Students' constructions of equivalent triangles 'in any possible way', using the tools of Cabri-Geometry II

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Abstract
This study focuses on the constructions of equivalent triangles developed by students aged 12 to 15 years-old, using the tools provided by the well known educational software Cabri-Geometry II (Laborde, 1990). Twenty-five students participated in a learning experiment where they were asked to construct several triangles and to transform them into other equivalent triangles 'in any possible way'. The analysis of the data shows that all the students were actively involved in this transformation task and they constructed strategies that fell into eight categories. Most of the students initially ignored the fact that two triangles can conserve their areas when their figures are altered. Despite this fact, all the students, while also exploiting the variety of the provided tools successively constructed equivalent triangles in more than one way, by expressing different pieces of knowledge they possessed. Most students viewed the conservation of the area of a triangle in a non-fragmented way in the context of the tools of Cabri-Geometry II. These tools provided the students with the challenge of viewing this concept in interrelation with the concepts of: regular polygons, area measurement using spatial units and area formulae. Some students also recognized the concept of conservation of area in families of equivalent triangles with common bases and equal altitudes. Hence, they all constructed a broader view of the concept of conservation of area regarding triangles. Students also used these tools in combination with the 'drag mode' and the 'automatic tabulation of numerical data' to explore the concepts of conservation of area in relation to the perimeter of the constructed triangles.

Introduction
Conservation of area is an essential and preliminary aspect in students' understanding of the concept of area measurement (Piaget, Inhelder & Szeminska, 1981; Hirstein, Lamb and Osborn, 1978; Maher and Beattys, 1986). Area is an invariable attitude – a definite measurable size of the plane surfaces enclosed by figures (Piaget, et al., 1981; Douady and Perrin, 1986). Conservation of area means that an area remains unaltered while its figure can be qualitatively new (Piaget, et al., 1981). Students have difficulties in understanding the possibility of equivalence of an area when it is represented in shapes of different forms (Carpenter, Coburn, Reys and Wilson, 1975). Students are restrained from understanding the concept of conservation of area since they confuse the area and the perimeter and they make conclusions based on their perceptions (Hughes & Rogers, 1979; Hart, 1989). Moreover, students' difficulties are also related to the form of the shapes to be conserved. More specifically, students may understand the possibility of conservation of area in squares and in parallelograms but they face difficulties in understanding this concept in irregular shapes and triangles (Hughes & Rogers, 1979; Maher and Beattys, 1986). Recently, Kordaki (in press) has investigated students' understanding of this concept regarding a specific group of triangles using the tools of a computer microworld, namely the C.AR.ME microworld (The Conservation of Area and its Measurement; Kordaki and Potari, 1998). In the said investigation, this specific group of triangles consisted of families of equivalent triangles with common bases and equal altitudes. In the study, students were confronted with difficulties in acknowledging that the triangles included in these families can be equivalent since their perimeters were different. Moreover, students presented difficulties in recognizing that the common properties of these triangles, are their common bases and equal altitudes. Students difficulties were overcome by using the tools of the microworld mentioned above. However, students understanding of the concept of conservation of area in arbitrary triangles has not yet been reported.

Many researchers acknowledge the role of computer tools in students' understanding of mathematical concepts (Noss and Hoyles, 1996; Kordaki and Potari, 2002, Kordaki in press). In interacting within a computer environment providing a variety of tools and having to deal with tasks asking to be solved 'in any possible way' students can construct a variety of approaches to the learning concepts (Kordaki and Potari, 2002, Kordaki in press). Each student can choose among tools the most appropriate for his/her cognitive development. By using a number of different tools, each student can express different pieces of knowledge he/she possesses thereby developing a broader view of the learning concept. The research has also shown that Cabri-Geometry II (Laborde, 1990) is a computer environment which provides a rich set of tools that help students in their understanding of Euclidean Geometry concepts (Laborde and Laborde 1995; Mariotti, 2001). In addition, this environment provides the possibility of forming dynamic views of the concepts above, using the 'drag mode' operation. This allows the possibility of altering the form of a geometrical construction while its basic properties are conserved.

In our attempt to investigate the concept of conservation of area in arbitrary triangles in a non-fragmented way, we chose to provide students with the opportunity to interact with the variety of tools of Cabri-Geometry II. To exploit this variety of tools in developing a broad view of the concept above, we asked students to perform 'in any possible way' a learning activity referring to the construction of equivalent triangles. Although the research literature has explored the impact of tools provided by Cabri in students' understanding of a diversity of geometrical concepts, the case of conservation of area in arbitrary triangles 'in any possible way' has not yet been reported.

In the following section of this paper the context of this study is presented. Next, the analysis of the data is demonstrated. Finally, the findings of this study are discussed and conclusions are presented.

The context of the study

The method and the focus of the study. This research is a qualitative study (Cohen and Manion, 1989) which focuses on: a) Students' strategies regarding the construction of equivalent triangles while interacting in the context of tools of Cabri and : b) The role of tools offered by Cabri affecting students' strategies.

The learning experiment. The learning experiment took place in a typical state secondary school of Patras, Greece. Three complete classes of students participated in this learning experiment. These classes consisted of: eight 1st grade students (13-year-old), nine 2nd grade students (14-year-old) and eight 3rd grade students (15-year-old). These students were asked to perform a task using the tools of Cabri II. The duration of the task was commensurate with the students needs. Each student participated for about two hours to complete this task. A familiarization phase using both basic operations of a computer and the tools of Cabri II took place before these students commenced the main study. This phase lasted about two months, two hours a week for each student. The familiarization with basic computer operations was decided to help students to be smoothly introduced to the Cabri II environment. The specific operations that students experienced were: turning on/off the computer, manipulating files (create, open, save and print a file), and finally using a word-editor. The aim of this phase was to introduce students to the use of these tools and not to get them involved in the solving of the specific task processes. More specifically, students were asked to try the specific tools of Cabri II that would enable them to experience those concepts of Geometry which are in the Greek school curricula. So, students were not introduced to the tools related to: conic, vectors, locus, dilation, inverse, graphics, animations, and macros.

Students were placed in groups of three and worked in a computer laboratory. During the familiarization phase these groups worked in rotation using three computers while in the main phase they also worked in rotation but using only one computer. This was due to technical limitations. The researcher participated in the study as an observer with minimum intervention. All interventions realized by the researcher are reported with reference to the specific cases in the results section of this paper. The data resources are the electronic files of students' geometrical constructions, the video recordings of all interactions and the field notes of the researcher.

The task. Students were asked to 'construct equivalent triangles in any possible way' using the tools of Cabri and to justify their solution strategies. By allowing and asking for a variety of solutions in a rich context of tools students have the opportunity to construct their individual approaches to the concept of equivalence of triangles as well as to express different pieces of knowledge they posses thereby constructing a broader view of this concept (Kordaki, 2003; Weir, 1992).

Students' previous knowledge. All students had been taught the basic area formulae regarding triangles: A=1/2(B*H), A= area of a triangle, B= a base of this triangle, H = the corresponding altitude to the base B. All students had also been taught the basic elements of a triangle such as sides, altitude, the segment that joints a vertex with the middle of the opposite side, angle, perpendicular bisector, parallel and perpendicular lines. Students of the 2nd and 3rd grade had also been taught symmetry and regular polygons. In addition, students of the 3rd grade had also been taught about equivalence and similarity of triangles.

The process of analyzing the data. Each individual group's multiple solution strategies to the given task were identified and reported. These strategies were analyzed in terms of students' conceptions regarding the concept of equivalence of triangles. In the next stage, the focus was on the entire group of students and a classification of strategies was constructed. Moreover, the students' strategies across grades were presented. Finally, the role of the provided tools in the construction of these strategies is studied.

Results

In the given task, all students recognized the equivalence of area in congruent triangles. Regarding the fact that triangles of different form can have equal areas, some students expressed that they had not thought about this before, and they weren't sure about this possibility, while other students (those who participated in group B1) expressed that it is possible.Students' solution strategies to the task were classified in eight categories which are presented in Table 1 and described in the following section.

Categories of students' strategies regarding the construction of equivalent triangles in the context of Cabri-Geometry II	**Number of groups**	**Number of Students**
Categories performed by		
C1: using the 'eye'	1A, 1B, 2G	2A, 3B, 4G
C2: focusing on the boundary of the original triangle	1B	3B
C3: experimenting while using the 'drag' mode in combination with automatic area measurement	3A, 3B, 2G	8A, 9B, 5G
C4: experimenting with right-angled triangles using the 'drag mode' in combination with automatic area measurement	1A	3A
C5: measuring areas using spatial units	1A, 2B, 2G	3A, 6B, 5G
C6: conserving the form of the original triangle using basic area transformations	2A, 3B, 2G	6A, 9B, 6G
C7: splitting polygons	3A, 2B, 1G	8A, 6B, 3G
C8: using area formula	2B	6B

Table 1. Categories of students' strategies regarding the construction of equivalent triangles in the context of Cabri.

C1: using the 'eye'. Here, students who performed this strategy (nine students) constructed an arbitrary triangle on the computer screen using the tool 'triangle'. Next, they used the same tool and constructed another triangle, which visually seemed equivalent to the original. To verify their solution strategy, these students measured the area of both triangles using the tool for automatic area measurement. As they realized that these areas were different, due to fact that the area-unit implied in using this tool is one pixel, they were confused and expressed that '*it is impossible to construct equivalent triangles using our visual perception'*. To overcome this conflict they moved on to exploiting the variety of the provided tools to construct other more sophisticated solution strategies.

C2: focusing on the boundary of the original triangle. Three students from the B grade performed strategies, which are presented in figure 1.

Strategy A: Conserving only the length of its sides

Focusing on the boundary of the original triangle

Strategy B: Conserving the length of its sides and its angles

Figure 1. Students' strategies focusing on the boundary of the original triangle

Here as well, students constructed an original triangle using the tool 'triangle'. Next, they attempted to construct another triangle equivalent to the original by conserving just the lengths of its sides but they had difficulty in forming it. To overcome this difficulty, they also realized the need to conserve the angles of the original triangle. At this point, students also did not manage to form a new triangle, as they did not conserve the order of the sides and angles of the original triangle.

C3: moving dynamically one vertex of the original triangle using the 'drag mode' in combination with automatic area measurement. Students' strategies in this category are presented in figure 2.

Strategy A
• Constructing two triangles ABC and ZKL • Measuring automatically the area of the triangles ABC and ZKL • Dragging the vertices of the triangle ZKL until it has equal area to the area of the triangle ABC
Strategy B
• Constructing: two parallel lines e1 and e2, a segment BC on the line e1, a point A on the line e2, a triangle ABC, different points K, L on the line e2, the triangles KBC and LBC • Automatically measuring the area of the triangles ABC, KBC and LBC.
Strategy C
• Constructing: two parallel lines e1 and e2, a segment BC on the line e1, a point A on the line e2, a triangle ABC • Automatically measuring the area of the triangle ABC • Dragging point A on the line e2 • Tabulating the area of the triangles constructed while point A is dragged

Figure 2. Students' strategies performed by moving dynamically one vertex of the original triangle using the 'drag mode' in combination with automatic area measurement.

Students who performed the strategy A (fourteen students, six from grade A, six from grade B and two from grade C), experimented with two arbitrary triangles of different form by dynamically transforming their figures and simultaneously observing the variation of their areas as it was demonstrated by the numerical results automatically produced. Suddenly, these students found an instance where these triangles had equal areas. At this point, they said: '*we are surprised viewing that two triangles of different form can have equal areas'*. Based on these visual and numerical data, these students had the opportunity to intuitively understand the concept of conservation of area in arbitrary triangles.

Students who performed strategies B and C in this category (Fourteen students) seemed to employ an intuitive way to construct equivalent triangles with a common base and equal altitudes. Despite the fact that these students didn't realize that the produced triangles had equal altitudes, they probably relied on their visual perception of the conservation of distance between parallel lines. Students who performed strategy B (five students from grade A and three from grade B) studied a limited number of triangles. However, students who performed strategy C (six from grade B and three from grade C) progressed to the construction of a large number of equivalent triangles using the 'drag mode'. Next, they automatically measured the areas of the triangles produced, thereby verifying strategy C as a way to construct a family of equivalent triangles.

C4: experimenting with right-angled triangles using the 'drag' mode in combination with automatic area measurement. Three students from the A grade performed the strategy presented in figure 3.

• Constructing two parallel lines e1 and e2 • Constructing a line e3 which is perpendicular to the lines e1 and e2 and meets them on the points Z and K respectively • Constructing a line e4, which meets the lines e1, e2 and e3 at the points A, B and C respectively • Constructing the triangles ACZ and BCK and measuring their areas automatically • Moving point C on the line e3 by dragging it until the triangles above have equal areas • Moving line e4 around the point C and exploring the equivalence of the area of the triangles ACZ and BCK

Figure 3. Students' strategies performed by experimenting with right-angled triangles using the 'drag mode' in combination with automatic area measurement.

Students, who realized this strategy performed a specific construction using parallel and perpendicular lines in a configuration where right-angled triangles were constructed. In our view, students tried to study these triangles as they were familiar to them from their school experience. Here, students experimented with the area of the similar triangles ACZ and BCK by dynamically rotating the point C across the segment ZK. During this experimentation these students found that when point C was located in the middle of the segment ZK the area of these triangles was equal. Next, they constructed a large number of pairs of right-angled triangles by dynamically moving line e4 around point C and tested the equivalence of their areas. Having performed this strategy, these students found a way to construct pairs of triangles with equal areas, and to verify this construction using a large number of visual and numerical data.

C5: measuring areas using spatial units. Fourteen students (three from the A grade, six from the B grade and five from the C grade) performed the strategy presented in figure 4.

- Constructing a triangle ABC
- Illuminating the square grid provided by Cabri.
- Measuring the area of this triangle by calculating the square units of the square grid
- Constructing another triangle ZKL with an area consisting of the same number of square units as the triangle ABC
- Automatically measuring the areas of the triangles ABC and ZKL respectively
- Using the drag mode to modify the triangle ZKL as the estimation of the square units performed by the students couldn't approximate the accuracy of the unit used by the computer

Figure 4. Students' strategies performed by measuring areas using spatial units.

By performing this strategy, students seemed to grasp that the area of a triangle can be conserved after splitting it into equal parts, (the spatial-units used) and recomposing these parts to produce equivalent triangles of different form. Students correctly counted the whole units placed inside the triangle while they faced difficulties in accurately estimating the parts of the units needed to cover the remaining shape. As they tried to verify the equivalence of the constructed triangles, using the automatic area measurement tool, they expressed that '*it's impossible to approximate the accuracy of the unit used by the computer'*. At this point, students made corrective adjustments to the second triangle using the 'drag mode' operation.

C6: *conserving the form of the original triangle using basic area transformations*. Fourteen students (six from each grade) performed strategies included in this category. These strategies are presented in figure 5.

Conserving the form of an original triangle using basic area transformations such as

- Parallel translation of the original triangle (**Strategy A**)
- Reflection about an axis (**Strategy B**)
- Rotational symmetry about a center (**Strategy C**)

Figure 5. Students' strategies performed by using basic area transformations.

Fourteen students performed strategy A (six students from grade A, three from grade B and five from grade C). These students used the '*copy*' and '*paste*' tools to construct a triangle congruent to the original. Twelve students performed strategy B (three students from grade A, six from grade B and three from grade C) constructing an axis of symmetry and consecutively using the '*reflection*' tool. Nine students performed the strategy C (three students from each grade) assigning a center of symmetry and subsequently using the '*rotation*' tool. By performing these strategies, students invented ways uncommon in school practices to construct equal triangles exploiting the variety of tools provided by Cabri and integrated their previous knowledge regarding basic area transformations, thereby developing a broader view of the concept of conservation of area in triangles. It is worthwhile mentioning that the students of grade A hadn't been taught about these area transformations in their school practices. They were inspired to try these transformations in the process of their familiarization with the tools of Cabri.

C7: splitting polygons. Seventeen students performed strategies included in this category. These strategies are presented in figure 6.

Strategy	Splitting …
A	a triangle, using the segment that joins a vertex with the midpoint of its opposite side
B	a right-angled quadrilateral (a rectangle & a square) into two equivalent triangles by using a diagonal
C	a regular hexagon into six equivalent triangles by using all its diagonals

Figure 6. Students' strategies performed by splitting polygons.

Twelve students performed strategy A (three students from grade A, six from grade B and three from grade C), using an isosceles triangle as the original and splitting it into two equivalent triangles using the perpendicular bisector from the vertex of the equal sides. Despite the fact that these students had been taught that each point of the perpendicular bisector of a segment is connected with its edges by a pair of segments of equal length, they did not feel safe enough, so they automatically measured these two triangles for verification. Nine students (six students from grade A, and three from grade B) also performed this strategy by splitting an arbitrary triangle into two equivalent ones using the segment that joins a vertex with the midpoint of its opposite side. These students used this specific segment intuitively so, they used the automatic area measurement tool to confirm the equivalence of these triangles. Eight students from grade A performed strategy B. Here, these students seemed to understand that a diagonal of a right-angled quadrilateral splits it into two right-angled congruent triangles. Three students from grade A performed strategy C. They intuitively split the regular hexagon, by using all its diagonals, into six equivalent triangles, and then, the automatic area measurement tool was also used to realize their equivalence. Here, these students also devised ways unusual in school practices to construct equivalent triangles.

C8: using area formula. Six students performed strategies included in this category which are presented in figure 7.

Strategy A

- Constructing a right-angled triangle ABC ($A=90^o$)
- Measuring the length of perpendicular sides AB and AC of this triangle by using the square grid provided by Cabri
- Calculating the area of this triangle by using the area formulae: Area =(½)* [(AB)*(AC)], *=multiplication
- Trying to find other pairs of numbers x, y and putting them in the position of AB and AC in the area formulae mentioned above to produce results equal to the calculated area of the original triangle
- Constructing other right-angled triangles with perpendicular sides AB and AC which have lengths that are produced in the following way: AB = x*a, and AC =y*a, where a= the length of the side of the square unit of the used grid.

Strategy B

- Constructing a triangle ABC and its altitude AD
- Measuring the length of the base BC and of the altitude AD
- Calculating the area of this triangle by using the area formulae: Area =(½)* [(BC)*(AD)], *=multiplication
- Constructing another triangle ZKL with base KL and altitude ZT equal to the length of the segments BC and AD respectively. The length of the segment BD was different from the length of the segment KT.

Strategy C

- Constructing a triangle ABC and its altitude AD
- Measuring the length of the base BC and of the altitude AD
- Calculating the area of this triangle by using the area formulae: Area =(½)* [(BC)*(AD)], *=multiplication
- Constructing another triangle ZKL with base KL and altitude ZT equal to the length of the segments AD and BC respectively.

Figure 7. Students' strategies performed by using area formula.

Three students from grade B performed both strategies A and C. By performing strategy A they seemed to understand how to use area formulae in right-angled triangles and that the two perpendicular sides of such a triangle can be considered as its base and its respective altitude. They also seemed to understand that the area of a right-angled triangle can be conserved when the product of the lengths of its perpendicular sides is conserved, despite the fact that the figure of the triangle is altered. The same students progressed to the construction of strategy C, where they experienced the conservation of area in arbitrary triangles. Here, these students also expressed an understanding of the use of area formulae regarding the previously mentioned triangles. Moreover, they appeared to understand the conservation of area of an arbitrary triangle as the conservation of the product of the lengths of its base and its respective altitude, even though the figure of this triangle can be transformed. Three other students from grade B performed strategy B. They also manipulated correctly the area formula in an arbitrary triangle. Moreover, they realized that when its base remains unaltered and the respective altitude slides on this base the area of this triangle is conserved. Despite the fact that students who realized the strategies mentioned above used the area formula correctly in triangles, they needed verification of their strategies by automatically measuring the equivalent triangles they constructed.

Discussion

A first glance at these results shows that the students have performed a variety of constructions of equivalent triangles in the context of tools of Cabri-Geometry II. Ninety eight constructions were performed by the 25 students who participated in this experiment. These constructions were classified into eight categories. Students were prompted to perform this plethora of constructions by exploiting the variety of the provided tools and by responding to the given task asking them to solve it 'in any possible way'. Two main points emerged from students' constructions: i) the approaches used in the learning concept and ii) the role of the provided tools on students' constructions.

i) Students used three alternative approaches to construct equivalent triangles. Firstly, the transformation of an original triangle to another equivalent one using a variety of strategies (those reported in categories C1, C2, C3, C5, C6, C8), secondly, the splitting of a polygon into equivalent triangles (performing the strategies reported in category C7) and thirdly, the specific construction of producing a pair of right-angled triangles (strategy reported in category C4). More specifically, students transformed the original triangle into another equivalent one by: using the 'eye', conserving its boundary, using the 'drag mode', measuring it using spatial units and recomposing them, using the area formulae, as well as by performing the basic area transformations such as parallel translation, reflection about an axis and rotational symmetry about a center (reported in categories C1, C2, C3, C5, C8 and C6 correspondingly). Moreover, students experienced the construction of equivalent triangles by splitting isosceles and arbitrary triangles, quadrilaterals and regular hexagons. Here, these students experienced the construction of two equivalent triangles by splitting an original triangle using the segment that joins one of its vertex with the middle point of the opposite side as well as by splitting an isosceles triangle using the perpendicular bisector of its base. In addition, these students expressed an understanding of the construction of equivalent triangles by splitting a quadrilateral and a hexagon using diagonals. Students also invented a specific construction of pairs of equivalent triangles using two parallel lines with a perpendicular segment and a secant line from the middle point of this segment to the previously mentioned parallel lines. By rotating the secant line around this middle point students had the opportunity to investigate the conservation of area in a large number of similar pairs of equal triangles.

By performing the variety of strategies mentioned above, students viewed the concept of conservation of the area of a triangle as an alteration of its position on the computer screen while conserving its figure (strategies reported in categories C1, C2, C6 and C8) as well as an alteration of its figure (strategies reported in categories C3, C5 and C8). By dynamically altering the figure of a triangle using the 'drag mode' (strategies reported in categories C3, and C4), provided as a basic option by Cabri, students had the opportunity to observe a large number of equivalent triangles of different figures. In this way they were supported in their understanding of the concept of conservation of area in an abstract way, thereby forming a dynamic view of this concept. This is impossible to be realized in the paper and pencil environment.

By performing strategies using the variety of approaches mentioned above, students had the opportunity to study a variety of triangles of different figures, such as, arbitrary, isosceles and right-angled triangles. Students also constructed equivalent triangles by using different pieces of knowledge they possessed leading to the opportunity to construct an integrated view regarding the construction of equivalent triangles. This knowledge refers to: a) area measurement using spatial units, b) area measurement using area formulae, c) basic area transformations d) basic elements of a triangle such as altitude, the segment that joins a vertex with the middle of its opposite side and the perpendicular bisector of the base of an isosceles triangle, e) basic properties of regular polygons and of typical quadrilaterals such as, rectangles and squares.

ii) The role of tools on students' constructions:

The 'drag mode' operation, the automatic area measurement and the automatic tabulation of the numerical data. By combining these features, students were given the challenge of observing a large number of non congruent triangles which conserved their area (categories C1, C3, and C4). Having constructed equivalent triangles using these features, students had the chance to construct a more abstract, dynamic and solid view of the concept of conservation of area regarding triangles. The tool for automatic area measurement was used in every strategy performed by the students, in order to verify their attempts at constructing equivalent triangles during this experiment. In most strategies this tool was used in combination with automatic tabulation of the produced numerical data. This tool was also used in combination with the ‘drag mode’ operation, as scaffolding elements to improve the construction of equivalent triangles based on students’ visual perception (category C1).

The tools for basic area transformations and for constructing regular polygons. Students availed of these Cabri-tools to construct equivalent triangles in ways uncommon in school practices (categories C6 and C7 correspondingly). By constructing equivalent triangles using basic area transformations and regular polygons, students exploited their previous knowledge.

The tools for constructing arbitrary, parallel and perpendicular lines. Students used these tools to form specific constructions of pairs of equivalent triangles (category C4).

Conclusions

This study demonstrates that students performed an abundance of strategies to construct equivalent triangles using the tools provided by Cabri. Students were inspired to use any provided tool by the context of the given task asking them to construct equivalent triangles in ‘any possible way’. The nature of the tools provided by Cabri affected students’ constructions of equivalent triangles. These Cabri-oriented constructions express different pieces of knowledge that students possessed. These pieces of knowledge referred to: area and its boundary, area measurement using spatial units, area formula, basic area transformations, polygons, basic elements of a triangle, different types of triangles as well as parallel and perpendicular lines. Students constructions used this knowledge in transforming an original triangle into other equivalent ones as well as in splitting a polygon into equivalent triangles. By expressing these different pieces of knowledge, students had the opportunity to construct an integrated view regarding the concept of conservation of area in triangles. Finally, by exploring the equivalence of a large number of equivalent triangles dynamically produced in the context of Cabri, students had the opportunity to construct a more abstract, solid and dynamic view of this concept. On the whole, the availability of a variety of tools by Cabri as well as its dynamic character makes it a competent environment in the teaching and learning of the concept of conservation of area in triangles.

References

Carpenter, T. P., Coburn, T. G., Reys, R. E. and Wilson, J. W. (1975). ‘Notes from National Assesment: basic concepts of area and volume’, *Arithmetic Teacher* 22 (6), 501-507.

Cohen, L. and Manion, L. (1989). *Research Methods in Education,* Routledge, London.

Douady, R. and Perrin, M-J. (1986). ‘Concerning conceptions of area (students aged 9 to 11)’, *Proceedings of the 10th International Conference, Psychology of Mathematics Education,* London, England, pp. 253-258.

Hart, K-M. (1989). ‘Measurement’, in J. Murray (ed.), *Students Understanding of Mathematics*: *11-16*, Athenaeum Press Ltd., G. Britain, pp. 9-22.

Hirstein, J., Lamb, C. E. and Osborn, A. (1978). ‘Student Misconceptions about area measure’, *Arithmetic Teacher* 25(6), 10-16.

Hughes, E. R. and Rogers, J. : 1979, ‘The concept of area’, in Macmillan Education (eds), *Conceptual Powers of Children: an Approach through Mathematics and Science*, Schools Council Research Studies, pp. 78-135.

Kordaki, M. and Potari, D. (1998). ‘A learning environment for the conservation of area and its measurement: a computer microworld’, *Computers and Education* 31, 405-422.

Kordaki, M. and Potari, D. ☹2002). ‘The effect of area measurement tools on student strategies : the role of a computer microworld’, *International Journal of Computers for Mathematical Learning 7(*1), 1-36.

Kordaki, M. (2003, in press). The effect of tools of a computer microworld on students’ strategies regarding the concept of conservation of area*. Educational Studies in Mathematics.*

Laborde, J-M. (1990). *Cabri-Geometry* [Software], Universite de Grenoble, France, 1990.

Laborde, C. and Laborde, J-M. (1995). What about a Learning Environment where Euclidean Concepts are manipulated with a mouse? In A. diSessa, C. Hoyles, R. Noss with L. Edwards (Eds), *Computers and Exploratory Learning* (pp.241-261), Berlin: Springer-Verlag.

Maher, C.A. and Beattys, C. B. (1986). ‘Examining the Construction of area and its Measurement by Ten to Fourteen Year old Students’, in E. Lansing, G. Lappan, and R. Even (eds), *Proceedings of the 8th International Conference, Psychology of Mathematics Education,* N. A., pp. 163-168.

Mariotti, M., A. (2001). Justifying and Proving in the Cabri Environment. *International Journal of Computers for Mathematical Learning* 6(3), 257-281.

Noss, R. and Hoyles, C. (1996). *Windows on mathematical meanings: Learning Cultures and Computers,* Kluwer Academic Publishers, Dordrecht.

Piaget, J., Inhelder, B. and Szeminska, A. (1981). *The child's conception of geometry*, Norton & Company, N.Y.