# The potential of multiple-solution tasks in e-learning environments: Exploiting the tools of Cabri Geometry II

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**Abstract:** This study focuses on the potential of multiple-solution tasks in e-learning environments providing a variety of learning tools. This was presented through a multiple-solution-based example for the learning of the mathematical notion of angle in the context of the well known e-learning environment Cabri-Geometry II (Laborde, 1990) dedicated for the learning of geometrical concepts. An a-priori task analysis showed that a variety of solution strategies could be invented by the students to face this type of task. In fact, students can select among the provided tools the most appropriate to express their knowledge. In the integrated context of such tasks and tools, students can express both inter-individual and intra-individual differences in the learning concepts in focus. In addition, students can consolidate these concepts, integrate the different kinds of knowledge they possess, enhance their learning styles and aquire advanced problem-solving skills.

## Introduction

As student inequality in school success has been related to their different learning styles, appropriately dealing with different student learning styles is clearly of major interest to all educators. To deal with these different styles, the proposed solutions fall into one of the following categories: a) changing the learning style of the learner so that they will better 'fit' the educational system (Corno & Snow, 1984; Bradley, 1985), b) changing the teaching and evaluation methods of the educational system so that it can match the variety of learner differences (Joyce & Weil, 1972; Bennett, 1977) and c) changing both the system and the learner so that every learner can master more than one learning style by providing more than one teaching and evaluative approach (Ramby, 1984; Lemerise, 1992). Nowadays, modern constructivist and social learning theories acknowledge the role of appropriately-designed tools and activities in order to help students express their inter-individual and intra-individual variety as well as acquire a variety of learning styles (von Glasersfeld, 1987; Vygotsky, 1978; Noss & Hoyles, 1996; Nardi, 1996).

Constructivist computer learning environments can act as catalysts in the whole learning context and can play an essential, crucial and exceptional role in student learning (Hillel, 1993; Dorfler, 1993; Noss & Hoyles, 1996). These environments can also act as scaffolding factors for the development of mathematical activity, while also helping students to reorganize their thinking (Hoyles & Noss, 1989; Hillel, 1993) and to make connections between informal and typical mathematics (Noss, 1988) as well as to experiment and make generalizations exploiting their experience (Hoyles & Noss, 1989). In particular, computer learning environments providing a variety of tools of different cognitive transparency could encourage students to select the most appropriate to express their knowledge. These different tools can provide students with opportunities to study the concepts in focus in multiple representation systems in order to express both inter-individual and intra-individual varieties (Dyfour-Janvier, Bednarz & Belanger, 1987; Janvier, 1987). Most learner difficulties are found in the gap between their intuitive knowledge and the knowledge they need to express in the proposed representation systems (RS) for use (Janvier, 1987). For example, prepositional, symbolic and abstract representation systems prevent some learners (usually beginners) from expressing their knowledge, the same systems being intended for use by advanced learners. Contrariwise, metaphors of everyday life and visual RS are more suitable for beginners. Multiple RS have been proposed to enable individual learner variety as well as to enable each learner to acquire a broader view of the subject to be learned (Kordaki, 2003).

Learning activities also play a significant and central role in the entire learning process. Constructivist design of learning activities emphasizes the fundamental concepts of the learning subject in focus and not its details (von Glasersfeld, 1987; Vygotsky, 1978; Nardi, 1996). Consequently, emphasis is placed on the necessary activities to be performed by students for their effective learning of the subject matter. Moreover, holistic learning activities can help learners to acquire a global view of the learning subject in focus. Problem-solving activities that put learners in an investigative mode can encourage them to construct their knowledge actively as well as to acquire certain essential problem-solving skills. In this way, learners could be encouraged to form and verify conjectures, hypotheses and formulas as well as to develop self-correction strategies. In addition, it is essential that student difficulties in the understanding of the concepts in focus be taken into account. Finally, the kind of activities that can help learners express their inter-individual and intra-individual varieties is of major interest. To this end, multiple-solutionactivities to be performed in contexts providing a variety of tools are the most appropriate. Students can select the tools that suit them in order to express their knowledge by constructing an individual solution to the given tasks. Thus, students can be encouraged to express their inter-individual differences. Furthermore, each individual student can express a different kind of knowledge regarding the learning subject in focus. As a result, they can express their intra-individual variety and acquire different learning styles.

With the above in mind, the design of a learning activity that could be performed by using a variety of solution strategies is presented in this paper. The aforesaid activity has been designed for the learning of the concept of angle in Euclidean plane geometry. This concept is of great significance as it is related to the whole context of Euclidean geometry. The concept of angle is a kind of 'inter- geometrical' concept, as it is related to a variety of concepts, such as straight lines, and a variety of geometrical figures such as triangles, polygons, circles, etc. To support the performance of this activity with a variety of tools, the well-known educational software Cabri-Geometry II has been used. This variety of tools can mediate student actions in constructing a variety of solution strategies for the aforesaid activity. Such design of multiple-solution learning activities for the learning of the concept of angle by primary and secondary level education students in the context of Cabri-Geometry II has not yet been reported.
In the next section of the paper, a brief description of Cabri-Geometry II as a learning environmt is presented, with special reference to the kind of activities that could be performed by using its tools and operations, followed by the description of the proposed task and an a-priori analysis of its multiple solutions. Subsequently, the importance of the proposed task is discussed and conclusions drawn.

## Cabri Geometry II as a learning environment

Cabri Geometry II (Laborde, 1990) is widely known educational software designed to support constructivist mathematical learning settings. In particular, Cabri provides students with potential opportunities in terms of: a) *means of construction*, providing a rich set of tools to perform a variety of geometrical constructions referring to a variety of concepts concerning Euclidean Geometry. These tools can be exploited by students to perform a number of different geometrical constructions and to deal with a variety of geometrical problems. *b) Tools to construct a variety of representations, both numerical and visual*, such as geometrical figures, tables, equations, graphs and calculations. These representations are of different cognitive transparency; consequently, students can select the most appropriate tools to express their knowledge. In this way, students have the possibility of expressing both inter-individual and intra-individual differences. The representation systems used also affect the kind of knowledge that students construct (Kordaki, 2003; Mariotti, 1995) c) *Linking representations*, by exploiting the interconnection of the different representation modes provided. d) *Dynamic, direct manipulation of geometrical constructions* by using the 'drag mode' operation. This operation gives learners the possibility of experimenting with geometrical constructions and forming and verifying hypotheses and conjectures by handling, in a physical sense, the theoretical objects which appear as diagrams on the computer screen (Laborde and Laborde, 1995). In these Cabri-constructions, their geometrical properties are retained under dragging, while their visual output differs. The 'drag mode' can be used in three modes: as an 'exploratory' mode, a 'verification' mode and an 'adjustment' mode (Kordaki and Balomenou, 2006). By using 'drag mode', students can also form dynamic views of the concepts in focus (Mariotti, 1995). e)*The possibility of collecting large amounts of numerical data*. Cabri provides the opportunity for automatic tabulation of large amounts of specific numerical data viewed as appropriate for the study of the geometrical concept in focus. In particular, the 'drag mode' can be used in combination with automatic measurements of specific elements of the geometrical constructions under study. These measurements can be automatically tabulated, providing learners with opportunities to reflect on them and form and verify conjectures about specific geometrical concepts and relationships. f) *Interactivity and feedback*; intrinsic visual feedback and extrinsic numerical feedback, providing learners with opportunities to form and verify conjectures as well as to self-correct their constructions. This is important as learner actions are closely connected with their consequences, contrary to the static and silent paper and pencil environment where there is no possibility of providing immediate response to those actions (Kaput, 1994), g) *Presenting information to the students in text form*, for example, the presentation of the tasks at hand*, h) Capturing the history of student actions* to provide teachers and researchers with a valuable amount of data for further studies, and *i) Extension.* Certain operations could be added as buttons on the Cabri interface following the formation of specific macros.

Cabri has strong capabilities for the design of learning activities that encourage learners to: take an investigative perspective, express their inter-individual and intra-individual learning differences, make self-corrections, formulate and verify conjectures and exploit the advantages from the negotiation of their knowledge with the knowledge of their classmates in cooperative settings (Straesser, 2001; Kordaki & Balomenou, in press). In addition, authentic meaningful real life learning activities can be integrated within the context of Cabri, activities that can develop strong learner motivation.

Six types of learning activities have been designed to be performed by students in the context of Cabri-Geometry II (Laborde, 2001, Kordaki and Balomenou, in press, Kordaki, 2005), namely: a) *Forming/verifying conjectures by focusing on the alteration of a geometrical construction using the drag-mode operation.* For example, when a student draws a triangle and its medians they can conjecture that 'the medians of a triangle always intersect internally in a single point'. In this way, students can also verify this conjecture, formed in some way during their experience. b) *Forming/verifying conjectures by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation.* A case in point is when a student draws an angle AOB, its bisector OD and the distances of a point K of OD from AO and BO, then automatically measures AO and BO, while at the same time dragging point K, and tabulates the data automatically produced by the aforementioned measurement operations. By focusing on these numerical data, this student can conjecture that 'each point of the bisector of an angle is equidistant from its sides'. In this way, students also can verify their conjectures formed in some way (eg. using their visual perception) during their experience. c) *Verifying a formula by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation.* For instance, students can verify the truth of the formula expressing the Pythagorean theorem a^2=b^2+c^2 in right-angled triangles, when 'a' is the hypotenuse of these triangles and 'b', 'c' the remaining two sides. d) *Black-box activities.* Students can participate in activities where they have to explore geometrical constructions with some of their properties hidden, which they then have to discover. To illustrate this, students can be asked to justify why the sum of the opposite angles of a quadrilateral included in a specific class of such figures is 180 degrees, as displayed by their automatic measurement. e) *Constructions simulating real life problems*. Such real life problems can help students to develop strong motivation in their learning and approach mathematics as a human activity (Bishop, 1988) as well as put mathematical concepts into an interdisciplinary context (Clements, 1989). f) *Multiple-solutionactivities.* As Cabri provides a variety of tools and operations, these can be effectively combined to support the performance of multiple-solution activities. In the context of these activities, it is possible to integrate all the possibilities provided by Cabri with any of the possible different types of activities. In the next section of this paper, the design of such a specific example and an a-priori of its potential solutions are presented.

## The design of the proposed learning task

**The task:** The proposed task could be presented to the students, verbally or in writing, asking them to: a) 'construct pairs of equal angles, in as many ways as possible, using any of the tools provided by Cabri' (at this point, students could be informed, by the researcher, that equal angles are angles with equal sizes), b) 'justify your solution strategy' and c) 'try another way of constructing another pair of angles with equal sizes. You can use other tools and the different kinds of knowledge you possess' (when students seem to be on the point of giving up, this intervention could involve them in the task and encourage them to continue). It is also essential to encourage students to work in groups, so as to exploit the advantages of cooperation.

The learning aims of this task are to enable students to: i) express their inter-individual and intra-individual differences regarding the notion of equal angles, ii) distinguish the concept of angle from the length of its sides by studying these concepts in relation to each other, and iii) link and integrate different kinds of knowledge about the concept of angle through using the diversity of the tools provided, at the same time enhancing their learning styles and developing a broad view of this concept.

**A-priori analysis of the possible solution strategies**

To document the importance of the task proposed in the aforementioned section, an a-priori analysis of the possible solution strategies (Si, i=1,…18) that could be performed by the students is presented in the following section. The possible learning outcomes that could be acquired by student performance for each strategy are also presented. To clarify the presentation of these strategies, some solutions are also presented in graphic form in Figures 1 and 2. The expected strategies are classified into the categories presented and reported below.

**Pairs of equal angles and students' visual perception:** These strategies could be formed by:
*S1: using the 'eye':* Starting from the construction of a first angle, construct a second angle, and by using perception and possible control by measurement tools, try to obtain an angle equal to the first one.
*Possible learning outcomes:* Strategy S1, which is mainly based on students' visual perception, is considered to be a primary approach to the concept of angle, the other strategies reported in the following section being considered more advanced.

**Pairs of equal angles and the direct manipulation of computational objects:** These strategies could be formed by:
*S2: using the drag mode operation in combination with automatic angle measurement:*
*S2a:*

- Constructing two angles ABC and ZKL
- Measuring the size of the angles ABC and ZKL automatically
- Dragging the vertices of the angle ZKL to find different instances where its size is equal to that of the angle ABC.

*Possible learning outcomes:* A primary estimation of equality in angles.
S2b:

- Constructing an angle ABC
- Measuring the size of the angle ABC and the length of its sides AB, BC automatically
- Dragging the sides AB and BC of the angle ABC and tabulating the numerical data produced.

*Possible learning outcomes:* To distinguish between the concept of angle and the length of its sides.

**Pairs of equal angles and basic geometrical transformations:** These strategies could be formed by:
S3: *using Cabri-commands for geometrical transformations*:

- Translation *(Strategy S3a)*
- Reflection about an axis *(Strategy S3b)*
- Symmetry *(Strategy S3c)*
- Rotation *(Strategy S3d)*

*Possible learning outcomes:* An understanding of the conservation of the size of an angle through basic geometrical transformations.

**Bisecting angles:** These strategies could be formed by:
*S4: splitting an angle into two equal angles using its bisector*
*Possible learning outcomes:* An understanding of the basic property of the bisector of an angle.

**Pairs of equal angles and straight lines:** These strategies could be formed by:
*S5: constructing vertical angles*

- Constructing two lines e1 and e2 intersecting at point A.
- Comparing the vertical angles produced by measuring them automatically

*Possible learning outcomes:* An understanding of the equality of vertical angles.

*S6: constructing pairs of acute angles with vertically intersecting corresponding sides.*
*Possible learning outcomes:* Understanding equality in angles produced by specific geometrical constructions.

*S7: constructing pairs of acute angles with corresponding parallel sides.*
*Possible learning outcomes:* Understanding equality in angles produced by specific geometrical constructions.

*S8: constructing a transversal which intersects two parallel lines and investigating the size of:*

- alternate interior angles
- interior-exterior angles on the same side
- alternate exterior angles

*Possible learning outcomes:* Understanding equality in angles produced by specific geometrical constructions.

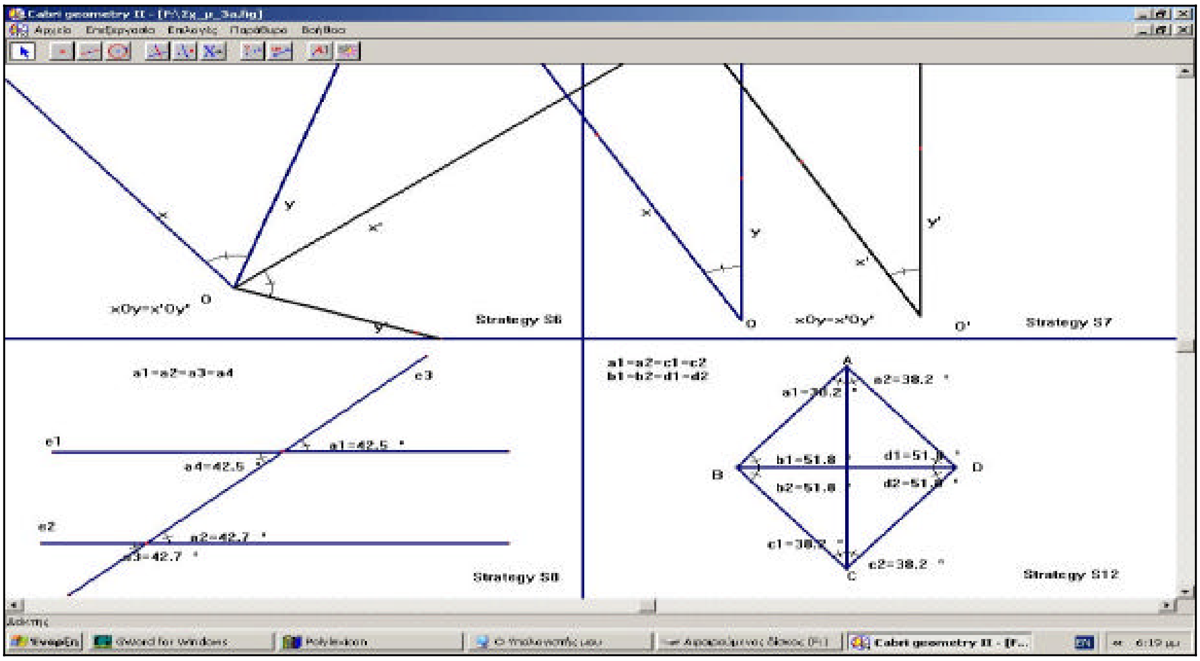


**Figure 1.** Examples of the possible solution strategies S6, S7, S8 and S12

**Pairs of equal angles and specific geometrical figures:** These strategies could be formed by:
*S9: constructing similar triangles and comparing the corresponding angles*
*Possible learning outcomes:* An understanding of similarity in terms of equality in angles.

*S10: constructing an isosceles triangle and comparing the angles of its base*
*Possible learning outcomes:* An understanding of isosceles triangles in terms of equality in angles.

*S11: constructing an equilateral triangle and comparing its angles*
*Possible learning outcomes:* An understanding of equilateral triangles in terms of equality in angles.

*S12: bisecting angles:* bisecting the angles of a rhombus using its diagonals
*Possible learning outcomes:* Understanding basic properties in a rhombus in terms of equality in angles.

*S13: regular polygons:* Contructing regular polygons and investigating the equality of their angles.
*Possible learning outcomes:* Understanding basic properties in regular polygons in terms of equality in angles.

**Pairs of equal angles and circles:** These strategies could be formed by:
*S14: central angles of a circle in equal arcs.* Constructing a circle (O, R), two equal arcs AB, CD and the corresponding angles AOB and COD.
*Possible learning outcomes:* Understanding equality in central angles of a circle.

*S15: inscribing angles in a circle on the same arc.*
*Possible learning outcomes:* Understanding equality in inscribed angles of a circle.

*S16: angle between a tangent and a chord of a circle:* constructing the angle between a tangent and a chord of a circle and the corresponding inscribed angle on the same arc of this chord. Here, two pairs of equal angles could be

considered: a) the acute angle and its corresponding inscribed angle on the minor arc of the chord in focus and b) the obtuse angle and its corresponding inscribed angle on the major arc of said chord.
*Possible learning outcomes:* Understanding specific geometrical constructions producing pairs of equal angles in a circle.

S17: *circle and its tangents:* constructing the pair of angles produced by constructing two tangents of a circle radiating from a point A (outside this circle) and the straight line through this point and the center of this circle.
*Possible learning outcomes:* Understanding basic geometric properties of a tangent of a circle.

*S18: Intercected circles:* Constructing two circles (O1, R1) and (O2, R2) intersecting at points A and B. Constructing segments O1A, O1,B, O2A, O2B and O1O2. Considering the equality of the following pairs of angles, namely: a) O1AO2, O1BO2, b) AO1O2, AO2O1, c) BO1O2, BO2O1.
*Possible learning outcomes:* Integrating equality in angles with intersecting circles.

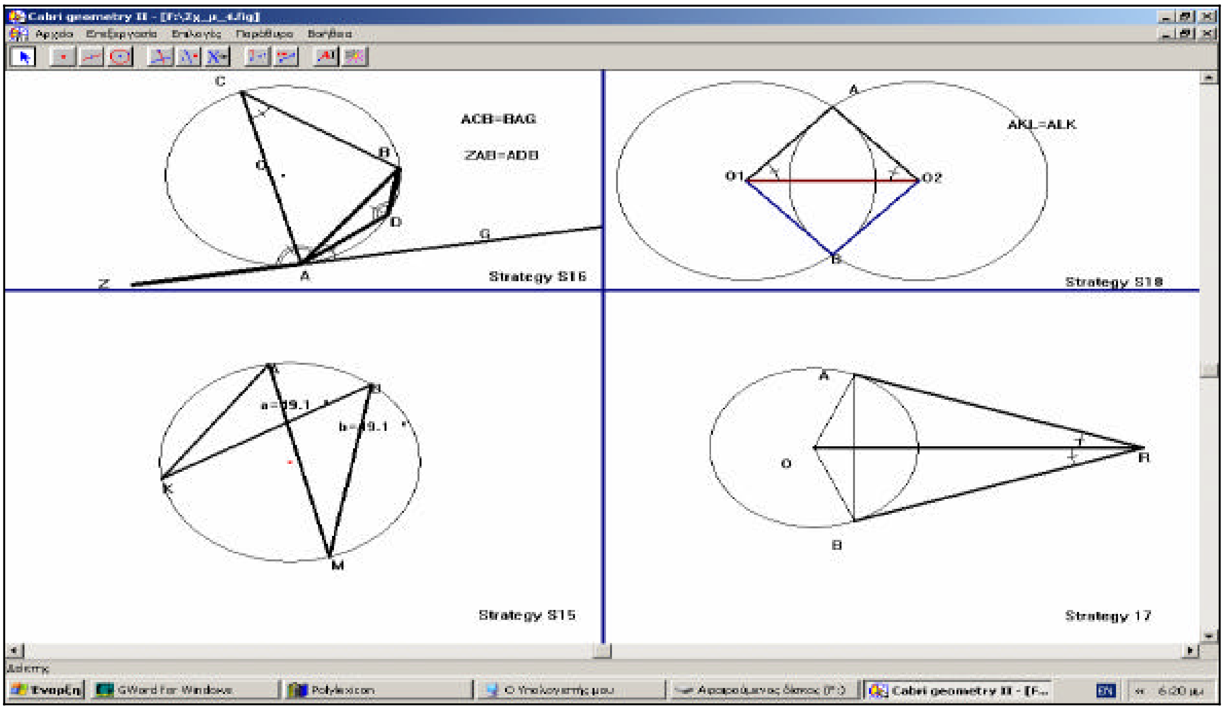

**Figure 2.** Examples of the possible solution strategies S15, S16, S17 and S18

## Discussion

The potential of multiple-solution tasks in the context of e-learning environments providing a plethora of tools is presented through a specific example in this paper. The potentiality of these tasks is that these can enable students not only to use all the provided tools in as many ways as possible but also to use all the knowledge they possess to construct an abundance of solution strategies. In fact, richness in the performance of these tasks entails a cornucopia of tools. Bearing this in mind, the e-learning environment used for posing the proposed task was the well-known Cabri-Geometry II, designed to provide a number of tools for the learning of a variety of concepts in Euclidean Geometry. The analysis of the potential of multiple-solution tasks was arrived at through the presentation of a specific task designed for the learning of the concept of angle. This particular concept was selected as it is of general interest and can be characterized as an inter-geometrical concept. The a-priori analysis of the possible solutions that could be constructed by students to perform this task shows that at least eighteen solution strategies could be performed by students exploiting the presence of the variety of tools in the Cabri interface. Obviously, a number of strategies were directly related to the conceptual meaning of an angle, to the relationship of an angle and its sides as well as to the properties of its bisector. However, a variety of potential solution strategies were related to a wide range of concepts in Euclidean geometry, namely: basic geometric transformations (e.g. symmetry, rotation, and parallel translation), specific geometrical constructions producing equal angles using different types of lines (e.g. parallel, perpendicular and arbitrary lines), different forms of triangles (e.g. scalene, isosceles and equilateral triangles), specific polygons (e.g. rhomb i and regular polygons), circles, tangents and chords as well as inscribed and central angles in a circle. Obviously, these activities could be used to help students in reviewing, consolidating and concentrating on the aforementioned concepts.

With the task at hand in mind, while at the same time acting within the context of the provided tools , students have opportunities to select from among the provided tools those most appropriate to express their inter-indivdual differences and to form at least one solution strategy. Consequently, their self confidence could be boosted, their perception of mathematics improved and, through being taught by their classmates, the mastering of more than one solution strategy achieved. In this way, students can enhance their learning styles regarding the concepts in focus. By demanding the construction of multiple solutions to this task, we allowed for the expression of both correct and incorrect solution strategies. By allowing students to express their intra-individual learning differences regarding a specific learning subject, we gave them the opportunity to clarify their solutions and correct their mistakes. Finally, in this problem solving process, students could reject the single-solution-based perspective to problems and establish a more flexible perspective based on alternative solutions and the formation of specific criteria for best solution selection. In addition, by demanding student production of a variety of solution strategies, we encouraged them to use higher mental functions and develop broader views on the learning concepts than those views they could aquire during a single-solution-based task.

## Conclusions and future work

This paper presented the potential of multiple-solution-based tasks performed in e-learning environments providing a variety of tools. This was presented through a multiple-solution-based example for the learning of the mathematical notion of angle in the context of the widely known e-learning environment, Cabri-Geometry II. An a-priori task analysis shown that at least 18 solution strategies could be invented by the students to deal with this task. To support the invention of a variety of strategies, an environment providing a variety of tools is necessary. In addition, the learning concept in focus has to be selected from among ones of global interest and value. In the context of this type of tasks, students have the opportunity to select from among the provided tools those most appropriate for expressing their inter-individual and intra-individual learning differences. Students can also integrate the different kinds of knowledge they possess, review and consolidate the learning concepts in focus and, as a result, form a broad view of these concepts. In addition, students can exploit the knowledge of their classmates to enhance their knowledge and to acquire different learning styles. Finally, by engaging students in multiple-solution-based tasks, they can use their higher mental functions and acquire advanced problem-solving skills. In the future, we plan to try this task in the field with real students.

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