techniques.

| Categories of pupils' strategies of area comparison |
|---|
| C2 : *automatic measurement*.(T4) |
| C5 : *use of spatial units (T2).* |
| C10: *simulation of sensory-motoric actions*. (T1) |
| C11:use of *automatic transformations*. (T3) |
| C3 : *use of the perimeter of the shapes*.(T4) |
| C4 :*automatic area measurement in combination with simulations of sensory-motor actions*.(T4, T1) |
| C6 :use of *units of area measurement in combination with automatic transformations* (T2, T3) |
| C7 :*automatic area measurement in combination to automatic transformations*.(T4,T3) |
| C8 : *use of area formulae in combination with simulations of sensory-motoric actions*. (form, T1) |
| C9: *automatic operation of area measurement in combination with use of area formulae*. (Form, T4) |
| C12: *Enclosing the non-convex polygon to its minimum convex superset. (T1, form)* |
| C13. *automatic transformations in combination with simulations of sensory-motoric actions*. (T4,T1) |

*Table 1. Categories of pupils' problem solving strategies associated to tools used*

Another interesting feature was the number of pupils who applied a specific solution strategy. The top strategies are shown in Table 2. From this table one can see that the most popular strategies are those involving one tool.

| Problem solving strategy (tools) | Number of pupils | % |
|---|---|---|
| C5 (T2) | 25 | 83% |
| C2 (T4) | 23 | 77% |
| C13 (T4,T1) | 16 | 53% |
| C10 (T1) | 14 | 47% |
| C11 (T3) | 10 | 33% |

*Table 2. The most popular strategies*

Also study of the average number of alternative solutions given using each strategy presents some interest. Strategies that have been re-applied present versatility. So in figure 5 it can be seen that the maximum value of this index is associated with C5 (2 solutions per pupil), while the rest are in the range [1..2]. This indicates that some strategies invited the user to re-apply them in alternative ways. For instance strategy C5 involved use of grids and spatial units of various shapes, so it was based on tools that were designed in such way to support experimentation and alternative representations.

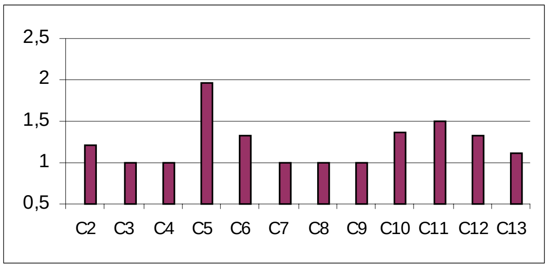


*Figure 5. Average number of solutions per strategy*

Another aspect concerns the order in which the strategies have been applied. Two indices have been defined the "First Strategy Index (FSI)" is assigned to the strategy that has been used first by a specific child, while the "Last strategy Index (LSI)" is assigned to the last strategy. The distribution of these indices in our user population is shown in figure 6.

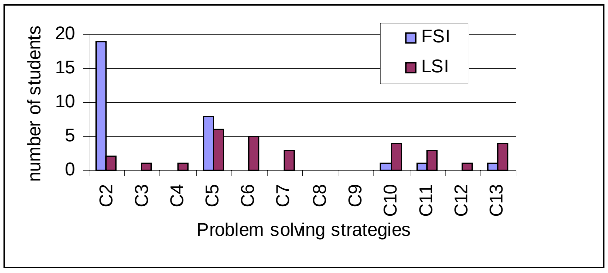


*Figure 6. First (FSI) and Last (LSI) strategy use*

From this figure one can deduce that while for the first strategy there is a bias towards the simplest strategies in terms of cognitive load and usability (C2, C5), the last strategy used is distributed more evenly among the problem-solving strategies.

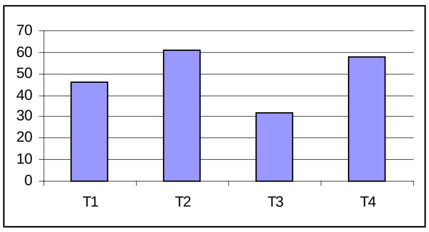


*Figure 7 Tools usage distribution*

**Analysis of tools usage**

An alterative approach in studying interaction of pupils with this environment involves analysis of the tools engaged in the above-described problem-solving strategies. As shown in Table 1, each strategy involved one or more

tools. By expressing the previous observations in terms of tools used, it is deduced that in 197 cases the pupils used the provided tools during this experiment, while in 6 cases they used additional tools from their background (mathematical formulas). The distribution of use of the four groups of tools is shown in figure 7.
From this figure a correspondence between the usability of the tools and the tools usage emerges. In the discussion of the introductory section it has been stated that the tools of group T3 present serious usability limitations. This is reflected in figure 7.
An alternative analysis of tools usage relates the number of cases a specific tool has been used by a pupil. As discussed in the previous section, a tool can be used in multiple strategies, so if a tool is used repeatedly in multiple problem solving strategies, this indicates that the student has mastered its use and felt comfortable to apply it in multiple alternative ways. This distribution is shown in the following Table 3.

| | *T1* | *T2* | *T3* | *T4* |
|---|---|---|---|---|
| *0* | 8 | 1 | 12 | 2 |
| *1* | 5 | 7 | 11 | 11 |
| *2* | 10 | 14 | 2 | 7 |
| *3* | 7 | 7 | 4 | 7 |
| *4* | 0 | 0 | 0 | 3 |
| *5* | 0 | 1 | 1 | 0 |
| *Σ* | 46 | 61 | 32 | 58 |

*Table 3. Distribution of number of times a certain tool has been used by a specific pupil*

The curves of distribution of tools in the users population, reflecting the contents of Table 3 are shown in figure 8.

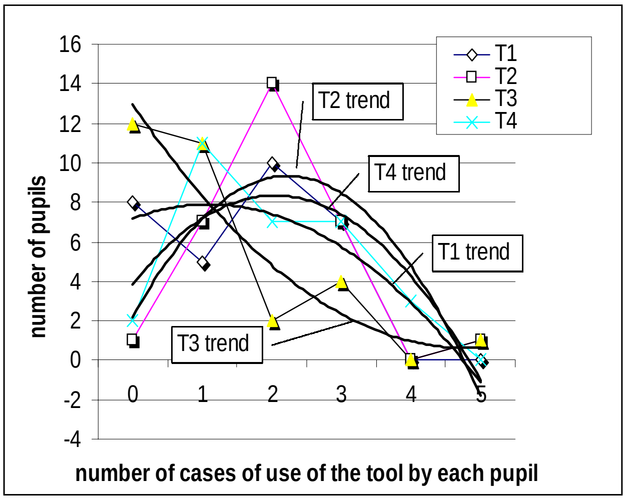


*Figure 8 Distribution of tools use in the pupils population*

The 3rd-order polynomial trend lines of these distributions indicate that while for tools T2, T4 and T1 a normal distribution of use is observed with a mean value around 2.1, the trend line of use of T3 is clearly different, as it has a peak at O and a negative slope.

## DISCUSSION

In this paper, we have attempted to study and discuss usage of multiple representations and tools by a group of young children in an open problem-solving environment. An extensive field study of use of the CARME learning environment is reported. An analysis of the findings of the experiment has been presented. Two complementary approaches during this analysis have been based on the problem solving strategies devised by the young children and the tools used in the frame of these strategies.
As shown in figure 6, the initial representation that most of the pupils used to face the task of comparison was the automatic measurement tool (77% of pupils used strategy C2, involving tool T4). These representations are 'opaque' fast and easily performed as automatical operations providing a numerical feedback, easily interpreted by the pupils. The most dominant representation, constructed by most of the pupils (83%) was based on the use of spatial units or grids to measure areas (strategy C5, tool T2). This representation is 'transparent' as it is spatial and visual and, as confirmed by the literature, matches the pupils' typical cognitive processes to face area measurement problems. Half of the pupils (strategy C13, tools T4, T1) used also the 'opaque' dynamic transformations of areas to equivalent ones in combination to the 'transparent' representations performed by using the simulation of pupils sensory motor actions in this computer environment The first of these representations used to overcome the non convex polygon's irregularity as well as to transform both areas under study in shapes of the same form in a fast way. The non-mathematical 'transparent' representation system that constitutes the simulations of pupils' sensory motor actions in this computer environment was used by half of the pupils (strategy C10, tool T1).
The automatic transformations of areas to other equivalent ones used by only one out of three pupils (category C11, tool T3). The representations of these equivalent shapes while they were 'opaque' but cognitively less complex to perform in terms of usability they presented serious limitations. So the pupils were less eager to apply them and to use them in combination with other tools in more complex problem solving strategies.
An interesting finding relates to the plethora of strategies invented by the pupils, combining the available tools. One out of three pupils combined the tools for automatic transformations with the use of spatial units (strategy C6, tools T2, T3). Representations as those described in strategies C4, C7, C8, C9 and C12 performed by fewer pupils. Representations corresponding to strategies C8, C9 and C12 included the use of area formulae that were not explicitly supported by any tool in this environment.

## CONCLUSIONS

This study shows that pupils used all the provided tools separately as well as in combination and constructed a variety of representations of area measurement. Pupils also constructed representations based on their previous knowledge despite the fact that they are not explicitly supported by the provided tools. As a conclusion, the design of tools should be done in such a way that provisions are made for non-anticipated use of them and for their inter-operability. The previous knowledge of the pupils must also not be ignored in this design.

This study also showed that pupils preffered the quick and easy constructed representations despite their 'opaqueness'. Moreover, the transparent representations were choosen by most of the pupils. The usability of the tools affects the way these are incorporated in problem solving, sometimes not matching the cognitive complexity of the underlying theoretical approach. It is therefore important to incorporate usability evaluation studies in design, in order to assure that not extra cognitive load is imposed on the young children due to usability issues.

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