Virtual laboratories within the context of Dynamic Geometry Systems

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This paper presents the concept of Virtual Laboratories (VL) within the context of Dynamic Geometry Systems (DGS). Despite the fact that a plethora of interactive constructions within DGS has been presented in the literature, the idea of forming virtual laboratories within DGS has not yet been reported. For the design of the proposed laboratories, a modelling methodology was developed. This methodology was based on the design of three models, namely: the learning model based on social and constructivist views regarding knowledge construction, the model of the knowledge domain based on the related specific literature and the student model describing his/her behavior while performing essential tasks for the learning of basic aspects of the previously mentioned knowledge domain. Various capabilities of DGS were also combined so as to assist the construction of the proposed laboratories, e.g. a) presentation of information in Multiple Representation Systems (MRS), b) direct manipulation of the geometrical constructions formed using the tools provided, and c) formation of appropriate buttons for illustrating/hiding the specific constructions viewed as appropriate/inappropriate for the learning of specific aspects of the concepts in focus. As a result of the modelling process, an architecture and a typical interface of the said labs were formed. The previously mentioned methodology and architecture were used for the design and implementation of specific VL for the learning of basic issues of Euclidean Geometry. To clarify the methodology and architecture proposed, a specific example regarding the design and implementation of a specific virtual laboratory for the learning of the mathematical concept of angle is also presented.

Keywords virtual laboratories; Dynamic Geometry Systems; modelling; angles, constructivism

1. Introduction

Diverse potential benefits can be provided by the adoption of e-learning in both formal and informal education. Among the major advantages of online education is its accessibility and convenience, in that learners can be logged on at any time and in any place [1-2]. Thanks to improved software and high-speed connections, but most of all to a new philosophy of learning that stresses interaction, engagement and customization rather than passive receptivity, e-learning has entered a new era. In fact, it is now a requirement for students to interact and communicate not only with their professor, but also, and mainly, with their virtual classmates. A successful e-learning experience is one that creates a sense of a learning community that can be as strong as – perhaps even stronger than – the face-to-face experience [3]. Needless to say, virtual learning communities seem friendly places. Appropriately designed online courses are also flexible, allowing learners to take control of their learning by choosing the flow of specific events that comprise their learning experience, for example, interaction with their peers, reading the provided materials, reading what others are saying in their drafts, practicing the provided exams, etc. Information also streams in on demand – which is to say that each learner has the chance to choose the content, and when, and from where, they pull it.

With the advent of modern computer technologies, web-based laboratories are used as an alternative or a supplement to physical labs [4]. Web-based laboratories have been also been termed virtual laboratories, or simply virtual labs, or cyber-labs. In fact, a computer simulation which enables essential functions of laboratory experiments to be carried out on a computer is called a VL [5]. An experiment is replaced by a computer model, and therefore takes place in the form of a simulation. Recently, VLs have been developed in a number of engineering and scientific disciplines such as Computer Science, Electrical and Mechanical Engineering, Natural Sciences, Chemistry, Statistics, Probability etc [6] using various technologies. The educational benefits of Web-based labs are based on their potential use in the "anytime, anyplace, online learning" world of learners as well as on the diversity of the representational capabilities of web-based technologies, such as dynamic, multiple and interlinked representations, at the same time enhancing learners' opportunities for experimentation and knowledge construction.

At this point, it is worth noting that experimentation by using hands-on experience is acknowledged as essential not only for the learning of experimental sciences but also for the learning of mathematics [7,8].

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Diverse educational software still exists to appropriately support learners' hands-on experimentation with visual representations of mathematical concepts. Among these pieces of software, Dynamic Geometry Systems and especially Cabri-Geometry II [9] hold a central position. In particular, Cabri is an e-learning context that provides *diverse tools* to support a *plethora of concepts of Euclidian geometry*. In addition, Cabri provides tools to construct *multiple representations*, both numerical and visual, such as geometrical figures, tables, equations, graphs and calculations which can also be *interlinked*. To this end, there is also the possibility of collecting *large amounts of numerical data* which can be used by students to form and verify conjectures regarding the geometrical concepts in focus. Furthermore, the geometrical *constructions within Cabri are dynamic* in that they can be directly manipulated by using the '*drag mode'* operation, thus conserving their basic properties while altering their form. Moreover, Cabri is a *highly interactive* environment that also provides learners with *multiple types of information* (text, figures, numbers) *and feedback* (intrinsic visual and extrinsic numerical) to facilitate the formation and verification of conjectures as well as self-correction of their constructions. Finally, the *history of student actions* is captured and possibilities for the *extension* of the environment are also available to teachers and learners through the addition of specific macros on the Cabri-interface.

Taking into account all the above, we exploited the extension capabilities provided by Cabri to form VLs for the learning of mathematical concepts concerning Euclidean Geometry. To this end, the modelling methodology described in the next section of this paper was adopted. As a result of this modelling process, the architecture of the proposed VLs was formed, and this is also presented below. The proposed methodology and architecture are presented through a specific example demonstrating the whole design process and the formation of a VL for the learning of the concept of angle by primary and secondary level education students. Potential dimensions of the design of the proposed lab are also discussed and, subsequently, there follows a summary, with future research plans also being considered.

2. The modeling methodology for the design of virtual labs; an example for the design of a VL for the learning of the mathematical notion of angle

For the design of the proposed VLs, a modelling methodology was adopted. Computer modelling is used in education to support learning in artificial worlds [10, 11], providing children with computer tools to enable them to create their own worlds, to express their own representations of the world and also to explore other people's representations. The said methodology involves the construction of three models, namely: the learning model based on modern social and constructivist views of learning [7, 12, 13] exploiting the advantages and the features of the educational software Cabri-Geometry II, the subject matter model based on the various aspects that constitute the concepts in question as have emerged from the literature and the learner model while performing essential tasks related to the subject matter in focus. In the following section of this paper, the entire modelling process for the design of a specific VL dedicated to the learning of the mathematical notion of angle, including the theoretical considerations taken into account during the construction of these models, as well as interpretations in terms of operational specifications of the software, is presented.

The learning model: The learning model was constructed after taking into account basic considerations of modern constructivist and social theories regarding knowledge construction [7, 12, 13]. Constructivist learning theories emphasize the active, subjective and constructive character of knowledge, placing students at the centre of the learning process. Constructivist learning also stresses the need for student experimentation through acquiring hands-on experience while using primary sources of data [14]. To this end, students have the opportunity to reflect on their experience and to articulate what they have learnt. Social theories of learning stress the importance of psychological tools as mediators for the development of students' higher mental functions [15]. In particular, computer tools and learning environments can help students to express their own knowledge, to explore the knowledge of others integrated in these environments, while also providing appropriate feedback, and to scaffold their thinking and actions in order to deepen their understanding [7, 16], in other words acting as mediators of socio-cultural development to help a student enhance their Zone of Proximal Development, which is essential in the development of their knowledge [13]. The mediating role of technology is based on the fact that computers can 'communicate' with learners through the variety of representations constructed and displayed on their screens [7]. The role of Representation Systems (RS) in helping students gain knowledge is acknowledged as significant by many researchers [17-19]. Specifically, Multiple Linked and Dynamic Representations of the concepts in focus can encourage learners to express their inter- and intra-individual differences and also to develop a broad view on the concepts in question. The expression of these concepts in Linked (RS) can also provide learners with opportunities to observe how the variations in one system can affect the other [18]. When learners interact with dynamic RS they have the chance to reflect and extract the basic structure and properties of the concepts represented by these systems. In general, external and visual representations can be used as supporting references of students' mental internal representations related to a specific learning concept. In some cases, RS can be used as students' primary references to support their reflection processes for the learning of a specific learning concept [18]. MRS can also motivate students to become actively involved in their own learning [19]. Furthermore, the understanding of a concept can be viewed as a process of giving meaning to its different representations. The basic requirements of the learning model described above can be effectively met by the basic properties of Cabri-Geometry II.

The learners' model: A dominant role in the modelling process was taken by the construction of the learner model, since the design perspective that was adopted emphasized a student-centred design [20]. The construction of this model is based on the specific literature on both the mathematical meaning of the concepts in focus and the learners' misconceptions and difficulties regarding these concepts. Specifically, a central role in the development of geometric knowledge is played by the understanding of the concepts of angle and rotation [21, 22]. However, students often encounter many difficulties in learning relevant aspects of these concepts [23, 24]. Previous research makes it clear that students have great difficulty coordinating the various facets of the angle embedded in various physical angle contexts involving slopes, turns, intersections, corners, bends, directions and openings [25]. Angles have been defined in two ways: as a part of the plane included between two rays meeting at their endpoints (the static definition) and as the amount of rotation necessary to bring one of its rays to the other ray without moving out of the plane (the dynamic definition) [26]. To this end, the dynamic definition of angle introduces the concept of directionality. It is important for students to grasp both of these definitions so as to avoid various misconceptions. At this point, it is important to note that - when the emphasis is on the static definition - students confuse angle measure with area [27]. Students also typically believe that angle measures are influenced by the lengths of the intersecting lines or by the angle's orientation in space [23]. Often children have little or no conception of angle as a measure of rotation. It seems likely that this may be due to a predominance of 'static' experiences of angle at the expense of 'dynamic' experiences. Common incorrect ideas expressed by children when trying to define angles include: a) 'The distance measured between two lines'; b) 'The gap where two lines meet'; c)'The area between two lines'; d) 'The spacing between two lines which meet at a point'; e) 'the size of an angle is determined by the length of the directional lines at either limit of the rotation', f) 'the size of an angle is dependent on distance of arc from centre of rotation' g) 'the length of the arc which labels the angle is the factor which determines the size of the angle' [28, 29]. Children also have difficulty recognising relationships between angles (e.g. supplementary angles). To help students successfully develop turn concepts and turn measurement, some approaches using LOGO have been reported, while other approaches have emphasized the use of concrete analogies as well as children's experience with physical rotations, especially rotations of their own bodies [23, 26, 30]. Despite this fact, the use of features of DGS to provide students with opportunities to explore the concept of angle in both definitions has not yet been reported.

The subject matter model: In the course of schooling, students need to encounter multiple aspects of the mathematical notion of angle, including [28]: a) conception of angle as movement, as in rotation or sweep; b) conception of angle as a geometric shape, a delineation of space by two intersecting lines; c) conception of angle as a measure, a perspective that encompasses both of the previous; d) classification of angles in terms of size, e.g.: acute angles; right angles; obtuse angles; straight angles and reflex angles; e) clarification of specific kinds of angles, such as: vertically opposite angles (when two straight lines intersect, pairs of angles which are opposite each other - across the intersection - are equal) and supplementary angles (such angles which lie adjacent on a straight line, f) appropriate estimation of the size and the relationships among the pairs of angles constructed when 2 parallel lines are intersected by another straight line, and g) specific statements related to some angle properties used in common geometrical shapes, such as: i) the angles in squares and rectangles are all 90°; ii) all the angles in an equilateral triangle are 60°; iii) an isosceles triangle has two equal angles; iv) an n-sided polygon has n angles; v) regular shapes have angles which are of equal size; vi) the angles in irregular shapes are not equal; vii) scalene triangles have three angles which are all different; viii) perpendicular lines are at right angles; ix) parallel lines never intersect to form an angle; x) congruent shapes have identical corresponding angles (as well as corresponding sides of the same length) but the orientation of the shapes may be different; xi) similar shapes have identical corresponding angles (but corresponding sides are only proportional in length and the orientation of the shapes may be different); xii) the sum of the interior angles of a triangle is 180°; xiii) the sum of the angles in any quadrilateral is 360°; ixv) the sum of the exterior angles of a n-sided polygon is 360°; xv) the sum of interior angles of a n-sided polygon is n*180°-360°; and xvi) the size of those angles inscribed in a semicircle is 90°. Based on the analysis above, it was considered appropriate to integrate into the proposed VL such possibilities that would explore all the mathematical aspects of the notion of angle described in this section.

3. Architecture and the interface design of the proposed virtual labs

Taking into account the analysis performed during the modelling process and especially during the construction of both the learners' model and the subject matter model, various aspects of the concepts in focus have been specified as essential for students to grasp. To help students grasp the diversity of these aspects, various interactive constructions were formed, each of which is dedicated to the learning of a specific aspect through the performance of a specific learning activity. As is shown in Figure 1, the interactive construction at hand appears in the centre of the screen and can be managed through both direct manipulation and a navigation bar. Each part of this navigation bar consists of specifically-designed buttons dedicated to the management of a specific interactive construction. By using each of these buttons, a specific construction can be illuminated or hidden.

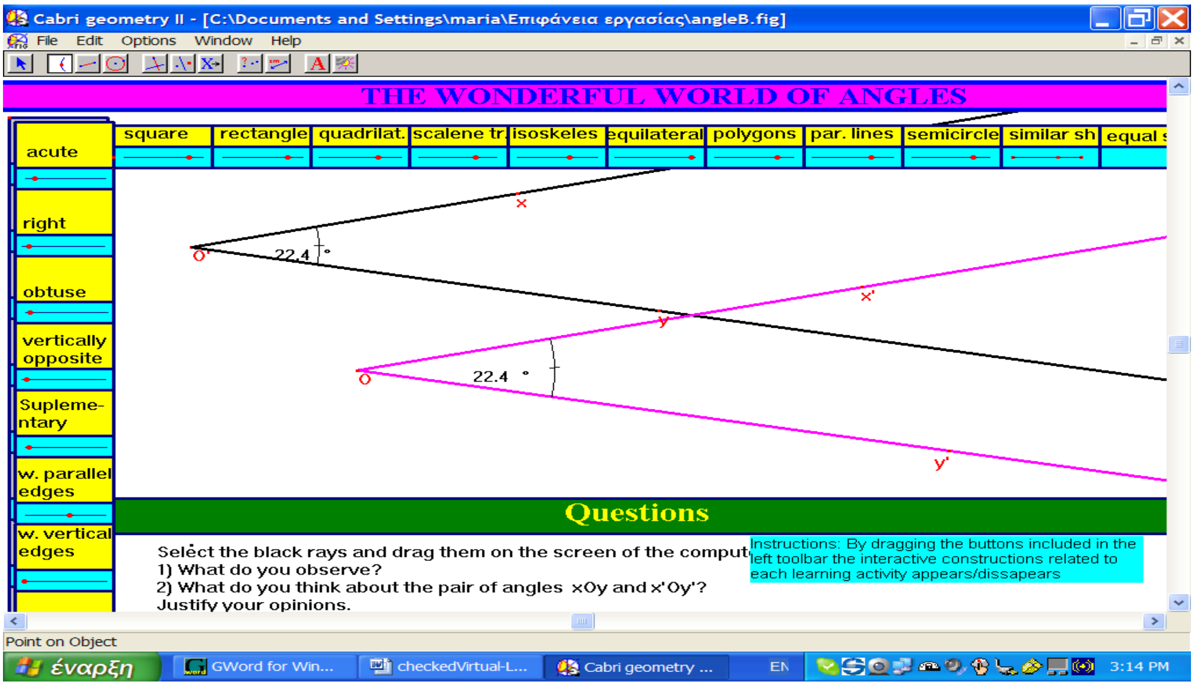

Figure 1. A virtual Lab for the learning of the concept of angle within DGS.

When a specific interactive construction is illuminated on the screen, specific information is also presented to learners, such as: a) text-based information on the aspects related to the sub-activity at hand; b) appropriate questions, so as to assess student knowledge and c) instructions to manage the construction in focus. All these interactive constructions can be directly manipulated and possibilities for expressions of the related concepts in MRS are also formed. The network of designed interactive constructions was implemented using the tools of Cabri Geometry II and all specific constructions were placed in the same interface. Then, this network of constructions was transformed into Jawa Applets. These Applets constitute the VLs proposed and were placed in a specifically-designed environment - using the MOODLE platform - so that they could be used by teachers and students remotely. In this way, teachers have at their disposal ready materials to use in their classrooms while their students can access these labs not only in their schools but also in their own time and space. As regards the formation of the specific VL dedicated to the learning of the mathematical notion of angle, various interactive constructions were formed to allow students to explore all aspects determined as essential during the construction of the 'subject-matter model' and the 'learners' model' presented in this paper.

4. Summary and plans for future work

This paper presented the idea of VLs within the context of DGS, and especially using the features of the well-known educational software Cabri-Geometry II. For the design of these labs, a modelling methodology was adopted consisting of the construction of 3 models, namely: the learning model based on social and constructivist views of learning, the learners' model and the subject matter model, as they emerged from the literature. Based on the analysis performed during this modelling process, various interactive constructions have to be formed so as to provide students with opportunities to explore all the essential aspects of the subject matter in question, and especially those aspects where students encounter difficulties. All these constructions have to be realized so as to support constructivist learning, i.e., they have to be interactive, direct manipulated, using MRS, and also provide students with opportunities for exploration, as well as formation and verification of mathematical hypotheses and conjectures. This network of constructions has to be fully integrated into the same interface manageable by specific buttons on a navigation bar. The said VLs consisted of the said network of interactive constructions translated into Jawa Applets. To clarify the methodology proposed, a specific VL for the learning of the mathematical notion of angle is presented. Field studies are necessary, to illuminate the impact of the design and implementation of this VL on student knowledge.

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