The concept of similarity in triangles within the context of tools of Cabri-Geometry II

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This paper presents the role of tools provided by the well known educational software Cabri-Geometry II [1] or the construction of the concept of similarity in triangles by the students. An a priori analysis showed that a variety of solution strategies could be invented by the students to tackle this concept in the context of the tools of Cabri-Geometry II. In fact, at least seventeen solution strategies can be performed by the students. For the construction of these strategies a variety of geometrical concepts could be integrated with the concept of similarity in triangles such as: the specific circles of a triangle (circumscribed circle, inscribed circle, and escribed circle), regular polygons, different forms of triangles (isosceles triangles, right-angled triangles), specific linear elements of a triangle (altitude of a triangle, bisector of an angle; medians of a triangle, parallel segments joining the mid points of two sides of a triangle), parallel and perpendicular lines, angles, as well as specific elements of a circle such as tangents of a circle and chords intersected outside and inside of it.

Keywords Similarity in triangles; Cabri-Geometry II; multiple representations; learning activities; multiple solution strategies

1. Introduction

The concept of similarity is essential for the students to grasp as it consists part of both mathematics and of their every-day life. Students have difficulty in understanding the concept of similarity as they confuse it with equality. Similarity constitutes a relationship between shapes/figures. A shape F1 is similar to a shape F2 if there is a transformations such as s(F1)=F2. In fact, the concept of similarity has a dynamic character as it implies an understanding of the fact that the size of the angles of a rectilinear figure could be conserved although the length of its sides and its area are altered according some specific ratio [2]. In addition, the concept of similarity is not an isolated geometrical concept but it is also interconnected to a variety of geometrical concepts. In fact, similar shapes can be produced as results of specific geometric transformations as well as results of specific geometrical constructions. To help students acquire a dynamic perspective regarding similarity as well as to integrate this concept in a wide context of geometrical concepts the role of Dynamic Geometry Systems (DGS) is crucial.

Cabri Geometry II [1] is a widely known DGS that provides students with potential opportunities in terms of: a) Means of construction, providing a rich set of tools to perform a variety of geometrical constructions referring to a variety of concepts concerning Euclidean Geometry. b) Tools to construct a variety of representations, both numerical and visual, such as geometrical figures, tables, equations, graphs and calculations. c) Linking representations, by exploiting the interconnection of the different representation modes provided. d) Dynamic, direct manipulation of geometrical constructions by using the 'drag mode' operation. In fact, Cabri-Geometry II provides possibilities for dynamic transformations of the geometrical constructions presented on the screen of the computer by using the 'drag mode' operation. These types of geometrical constructions retain their geometrical properties through dragging. By experimenting with this type of dynamic transformations, students can form hypotheses, generalizations and abstractions regarding the learning of the concepts in focus [3]. e)The possibility of collecting large amounts of numerical data. f) Interactivity and feedback; intrinsic visual feedback and extrinsic numeri-

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cal feedback, providing learners with opportunities to form and verify conjectures as well as to self-correct their constructions. g) Presenting information to the students in text form, for example, the presentation of the tasks at hand, h) Capturing the history of student actions to provide teachers and researchers with a valuable amount of data for further studies, and i) Extension. Certain operations could be added as buttons on the Cabri interface following the formation of specific macros ([4], [5], [6], [7], [8]).
Cabri-Geometry II has been also designed to support constructivist mathematical learning settings. In these settings the role of learning activities in student motivation to be actively engaged in the construction of their knowledge is crucial ([9], [10]). In the context of Cabri-Geometry II, six types of learning activities have been designed to be performed by students ([3], [11], [12], [13], [14], [15]), namely: a) Forming/verifying conjectures by focusing on the alteration of a geometrical construction using the drag-mode operation. b) Forming/verifying conjectures by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation. c) Verifying a formula by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation. d) Black-box activities where students are asked to discover geometrical properties which are implied in the geometrical construction at hand. e) Constructions simulating real life problems. f) Multiple-solution activities where students are asked to face the problem at hand in 'as many ways as possible' by using any of the tools provided. These activities can be used to support students to select among the provided tools the most appropriate to express their knowledge. In the integrated context of such tasks and tools, students can express both inter-individual and intra-individual differences regarding the learning of the concepts in focus. In addition, students can consolidate these concepts, integrate the different kinds of knowledge they possess, enhance their learning styles and acquire advanced problem-solving skills.
With the above in mind, we designed a multiple solution based task to be performed by exploiting the variety of tools provided by Cabri-Geometry II aiming at the encouragement of high school students to grasp the concept of similarity in triangles. These shapes were selected to be studied as they are fundamental in the context of Eucledian geometry. Despite the fact that, literature has shown the use of DGS in teaching similarity – thgrough symmetry and transformations that can occur in a shape [16]- such a task has not yet been reported. In the next section of this paper the design of the said task is presented followed by an a priori analysis of the possible solutions to be performed by the students. Finally, conclusion remarks and proposals for future research are drawn.

2. The design of the task

The task: The proposed task could be presented to the students verbally or in paper and pencil, asking them to: a) 'construct pairs of similar triangles, in as many ways as possible, using any of the tools provided by Cabri' (at this point, students could be informed, by the researcher, that similar triangles are triangles with equal angles), b) 'justify your solution strategy' and c) 'try another way of constructing another pair of similar triangles. You can use other tools and the different kinds of knowledge you possess' (when students seem to be on the point of giving up, this intervention could involve them in the task and encourage them to continue). It is also essential to encourage students to work in groups, so as to exploit the advantages of cooperation.
The learning aims of this task are to enable students to: i) express their inter-individual and intra-individual differences regarding the notion of similarity in triangles, ii) distinguish the concept of equality from the concept of similarity in triangles by studying these concepts in relation to each other, and iii) link and integrate different kinds of knowledge about the concept of similarity in triangles through using the plethora of the tools provided, at the same time developing a broad view of this concept and consequently enhancing their learning styles. To document the importance of the task proposed in the aforementioned section, an a priori analysis of the possible solution strategies (Si, i=1,...17) that could be performed by the students is presented in the following section. The possible learning outcomes that could be acquired by student performance for each strategy are also presented. To clarify the presentation of these strategies, some solutions are also presented in graphic form in Figures 1 and 2. The expected strategies are classified into the categories presented and reported below.

A-priori task analysis

The potential solution strategies for the construction of similar triangles are presented below:

Pairs of similar triangles and students' visual perception: These strategies could be formed by:

S1: Starting from the construction of a first triangle, construct a second triangle, and by using perception and possible control by measurement tools, try to obtain a triangle equal to the first one.

Possible learning outcomes: This strategy is considered to be a primary approach to the concept of similarity in triangles as it is mainly based on students' visual perception, the other strategies reported in the following section being considered more advanced.

Abundance of similar triangles by dynamically transforming a triangle's specific circles: circumscribed, inscribed, escribed. These strategies could be formed by:

S2: Constructing a plethora of similar triangles with their sides as tangents in a circle and dynamically altering the radius of this circle. This circle is inscribed in the constructed triangles.

S3: Constructing a diversity of similar triangles with their sides to be tangent in a circle so that this circle become escribed in these triangles and dynamically altering its radius.

S4: Constructing a variety of similar triangles by dynamically altering the radius of its circumscribed circle.

Possible learning outcomes: Understanding similarity in triangles in terms of tangents in a circle, perpendicular lines, bisector of an angle (S2 and S3), the inscribed circle in a triangle (S2), the escribed circle in a triangle (S3) as well as angles of a triangle, inscribed and central angles on the same arc in a circle, isosceles triangles, and circumscribed circle in a triangle (S4).

Dynamic transformations of pairs of triangles with corresponding parallel and perpendicular sides. These strategies could be formed by:

S5: Constructing a variety of pairs of similar triangles with parallel sides and dynamically altering the length of these sides.

S6: Constructing a variety of pairs of similar triangles with perpendicular sides and dynamically altering the length of these sides.

Possible learning outcomes: Understanding similarity in triangles in terms of parallel (S5), perpendicular lines and angles (S6).

Dynamic transformations of a regular polygon and focusing on the triangles constructed by using its diagonals. These strategies could be formed by:

S7: Forming a variety of similar triangles by constructing regular polygons, focusing on the triangles constructed by using their diagonals and next, dynamically altering the radius of its circumscribed circle.

Possible learning outcomes: Understanding similarity in triangles within the context of regular polygons.

Dynamic transformations of an original triangle and the triangles produced by exploiting the properties of the mid points and the medians of the original. These strategies could be formed by:

S8: Forming a variety of pairs of similar triangles by constructing an original triangle and a triangle whose vertices are the midpoints of the original and dynamically altering its figure.

S9: Forming a plethora of similar triangles by constructing an original triangle ABC and another triangle whose vertices are the points at a distance equal to 1/3 of the correspondent median from each vertex of ABC. Next, dynamically altering the figure of ABC.

Possible learning outcomes: Understanding similarity in triangles in combination with basic points of a triangle (S8) and basic lines of a triangle (S9).

Dynamic transformations of pairs of triangles and of families of triangles produced by exploiting the geometrical properties of perpendicular lines and of a triangle's altitudes. These strategies could be formed by:

S10: Forming a variety of pairs of similar triangles by constructing an original right triangle ABC (A=90°) and the right triangle EDC, where; D is an arbitrary point on AC or AB and the DE is a perpendicular segment from point D to the hypotenuse of ABC. Then, dynamic alteration of ABC.
Possible learning outcomes: Understanding similarity in right triangles in relation to perpendicular lines.

S11: Forming a variety of pairs of similar triangles by constructing an original right-angled triangle ABC (A=90°) and the triangles produced by the altitude to the hypotenuse. It is worth noting that, this procedure can be realized in the produced triangles. In this way, an unlimited number of similar triangles can be produced. In addition, by dynamically altering the figure of ABC an unlimited number of families of similar triangles can be produced.

Possible learning outcomes: Understanding similarity in right-angled triangles in combination with the properties of their altitudes. Students can also form an algorithm of construction of similar triangles.

S12: Forming a variety of pairs of similar triangles by constructing an original triangle ABC and the triangle DBE where D and E are the intersection points of the altitudes AD and CE with BC and AB respectively. It is worth noting that in this construction the triangles ADB and CEB are also similar. By dynamically altering ABC an unlimited number of pairs of similar triangles can be produced.

Possible learning outcomes: Understanding similarity in scalene triangles in combination with the properties of their altitudes.

S13: Forming a pair of similar triangles by constructing an original triangle ABC (B>90°, and B=90+C) and the triangle ABD where D is the intersection point of the altitude AD to BC.

Possible learning outcomes: Understanding similarity in obtuse specific triangles in combination with the properties of their altitudes.

Dynamic transformations of specific geometrical constructions producing pairs of similar triangles by exploiting the geometrical properties of a circle. These strategies could be formed by:

S14: Constructing two chordes AB, CD on a circle (O, K) intersecting on point R outside this circle. Next constructing the pair of similar triangles ADR and BCR. Forming a variety of pairs of similar triangles by dynamically altering the radius of the circle (O, K).

S15: Constructing two chordes AB, CD on a circle (O, K) intersecting on point R outside this circle. Next, constructing a pair of similar triangles BDR and ACR. Forming a variety of pairs of similar triangles by dynamically altering the radius of the circle (O, K).

S16: Constructing a triangle ABC inscribed on a circle (O, K) and the bisector of angle A intersecting the circle (O, K) on point E and the segment BC on point D. The triangles ABE and BDE are similar. Forming a variety of pairs of similar triangles, by dynamically altering the radius of the circle (O, K).

S17: Constructing two circles (K, R1) and (O, R2) and two tangents KA, KB from K to circle (O, R1) which intersect circle (K, R1) on points C and D respectively . Triangles ABK and CDK are similar. Forming a variety of pairs of similar triangles, by dynamically altering the radius of the circles (K, R1) and (O, R2).

Possible learning outcomes: Understanding similarity in triangles in combination with basic properties of a circle such as tangents, inscribed and central angles.

3. Conclusions

This paper presented the role of tools of Cabri Geometry II for the construction of the concept of similarity by students at secondary education level. This was presented through the design and an a priori analysis of a task to be solved 'in as many ways as possible' by exploiting the variety of tools provided by Cabri-Geometry II. The analysis of possible solutions showed that at least seventeen solutions can be performed by the students. These strategies fall in the following categories: a) Dynamic transformation of a triangle's specific circles: circumscribed, inscribed, escribed (3 strategies), b) Dynamic transformations of pairs of triangles with corresponding parallel and perpendicular sides (2 strategies), c) Dynamic transformations of a regular polygon and focusing on the triangles constructed by using its diagonals (1 strategy), d) Dynamic transformations of an original triangle and the triangles produced by exploiting the properties of the mid points and the medians of the original (2 strategies), e) Dynamic transformations of geometrical constructions of pairs of triangles and of families of triangles produced by exploiting the geometrical properties of perpendicular lines and of a triangle's altitudes (4 strategies), f) Dynamic transformations of specific geometrical constructions producing pairs of similar triangles by exploiting the geometrical properties of chords intersected inside and outside a circle and its tangents, bisector of an angle and inscribed angles (4 strategies) and g) Similar triangles constructed by using visual perception. By performing this variety of solution strategies, students can integrate similarity in triangles with a variety of geometrical concepts, namely: the specific circles of a triangle (circumscribed circle, inscribed circle, and escribed circle), regular polygons, different forms of triangles (isosceles triangles, right-angled triangles), specific linear elements of a triangle (altitude of a triangle, bisector of an angle; medians of a triangle, parallel segments joining the mid points of two sides of a triangle), parallel and perpendicular lines, angles, as well as specific elements of a circle such as tangents of a circle and chords intersected outside and inside of it. However, it is our suggestion that more research should be carried out with a view to illuminating the potential of this task by testing it in the field with real students.

4. References

[1] Laborde, J-M. (1990). *Cabri-Geometry* [Software]. France: Universite de Grenoble.

[2] Vollrath, H.J. (1977). The understanding of Similarity and Shape in Classifying Tasks. Educational Studies in Mathematics, 8, 211-224.

[3] Laborde, C. and Laborde, J-M. (1995). What about a Learning Environment where Euclidean Concepts are manipulated with a mouse? In A. diSessa, C. Hoyles, R. Noss with L. Edwards (Eds), *Computers and Exploratory Learning* (pp.241-261), Berlin: Springer-Verlag.

[4] Hillel, J. (1993). CA as Cognitive Technologies: Implication for the Practice of Mathematics Education. In C. Keitel and K. Ruthven (Eds), *Learning from computers: Mathematics Education and Technology* (pp. 18-47). Berlin: Springer-Verlag.

[5] Dorfler, W. (1993). Computer use and views of the mind. In C. Keitel & K. Ruthven (Eds), *Learning from computers: Mathematics Education and Technology* (pp.159-186). Berlin: Springer - Verlag.

[6] Dyfour-Janvier, B., Bednarz, N.,& Belanger, M. (1987). Pedagogical considerations concerning the problem of representation. In C. Janvier (Eds), *Problems of representation in teaching and learning of mathematics* (pp. 109-122). London: Lawrence erlbaum associates.

[7] Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings: Learning Cultures and Computers*. Dordrecht : Kluwer Academic Publishers.

[8] Straesser, R. (2001). Cabri-Geometre: does Dynamic Geometry Software (DGS) change geometry and its teaching and learning?. *International Journal of Computers for Mathematical Learning*, 6, 319-333.

[9] Nardi, B.A. (1996). Studying context: A comparison of activity theory, situated action models, and distributed cognition. In B.A. Nardi (Ed.), *Context and consciousness: Activity theory and human-computer interaction*, Cambridge, MA: MIT Press.

[10] von Glasersfeld, E. (1987). Learning as a constructive activity. In C. Janvier (Eds), *Problems of representation in teaching and learning of mathematics* (pp.3-18). London: Lawrence Erlbaum.

[11] Kordaki, M. (2003). The effect of tools of a computer microworld on students' strategies regarding the concept of conservation of area. *Educational Studies in Mathematics*, 52, 177-209.

[12] Kordaki, M. (2005). Modeling in the design of learning activities for Thales theorem in the context of Cabri-Geometry II. 1st *Pan-Hellenic Conference on Mathematics Education. Greek Association of Researchers in Mathematics Education (with International Participation)*. University of Athens Dept of Philosophy, Pedagogy and Psychology. Athens, Greece, December, 9-11, 2005. pp. 8-11.

[13] Kordaki, M. & Balomenou, A. (2006). Challenging students to view the concept of area in triangles in a broader context: exploiting the tools of Cabri II. *International Journal of Computers for Mathematical Learning*.

[14] Laborde, C. (2001). Integration of Technology in the design of geometry tasks with Cabri-Geometry. *International Journal of Computers for Mathematical Learning*, 6, 283-317.

[15] Mariotti, M., A. (1995). Images and concepts in geometrical reasoning. In R. Sutherland & J. Mason (Eds), *Exploiting Mental imagery with Computers in Mathematics Education* (pp. 97-116). Berlin: Springer-Verlag.

[16] http://standards.nctm.org/document/eexamples/chap6/6.4