ANGLE BISECTOR THEOREMS: A REAL LIFE APPROACH WITHIN THE CONTEXT OF DYNAMIC GEOMETRY SYSTEMS

Maria Kordaki* and Alexios Mastrogiannis**
*Dept of Computer Engineering and Informatics,
Patras University, 26550, Rion Patras, Greece, e-mail: kordaki@cti.gr
**Dept of Mathematics, Patras University, 26550, Rion Patras, Greece,
e-mail: alexmastr@upatras.gr

ABSTRACT

This paper presents a dynamic, multi-representational, real-life framework for the learning of bisector theorems by students in secondary education. In the formation of this framework, social and constructivist theories of learning were taken into account. The features of Dynamic Geometry Systems (DGS) – especially the well-known educational software Cabri-Geometry II (Laborde 1990) - were also exploited. As a result, a polymorphic, interactive construction was modeled. This construction provides learners with opportunities to acknowledge the significance of bisector theorems through their exploration of a real life simulation and subsequently to progress smoothly to its geometrical transformation in order to be able to carry out mathematical investigations.

INTRODUCTION

The angle-bisector is a basic concept within the context of Euclidean geometry. Understanding this concept presupposes an understanding of the concept of angle that is central to the development of geometric knowledge (Clements and Battista, 1992; Krainer, 1991; Mitchelmore, 1998). Understanding the basic property of an angle bisector is essential for students to grasp as it is used in fundamental geometrical constructions such as the inscribed and escribed circles of a triangle. The angle bisector is also a significant segment in a variety of geometrical shapes such as triangles, squares, rhombuses etc. In higher secondary education, the bisector theorem in triangles is also important. In fact, the internal bisection theorem relates the length of the line segments of the side opposite one angle (A) of a triangle (ABC) to the lengths of the other two sides of the triangle. In particular, in a triangle ABC, the ratio of the length of side AB to the length of side AC is equal to the ratio of the length of the line segment BD to the length of segment DC, where D is the intersection point of the internal bisector AD of angle

A with the side BC: AB/AC=DB/DC. As regards intersection point E of the external bisector AE of angle A with side BC, the relationship AB/AC=EB/EC is also satisfied. Despite the above, students often harbor many misconceptions and experience difficulties in learning relevant concepts and skills in these domains (Clements and Battista, 1989; Kieran, 1986; Magina and Hoyles, 1997).

Constructivist learning environments can play a crucial role in students' learning. In these environments, learning is acknowledged as an active, subjective and constructive activity (von Glasersfeld, 1987). The role of both appropriately-designed learning activities and tools to perform these activities is also acknowledged by many researchers as being crucial in motivating learners to be actively involved in their learning (von Glasersfeld, 1987; Vygotsky, 1978; Noss & Hoyles, 1996; Nardi, 1996). Constructivist computer learning environments can act as catalysts and distinctive participants in the whole learning context and can play a significant, crucial and unique role in student learning (Hillel, 1993; Dorfler, 1993; Laborde, 1993; Noss & Hoyles, 1996).

Dynamic geometry systems such as the well-known educational software Cabri-Geometry II can play a crucial role in students' thinking. In fact, Cabri-Geometry II provides learners with a variety of tools regarding a plethora of concepts in Euclidean geometry. Geometrical constructions can acquire a dynamic character within Cabri, which means that their form can be altered while their geometrical properties are conserved. Thus, students have the opportunity to experiment with a plethora of forms of a specific geometrical construction and can shape or verify relevant hypotheses and conclusions. Interactive real-life simulations can also be formed in the context of Cabri (Kordaki, 2005). These real life simulations can be automatically transformed into their corresponding geometrical constructions using appropriately designed buttons. To this end, Integrated Interactive Constructions supporting Multiple Learning Activities (IIC-MLAs) can be formed which can be manipulated through the formation of activity-based specific interfaces (Kordaki, 2006a; 2006b).

Student constructions of angle-bisectors using Cabri-Geometry II tools have been investigated (Trigo, 1999), as have student attempts to construct the intersection point of the three bisectors of a triangle in order to determine the center of the inscribed circle. In some cases, students have also recognized that a rectangle can become a square when the angle bisectors are the same as the diagonals. Student experimentation with the intersection points of the four bisectors in quadrilaterals as well as the intersection points of the three bisectors of a triangle in DGS has also been reported (Gutierrez, Laborde, Noss and Rakov, 1999).

In our attempt to provide students in secondary education with opportunities to give meaning to the concept of the internal and external bisector of an angle and of the corresponding theorems, we designed a learning activity using a specifically designed IIC-MLA, at the same time exploiting the advantages provided by Cabri-Geometry II. This construction is interactive and dynamic and supports students' multiple learning activities. In fact, this construction supports student interactions through a simulation of a real life problem and also places their activities in a pure geometrical context where the real life simulation can also be dynamically transformed. Such learning activity has not yet been reported in the literature.

In the following section of the paper, we present the proposed learning activity with special reference to the specific interactive constructions formed. The features of this activity in relation to potential student learning are also discussed. Finally, summarizing remarks and proposals for future research plans are made.

THE PROPOSED ACTIVITY

The proposed learning context consisted of three real life learning activities. These activities have been designed to encourage students to be actively involved in the learning of internal and external bisector theorems as well as the basic property of an angle-bisector, respectively. These activities can be presented to the students through appropriate scenarios and can be performed within Cabri by interacting with specific interactive constructions. All these constructions can be dynamically manipulated using the 'drag mode' operation. All favorable lengths can also be automatically measured and tabulated to provide students with opportunities to reflect on a plethora of numerical data and to form relative hypotheses. All these specific activities are not alienated but part of a main activity based on real life. The associated interactive constructions are also integrated into a main construction. Each specific construction can be manipulated through respective buttons on a navigation bar on the interface of Cabri (Figure 1a). In the following section, the previously mentioned scenarios are presented in relation to the correspondent interactive constructions.

Scenario 1: Acknowledging the significance of the internal bisector theorem

The real life problem can be presented to students in the following way: 'In a small mountain village, there are two neighbors (neighbors B and C) that produce different products. Neighbor B is a dairy farmer while neighbor C cultivates fruit and vegetables. They need to exchange their goods in order to live in balance. However, there is no direct route between their houses. In fact, they have to meet each other by driving through another village named C. As this procedure was expensive and time consuming, they thought that it would be a better idea to simplify the route by constructing a road directly between their houses. After fostering this idea, they negotiated the financial issues related to the construction of the road. After some discussion, they agreed that it is appropriate and fair for each one to pay money in proportion to the distance of their houses from village C. So, they looked for a point D on segment BC to fulfill this agreement'.

At this point, students are provided with an interactive construction (Figure 1a) where the village A, the houses B and C and the roads AB and AC are illustrated. Students can be informed as follows: 'On the screen of your computer you can see the houses belonging to neighbors B and C. You can also see the small village A and the roads that connect B and C with A. On segment BC there is a point D. Could you find the precise position of point D so that the ratio of the segments DB/DC are equal to the ratio of AB/AC. To find the appropriate position of D, you can experiment by dragging point D on segment BC and automatically tabulate the numerical data regarding segments AB, AC, DB, DC and AB/AC and DB/DC' (Figure 1b).

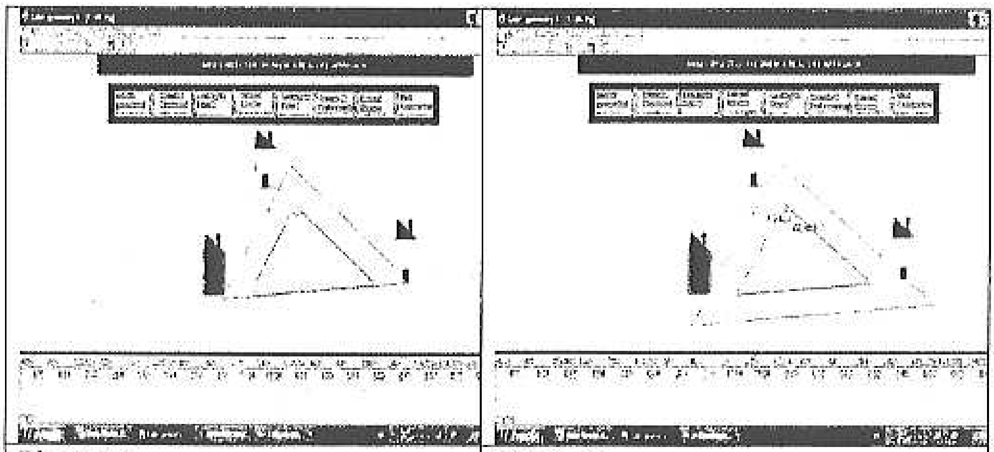

Figure 1a. Figure 1b.

Figure 1. *A real life problem to assist students in their understanding of the internal bisector theorem*

When students have found the position of D that fulfills the relationship AB/AC = DB/DC they then have to be asked to see if point D is arbitrary or if it belongs to some specific elements of the triangle ABC (Figure 1b). At this point, students have to be asked to define which elements these are. Next, they are provided with the opportunity to construct these specific arbitrary segments, namely; the height and the median from vertex A as well as the bisector of BAC. In this way, students have the chance to clarify the differences among concepts related to these specific elements and to be aware that AD is the bisector of BAC. At this point, students can enhance this awareness by experimenting (using the 'drag mode' operation) with a variety of triangles such as scalene, isosceles, equilateral, acute-angled, right-angled and obtuse-angled triangles. Consequently, students can acknowledge both the internal bisector theorem and its significance.

Scenario 2: Acknowledging the significance of the external bisector theorem

Here, students can be informed about an associated real life problem in a similar way: 'In the summer, neighbors B and C used to go swimming in a swimming pool in the small village, as it was too far to get to the beach. As they constructed the direct road between B and C, they considered the further possibility of constructing a swimming pool on the line BC, and especially in the direction CB, as the area around C is rocky and inaccessible. Moreover, after discussions and negotiations, they came to the conclusion that the appropriate position of this pool should be far from B to avoid the noise and fuss of children. They resolved to extend the road CB they had previously constructed and to double its width. As regards the financial aspect of this construction, neighbors C and B again agreed that it is right and fair to keep to their previous arrangement. That means they have to pay in proportion to the distance of their houses from village C'.

At this point, students are provided with an interactive construction (Figure 2a) where the village A, the houses B and C and the roads AB, AC and BC are illustrated. Students can be informed as follows: 'On the screen of your computer you can see the houses belonging to neighbors B and C. You can also see the small village A, and the roads that connect B, C and A. On the extension of segment CB there is a point E. Could you find the precise position of point E so that the ratio of the segments EB/EC is equal to the ratio of AB/AC. To find the appropriate position of the pool (point E), you can experiment by dragging point E onto the extension of CB and automatically tabulate the numerical data regarding segments AB, AC, EB, EC and AB/AC and EB/EC' (Figure 2b).

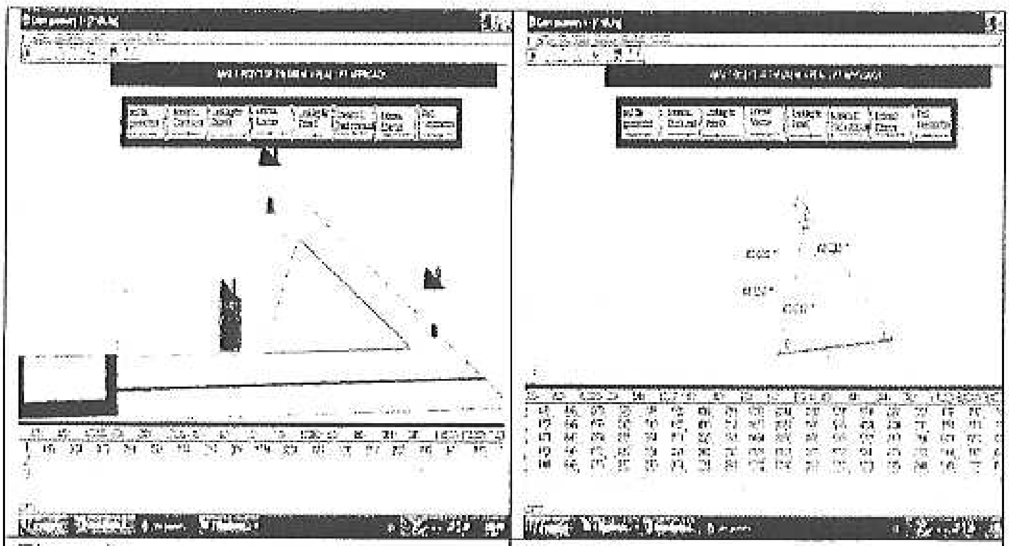

Figure 2a Figure 2b

Figure 2. A real life problem to assist students in their understanding of the external bisector theorem

After students locate the position of E that satisfies the relationship AB/AC = EB/EC, they are asked to investigate if point E is arbitrary or some specific point of ABC. Here, students have to be asked to form certain hypotheses about the specific segments of ABC to which E could possibly belong. At this point, students are provided with the opportunity to construct the height from the vertex and the external bisector of BAC. In this way, students have the chance to clarify that the only external specific elements of a triangle are the bisectors of its external angles and possibly one of its heights. By experimenting with this interactive construction, students also have the opportunity to recognize that AE is the external bisector of BAC. At this point, students can also enhance this awareness by experimenting (using the 'drag mode' operation) with a variety of triangles of different form. In this way, students can acknowledge both the external bisector theorem and its significance.

Scenario 3: Properties of bisectors in a triangle
At this point, students can be informed of the following real life problem: 'As the time passed, the exploitation of the triangular land ABC became the focus of neighbors ABC. Actually, they considered cultivating this land in some way. So, they started to think of ways of sharing this area. As they shared the costs of the previously mentioned communal constructions proportionally, they resolved to divide the triangular land in the same manner. In this way, the whole sharing procedure would remain fair'.

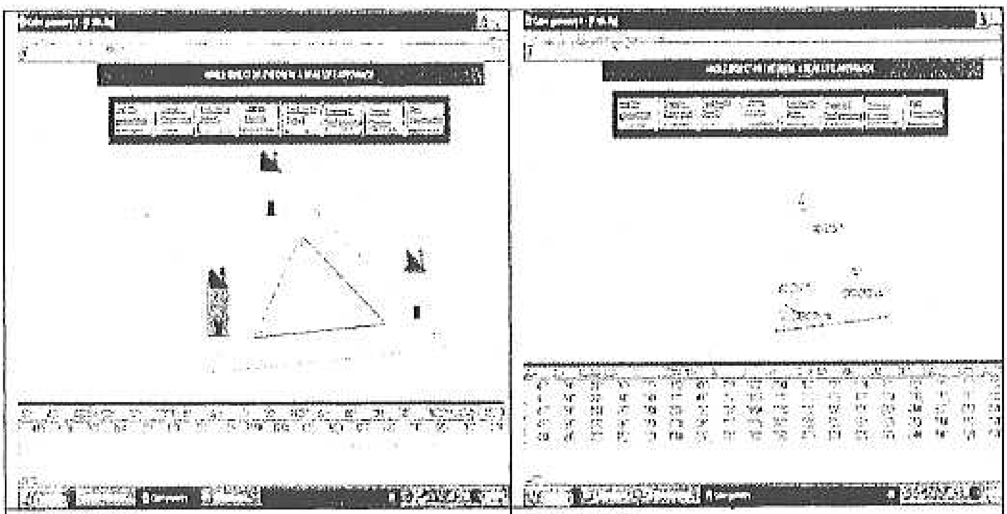

Figure 3a Figure 3b

Figure 3. *A real life problem to assist students in their understanding of the bisector basic properties*

Here again, students are provided with an interactive construction (Figure 3a) where the village A, the houses B and C and the triangular land ABC are illustrated. Students can be instructed as follows: 'On the screen of your computer you can see a small village A and the houses of neighbors B and C. On segment CB, there is a point D. Could you find the precise position of point D so that the ratio of the areas ABD/ACD is equal to the that of AB/AC. To find the appropriate position of D, you can experiment by dragging it onto CB and automatically tabulate the numerical data regarding segments AB, AC, DB, DC, ABD, ACD and AB/AC and ABD/ADC' (Figure 3b).

When students have located the position of D that satisfies the relationship AB/AC = ABD/ACD, they have to be asked to investigate if point D is arbitrary or some specific point of ABC. Here, we can expect students to be able to exploit their experience acquired in performing the activity described in Scenario 1 and to recognize that this point belongs to the internal bisector of BAC. Indeed, the aim of this task is to provide students with opportunities to experiment in order to construct the basic property of the internal bisector of an angle that means that: each point of a bisector of an angle is the same distance from its sides. Here, students have to be encouraged to form certain hypotheses to justify why the relationship AB/AC = ABD/ACD is correct. At this point, students have the opportunity to construct the height AE of ABD from vertex A that is common to both triangles; ABD and ACD. Next, they can calculate the area of ABD=(AB*AE)/2 and the area of ACD=(AC*AE)/2. They may thus conclude that AB/AC = ABD/ACD. At this point, students can also calculate the area of the previously mentioned triangles by using segments AB and AC as bases and segments DZ_1 and DZ_2 as corresponding heights. Consequently, students can produce the equation $[(AB*DZ_1)/2]/[(AC*DZ_2)2]=AB/AC$. Based on this, students can deduce that $DZ_1= DZ_2$.
Students also have the chance to construct and measure the heights of triangles ABD and ACD from point D (the segments DZ_1 and DZ_2) and to recognize that these are equal. From this, they can infer that AB/AC = ABD/ACD.

Finally, students are provided with another interactive construction illustrating an angle BAC, its bisector AD, a point on it (point K) and its distances (KM_1 and KM_2) from the sides of the aforementioned angle. By dragging point K onto AD students have the chance to gather the basic property of the bisector of an angle: each point of the bisector of an angle is equidistant from its sides.

SUMMARY AND PLANS FOR FUTURE WORK

This paper presented a real-life problem-based activity for the learning of both the internal and external bisector theorems as well as the basic properties of an angle bisector. Social and constructivist views of learning were taken into account in designing this activity (Vygotsky, 1978; von Glasersfeld, 1987; Noss & Hoyles, 1996). This activity consists of three interrelated scenarios which can be realized by students during their interactions with a specifically-designed Integrated Interactive Construction providing possibilities for performance of Multiple Learning Activities (IIC-MLA). In the construction of this IIC-MLA, the potential features of Cabri-Geometry II were exploited. These features emphasize the dynamic character of the geometrical constructions on the screen of the computer using the 'drag mode' operation. The possibility of constructing specific buttons – by forming specific macros- to manipulate these constructions was also exploited.

In the context of the proposed activities, students can be encouraged to be actively involved in their learning, as these activities are related to real life problems. In fact, by interacting within the context of these activities, students can attach real life meanings to the angle bisector theorems as well as to the angle bisector basic properties. Students can also be actively involved in experimentation and investigation that verifies, in the case of a triangle ABC, that the internal and external bisectors of one angle (A) relate the length of the line segments of the side opposite this angle (A) of the aforementioned triangle ABC to the lengths of its other two sides. In addition, students have the opportunity to experiment in order to form hypotheses about the basic property of an angle bisector. Students can also be attracted by the colorful pictures surrounding the context of the proposed activities on the computer screen.

Finally, it is worth noting that more research is needed to test the proposed learning context with real students in order to illuminate what learning constructions there are during their involvement in the proposed learning activities.

REFERENCES

Clements, D.H. and Battista, M.T. 'Learning of geometric concepts in a Logo environment', Journal for Research in Mathematics Education 20, 450–467, (1989).

Clements, D.H. and Battista, M.T. 'Geometry and spatial reasoning', in D.A. Grouws (ed.), Handbook of research on mathematics teaching and learning, Macmillan, New York, pp. 420–464, (1992).

Dorfler, W. Computer use and views of the mind. In C. Keitel & K. Ruthven (Eds), Learning from computers: Mathematics Education and Technology (pp.159-186). Berlin: Springer – Verlag, (1993).

Guterrez, A., Laborde, C., Noss, R. and Rakov, S. Tools and Technologies. In Inge Schwank (Ed), Proceedings of European Research in Mathematics Education I: Group 2, pp.183-189, Osnabruck, (1999).

Hillel, J. CA as Cognitive Technologies: Implication for the Practice of Mathematics Education. In C. Keitel and K. Ruthven (Eds), Learning from computers: Mathematics Education and Technology (pp. 18-47). Berlin: Springer-Verlag, (1993).

Kieran, C., Hillel, J. and Erlwanger, S. 'Perceptual and analytical schemas in solving structured turtle-geometry tasks', in C. Hoyles, R. Noss and R.

Sutherland (ed.), Proceedings of the Second Logo and Mathematics Educators Conference, University of London, London, England, pp. 154–161, (1986).

Kordaki, M. The effect of tools of a computer microworld on students' strategies regarding the concept of conservation of area. Educational Studies in Mathematics, 52, 177-209, (2003).

Kordaki, M. Modeling in the design of learning activities for Thales theorem in the context of Cabri-Geometry II. 1st Pan-Hellenic Conference on Mathematics Education. Greek Association of Researchers in Mathematics Education (with International Participation). University of Athens Dept of Philosophy, Pedagogy and Psychology. Athens, Greece, December, 9-11, 2005. pp. 8-11, (2005).

Kordaki, M. Integrated Interactive Constructions for Multiple Learning Activities Within E-Learning Contexts. In T. Reeves & S. Yamashita (Eds), Proceedings of World Conference on E-Learning in Corporate, Government, Healthcare & Higher Education (E-Learn 2006), October, 13-17, Honolulu, Hawaii, USA, pp.1268-1273. Chesapeake, VA: AACE, (2006a).

Kordaki, M. Activity based interfaces in the context of Cabri Geometry II: exploiting the results of a field study. In P. Kommers & G. Richards (Eds.), Proceedings of World Conference on Educational Multimedia, Hypermedia and Telecommunications (ED-MEDIA) 2006 (pp. 672-679). Chesapeake, VA: AACE, (2006b).

Krainer, K. 'Consequences of a low level of acting and reflecting in geometry learning – Findings of interviews on the concept of angle', in F. Furinghetti (ed.), Proceedings of the Fifteenth Annual Conference of the International Group for the Psychology of Mathematics Education, Program Committee, 15th PME Conference, Assisi, Italy, pp. 254–261, (1991).

Laborde, J-M. Cabri-Geometry [Software]. France: Universite de Grenoble.

Laborde, C. (1993). The computer as part of the learning environment: the case of geometry. In C. Keitel & K. Ruthven (Eds), Learning from computers: Mathematics Education and Technology (pp. 48-67). Berlin: Springer – Verlag, (1990).

Magina, S. and Hoyles, C. 'Children's understandings of turn and angle'. In T. Nunes and P. Bryant (Eds.), Learning and teaching mathematics: An international perspective, Psychology Press, East Sussex, England, pp. 99–114, (1997).

Mariotti, M.A. Justifying and Proving in Geometry: the mediation of a microworld. In Hejny M., Novotha J. (eds.) Proceedings of the European Conference on Mathematical Education (pp. 21-26), Prague: Prometheus Publishing House, (1997).

Mitchelmore, M.C. 'Young students' concepts of turning and angle', Cognition and Instruction 16, 265–284, (1998).

Nardi, B.A. Studying context: A comparison of activity theory, situated action models, and distributed cognition. In B.A. Nardi (Ed.), Context and consciousness: Activity theory and human-computer interaction, Cambridge, MA: MIT Press, (1996).

Noss, R., & Hoyles, C. Looking Back and Looking Forward. In C. Hoyles and R. Noss (eds), Learning Mathematics and Logo (pp. 431-470). Cambridge, Ma: MIT Press, (1992).

Straesser, R. Cabri-Geometre: does Dynamic Geometry Software (DGS) change geometry and its teaching and learning?. International Journal of Computers for Mathematical Learning, 6, 319-333, (2001).

Trigo M.S. The use of technology as a means to explore mathematics qualities in proposed problems. In Fernando Hitt & Manuel Santos (Eds), Proceedings of the 21st Annual Meeting of the International Group for the Psychology of Mathematics Education, pp. 139-146, Cuernavaca, Morelos, México, October 23 – 26, (1999).

von Glasersfeld, E. Learning as a constructive activity. In C. Janvier (Eds), Problems of representation in teaching and learning of mathematics (pp.3-18). London: Lawrence Erlbaum, (1987).

Vygotsky, L. Mind in Society. Cambridge: Harvard University Press, (1978).