VOLUME: A COMPUTER MICROWORLD FOR THE LEARNING OF THE CONCEPT OF VOLUME

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ABSTRACT

This paper presents the design and basic features of a computer microworld for learning the concept of volume, for use by both primary and secondary level education students. The design of this microworld is the result of a modelling process in which three models were constructed, namely: the learning model, the subject matter model and the learner model. The construction of the learning model was based on constructivist and social theories of learning interpreted within the context of the computer. The construction of the subject matter model was based on the two fundamental aspects that constitute the concept of volume, namely: conservation and measurement of volume. The construction of the learner model was based on the literature referring to student behaviour regarding the learning of the concept of volume. All the models above are in the form of hierarchical trees, where educational specifications were gradually transformed into operational specifications which were then used for the implementation of the software.

In the context of VOLUME, students can study any parallelepiped. The basic features of VOLUME are: a) the construction of any parallelepiped by the students, just by drawing its dimensions, b) the dynamic transformation of a parallelepiped into a plethora of other parallelepipeds of equal volume, c) the construction of a cubic-unit for volume-measurement by the students, simply by drawing its dimensions, d) the iteration of the constructed cubic-unit, e) the automatic projection of the constructed unit onto the figure of a parallelepiped, f) the automatic measurement of the volume of the parallelepiped under study, using as a unit the cubic-unit constructed by the student, g) the automatic measurement of the volume of the parallelepiped under study, using the volume formulae, and h) the automatic measurement of the length of the edges of a parallelepiped using the length of the volume-unit constructed by the student as the length unit.

The learning activities that students can perform in the context of VOLUME are: a) the transformation of a parallelepiped into other parallelepipeds with equal volume, b) the comparison of volume for a variety of parallelepipeds, c) the investigation of the basic properties of a parallelepiped, e.g. the length of its edges, the area of its sides, its angles etc., and d) the investigation of equivalence regarding the volume of parallelepipeds.

KEYWORDS

Volume, Primary and Secondary Education, Educational Software

INTRODUCTION

Measurement is a universal and fundamental activity in all cultures (Bishop, 1988). Measurement of volume plays a major role in the culture of all societies, not only in science and technology but also in daily life. Volume, in a geometrical sense (sometimes termed occupied volume or external volume), is the amount of space occupied by an object (a 3D shape) as a whole in relation to other objects round about it (Piaget, Inhelder and Szeminska, 1981, p.360). Volume can be conserved when the figure of its shape is altered. Understanding conservation of volume is essential for its conceptualization. Generally speaking, conservation is an essential preliminary to all Euclidean constructions (Piaget, et al., 1981). Moreover, understanding conservation of volume is prerequisite for the understanding of measurement of volume, which presupposes the construction of a spatial continuum and its quantification in terms of multiplicative matrices. The elaboration of metrical relations between the volume under consideration and the surfaces by which it is bounded is also crucial for its conceptualization. Understanding volume formula is based on all the above while, on the whole, the understanding of volume in terms of: interior volume, external volume, geometrical volume, physical volume, capacity, displacement, conservation of volume, measurement of volume using cubic units and volume formulae is essential to its conceptualization (Piaget, et al., 1981; Potari and Spiliotopoulou, 1996). It is also acknowledged that essential tasks for students to be able to conceptualize the notion of volume include transformation and comparison of 3D shapes as well as the construction of a variety of 3D shapes using cubic bricks (Piaget, et al., 1981).

A large amount of research focuses on children's difficulties with the concept of volume. Despite the fact that pupils can understand the notion of conservation with regard to 'interior volume' (in other words, they can understand the invariance of the amount of matter which is contained within boundary surfaces), they do encounter difficulties in understanding conservation in terms of the 'occupied space' mentioned above. More specifically, children perform transformations and reconstruction or comparisons of volume by focusing on one dimension only, usually the largest. Children's primary understanding of conservation of volume is usually limited to its conceptualization as something bounded by surfaces (Piaget, et al., 1981). Moreover, children present difficulties in understanding the accurate relationship between variation of length and variation of volume in cubes, when performing activities regarding the doubling and tripling of volume (Battista and Clements 1996; Battista, 1999; Potari and Spiliotopoulou, 1996; Kouranou and Potari, 2003; Piaget et. al., 1981). In the process of doubling and tripling, students usually do not succeed in formulating an accurate quantitative relationship; rather, they consider this relationship in a primitive and intuitive way, simply acknowledging that the cube with the bigger edge has the bigger volume. Students also face great difficulties in measuring volume using cubic units and subsequently in understanding volume formulae.

For the understanding of the concept of volume, a number of different objects were used, such as: real objects, objects represented in a paper and pencil environment (Piaget et al., 1981; Potari and Spiliotopoulou, 1996; Markopoulos and Potari, 1999) as well as objects represented in computer based environments, for example Geometria, Sketchpad-Hypercube and Cabri-Geometry II (Kouranou and Potari, 2003; Markopoulos, 2003). Despite the fact that students were provided with great opportunities when studying real objects as well as when studying 3D objects in the paper and pencil environment, computer learning environments can provide the students with even wider opportunities. The aforementioned computer environments present students with opportunities to create 3D hexahedron and to transform them by dragging their segments (Cabri-Geometry II, Sketchpad-Hypercube) or by numerically altering their dimensions (Geometria). Students also have the ability to calculate the size of the sides, the angles and the volume of the shape drawn on the computer screen. However, computer learning environments that support students in their understanding of the concept of volume in a geometrical sense by allowing them to experiment with the conservation of volume, as well as with measurement of volume using cubic units, have not yet been reported.

It is widely accepted that appropriately designed computer learning environments can support constructivist and exploratory learning. Designers of educational software can exploit the advantages of computer technology, in particular object-oriented design, as well as the computer's ability to represent a concept in multiple and linked external representation systems in the design of open problem solving environments. The dynamic character of computational objects and the intrinsic visual feedback of these environments have been acknowledged by many researchers as crucial for students in their attempts to reflect on their actions and form abstract concepts, especially in Geometry. Well known examples of such environments are Cabri-Geometry II, The Geometers' Sketch-Pad etc. Of course, the role of the tasks and the intervention of the teacher are also crucial to student learning, as each of these environments and tools is not effective by itself.

In our attempt to provide the students with the opportunity to exploit the previously mentioned advantages of computer technology in their learning regarding the concept of volume, in a geometrical sense through its conservation and its measurement, we designed and implemented a computer microworld that we named VOLUME. Such a microworld has not yet been reported. In this paper, the rationale of the VOLUME microworld is presented in the next section, followed by a demonstration of the specific features of this microworld and a description of the categories of activities that students can perform within the context of VOLUME. Finally, there is a discussion of the features of this microworld and proposals for future research work are given.

THE RATIONALE OF THE VOLUME MICROWORLD

By the term microworld we mean a conceptual world that consists of a set of objects, a set of rules about their relations and a set of operations that can act on them (Puffal, 1988; Laborde, 1990). In the case of VOLUME, the objects are parallelepipeds constructed by the student while interacting with this microworld. The relations between the objects are the conservation of their volume through dynamic transformations into a variety of parallelepipeds with equal volume. The available system operations are tools for transforming parallelepipeds into other ones of equal volume, as well as tools for volume measurement.

The design of VOLUME has been based on a modeling process consisting of the design of three specific models, namely: the learning model, the subject-matter model and the learner model (Kordaki and Potari, 1988). In our attempt to construct the learning model, we tried to explain our view regarding knowledge construction in terms of software specifications. The subject-matter model refers to the concept of volume in terms of conservation and measurement of parallelepipeds. This model has been designed taking into account the psychological foundations of the above-mentioned concepts as these emerged from the literature. Learner difficulties with the concept of volume have also been taken into account in the construction of this model. The design of the learner model aims to describe the possible behavior of the learner in terms of his/her possible actions while implementing essential activities for the learning of the concepts in focus. The design of this model was also based on the specific literature. Additionally, features of electronic media - specifically the possibilities for using and manipulating computational objects as well as the possibility for using multiple and linked representation systems such as: static visual, dynamic visual, numerical - had a significant impact on the design of each specific model. These representation systems were used to provide the student with the opportunity to express the different kinds of knowledge she/he possesses, such as: primary intuitive knowledge, spatial knowledge using volume units and volume formulae (Kordaki and Potari, 2002; Kordaki, 2003). By expressing these different kinds of knowledge, students can construct a broader view of the concept in focus (Kordaki, 2003). Moreover, our design aimed to support student work in essential tasks, enabling these to be performed in a variety of contexts. This is appropriate for the support of students in their conceptualization of volume in a geometrical sense.

In the following section, the aforementioned models are presented.

The learning model

For the construction of this model modern constructivist and social considerations of learning have been taken into account (von Glasersfeld, 1990; Crawford, 1996a; 1996b). Constructivist and social views of knowledge stress that learning is viewed as a subjective, active and constructive activity within a context rich with tools - especially computer tools - that support learners in expressing their knowledge as well as in exploring the knowledge of others. Computer tools can be used to support learners in expressing their knowledge through external representation systems. Different representation systems that are cognitively transparent to learners can support them in expressing both inter-individual and intra-individual differences, as they can select from the provided representations those most appropriate to

their knowledge. In addition, different representation systems - especially those non-cognitively transparent to learners - can help learners to explore the knowledge of others as represented in these systems, in this way enabling them to enhance their Zone of Proximal Development (Vygotsky, 1978).
In Table 1, an attempt has been made to express the basic aspects of the aforementioned modern theories of learning in terms of design principles.

Table 1. The learning model: from theoretical considerations to design principles

The 'learning' model	
Theoretical considerations	**Design principles**
The active role of the learner	• Interactivity of the environment • Availability of tools
The subjective character of learning	Availability of tools: • to construct different representations of the concepts to be learned, • to help students construct and study their own computational objects related to the concepts in focus
The constructive character of learning	Availability of tools: • to explore the knowledge of others e.g. simulations, computational objects • to be used in solving different problems • to be used in making constructions, acquiring hands-on experience related to the subject matter, • to give intrinsic visual feedback on learner actions

The subject matter model
In the first column of Table 2, the essential aspects for the understanding of the concept of volume, namely: conservation of volume, volume measurement (the concept of cubic unit, the iteration of units and the counting of units), and the relationships between volume and the various properties of the shapes under study are presented. Activities essential for students to grasp these aspects are also presented in this column. These aspects are interpreted in terms of design specifications in the second column of this table.

Table 2. The model of volume: from theoretical considerations to design principles

The model of the 'subject matter'	
Theoretical considerations: Students can construct the concept of volume in a geometrical sense by…	**Design principles:** Tools needed to…
Conservation of volume	
conserving the volume of a parallelepiped actively, splitting it into 3D-parts and recomposing these parts to produce other 3D-shapes with equal volumes	split and recompose parallelepipeds : tools to select, copy, cut and paste entire parallelepipeds or parts of them
constructing a 3D-shape composing cubic-units and then *conserving the volume* of the produced shape by recomposing the cubic-units actively.	construct a variety of learner-made cubic-units and to iterate these units and to iterate them
exploring *conservation of volume* in a number of different parallelepipeds of equal volume dynamically represented by the system.	automatically transform parallelepipeds into others of equal volume
exploring *conservation of volume* in classes of parallelepipeds with equal volume	automatically produce classes of parallelepipeds of equal volume and equal dimensions (length, height and width)
Relationships between volume and various properties of the shapes under study	
realizing the *relationship between different properties of parallelepipeds* such as: edges, angles, sides with their volume. Students can also see the impact of the variation on an element (side, edge, angle) of a parallelepiped to the variation of its volume.	measure the edges, the sides, the angle and the volume of a parallelepiped
Volume measurement	
measuring the volume of parallelepipeds by splitting them into a variety of volume-units and counting the total units	construct a variety of learner-made cubic-units and a variety of grids, and to project these grids onto the hexahedron's sides.

The learner model
The learner model consists of possible actions a learner can take in performing the tasks that can be performed within this computer environment. These tasks are proposed by the literature as being essential

if learners are to grasp the concept of volume in a geometrical sense. The proposed tasks are: a) comparison of 3D-shapes not easily comparable by sight, and b) transformation of 3D-shapes into others of equal volume. In the first column of Table III, these tasks are presented, while in the second column possible learner actions in the performance of these tasks are demonstrated.

Table 3. The learner model of volume: from theoretical considerations to design principles

The learner model in performing essential tasks	
The task of comparison	By measuring the volume of shapes in comparison: • automatically • using volume units or grids By superimposing the shapes in comparison using : select, copy, cut and move operations
The task of transformation	Transforming parallelepipeds into others of equal volume: • automatically • by splitting them up and recomposing the parts to produce equivalent shapes using the copy, cut, paste and move operations • using volume-units or grids, counting the cubic units and recomposing these units to produce parallelepipeds of equal volume

THE FEATURES OF THE VOLUME MICROWORLD

This section presents the features of the VOLUME microworld:

- Drawing parallelepipeds
- Manipulation of parallelepipeds without the use of numbers
- Measuring volumes using a variety of tools
- Transforming parallelepipeds automatically into other ones of equal volume
- Measuring automatically: sides, edges, angles and volumes
- Providing material for off-line study
- Comprehensive on-line help

Invoking VOLUME brings up a dialogue box in which the student enters their name; the system then automatically saves all their interactions with the microworld operations in a log-file, for off-line study. Then the main menu is shown (Figure 1).

File	**Draw**	**Edit**	**Compute**	**Equal Volumes**	**Metric Unit**	**Help**
Save as	New Shape	Select Shape	Volume	Cube	Show Unit	How to ..
Print	Background Grid	Delete Shape	Show Results	Rectangular	New Unit	Index
Return to Main Menu	Clear Screen	Delete Last Shape	Shape Info	Parallelepipeds	Restore Unit	
Exit		Move To..			Iterate Unit	
					Shape Grid	

Figure 1. General design of the VOLUME interface

Drawing parallelepipeds. Students can draw parallelepipeds with the following tools:
New Shape: Clicking on this command causes the program to begin creating a new parallelepiped by asking the student for the lengths of its edges and the angles between them. At this point, the student must draw on the screen the segments that she/he wishes to correspond to the length, the width and the height of the new shape. The system then asks for the angles between the edges of the new parallelepiped. At this point, the student must give the measurement of these angles in degrees. Next, the system asks for the position on the screen where the new shape is to be displayed.

Background Grid: Displays a background square grid in the drawing area, dependent on a metric unit constructed by the student. This grid can be used by the student to draw the length, the width and the height of a parallelepiped by measuring them in a qualitative way.
Clear Screen: By clicking on this command, the student instructs the program to clear the screen

Manipulation of parallelepipeds without the use of numbers. The following Edit commands are available:
Select Shape: When activated, a shape can be selected. In order to select a shape, the learner must select a corner point. In this case, the selected shape changes its colour.
Delete Shape: Deletes the selected shape.
Delete Last Shape: Deletes the latest shape created.
Move To..: Moves the selected shape to a position indicated by the learner. The corner point in the indicated position corresponds to that which was used for the selection of the shape. You can see an example of this feature in Fig.2. where Shape 1a is selected and then moved as Shape 1b into Shape 2 for comparison.

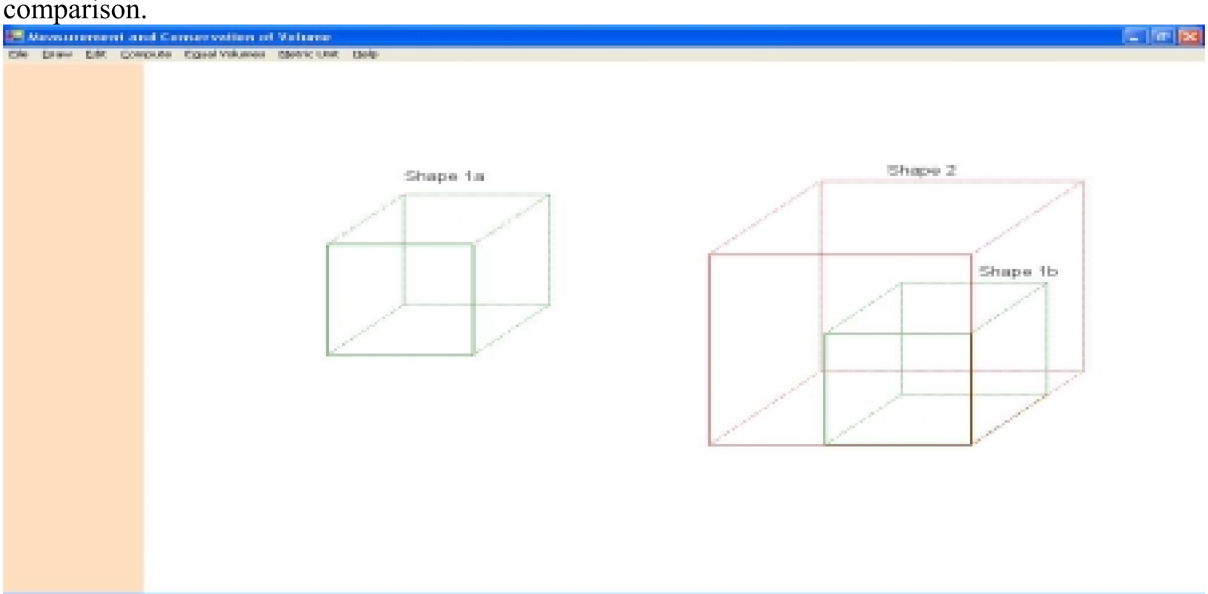

Figure 2. Manipulating a parallelepiped using Edit commands

Measuring automatically: sides, edges, angles and volumes. The following computing commands are provided:

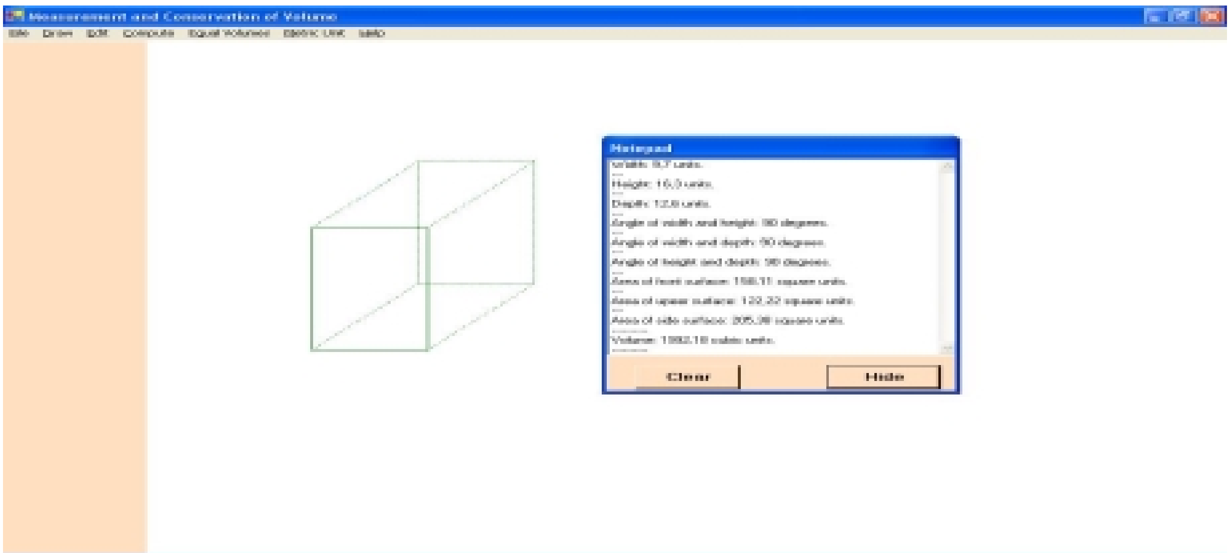

Figure 3. Providing numerical information for a selected shape

Volume: Computes the volume of the selected shape, using the current metric unit.
Shape Info: Displays a message-box with all the numeric data of the shape selected (edges, lengths and angles between them as well as surface areas). You can see an example of this feature in Fig.3.
Show Results: Brings the notepad to the foreground; this contains the history of all the numerical characteristics of all shapes constructed by the learner: i.e. volume, edges, surfaces etc.

Transforming parallelepipeds automatically into others of equal volume. The available features are:
Cube: Draws a cube of volume equal to that of the selected shape.
Rectangular Parallelepipeds: Asks for the width and height and then draws a rectangular parallelepiped equal to the selected one. The program asks the learner for the position of this parallelepiped on the computer screen. You can see an example of this feature in Fig.4.

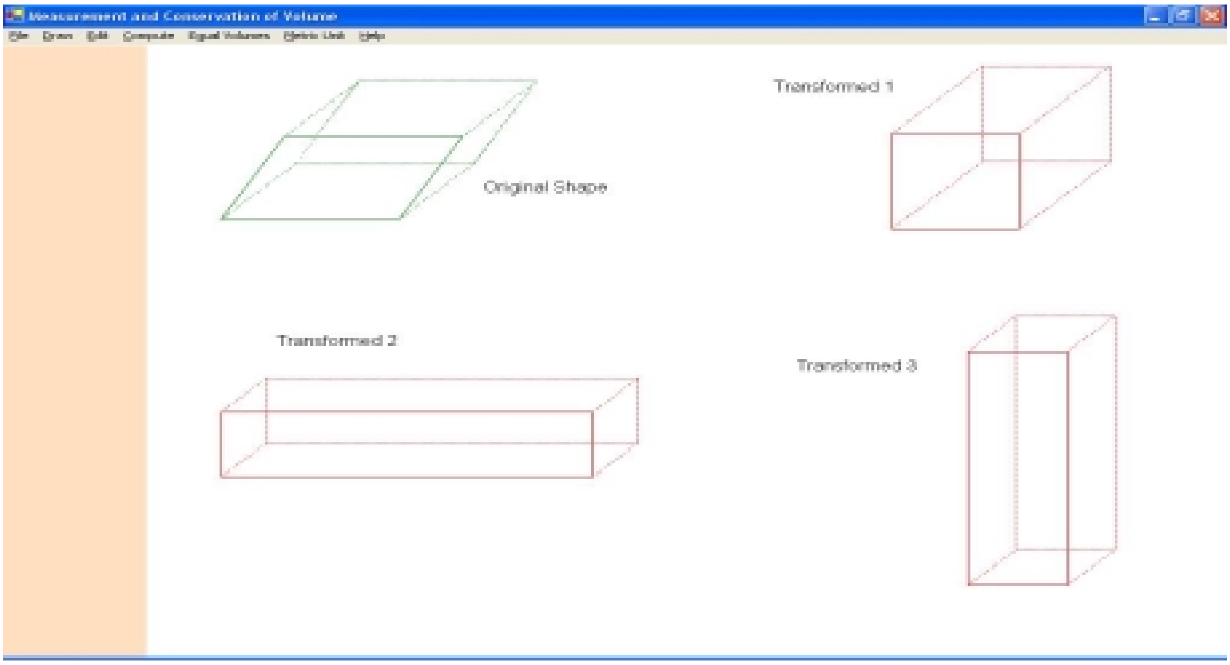

Figure 4. Transformations of an original parallelepiped into others of equal volume

Measuring volumes using a variety of tools. The tools provided for volume measurement are presented below:
Show Unit: Displays - in the upper left corner of the drawing area - the default volume metric unit and its length.
New Unit: Makes it possible to create a new cubic metric unit by asking the student to draw its edge.
Restore Unit: Restores the metric unit to its initial form (cubic metric unit with 10 pixel edge)
Unit Iteration: Places the selected cubic-unit on the screen at a specific point indicated by the learner by clicking. You can see an example of this feature in Fig.5.
Shape Grid: Displays a square grid (dependent on the metric unit used) on two adjacent surfaces of the shape. You can see an example of this feature in Fig.5.

Providing material for off-line study
The environment includes typical File operations, some of which can be used to provide the student, the teacher and the researcher with extra materials (remember that student interactions with the software are automatically saved by the program in log files) for off-line study. These operations are:
Save as: Saves the drawing area as a JPEG image file.
Print: Prints the drawing area to a default printer
Return to Main Menu: Closes the current form and returns to main menu
Exit: Quits the program

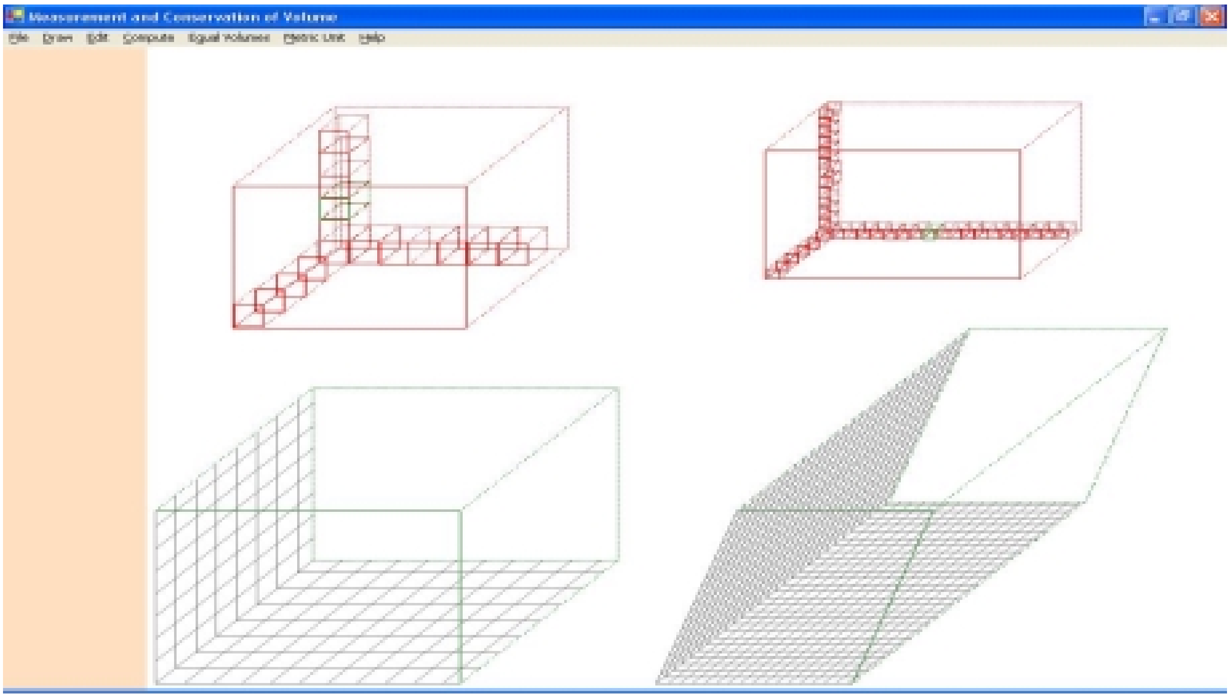

Figure 4. Measurement of volume using learner-made: cubic-units (top) and grids (bottom)

Comprehensive On-line Help

An On-line Help component is useful in preparing the students to interact with the microworld. The information provided is broken down into small pieces which are presented in alphabetical order. Each piece of information refers to each submenu operation provided by the microworld.

Technical information: The microworld has been implemented in Visual Basic.NET, running with Windows XP or later versions.

CONCLUDING REMARKS

The design and specific features of a computer microworld for the learning of the concept of volume by primary and secondary level education students has been presented in this paper. The design of this microworld has been based on a modelling process consisting of the construction of three models: a) the learning model, based on modern constructivist and social views of learning, b) the subject matter model, based on the information provided by the literature regarding the concept of volume and its conceptualisation by students and c) the learner model, based on the designers' views on the possible actions of learners while performing essential tasks in order to grasp the concept of volume, as having emerged from the literature. In the construction of the aforementioned models, an attempt has been made to interpret theoretical principles in design specifications. An attempt has been also made to exploit the advantages of computer technology in terms of the use of computational objects and multiple and linked representation systems in the design of this microworld. The specific features provided by this microworld have not yet been reported by other researchers.

In the context of VOLUME, students can study any parallelepiped. Students can use the basic features of VOLUME to:

a) construct any parallelepiped, simply by drawing its dimensions,
b) dynamically transform a parallelepiped into a plethora of other parallelepipeds of equal volume,
c) compare parallelepipeds without the use of numbers (placing one parallelepiped within another)
d) construct a cubic-unit just by drawing its dimensions, and to iterate this unit for measuring the volume of a parallelepiped,

e) construct a square grid simply by drawing its dimensions, automatically projecting this grid onto the neighboring sides of a parallelepiped,
f) automatically measure the volume of the parallelepiped under study, using as a unit the cubic unit constructed by the student,
g) automatically measure the volume of the parallelepiped under study, using the volume formulae,
h) automatically measure the length of the edges of a parallelepiped using the length of the cubic-unit constructed by the student as the length-unit, and
i) study material provided off-line (the automatically saved log files capturing learner interaction with the provided tools and the saved materials using the typical File operations for saving and printing files)

The learning activities that students can perform in the context of VOLUME are:
a) transformation of a parallelepiped into other parallelepipeds of equal volume,
b) doubling and tripling the volume of a parallelepiped by appropriately transforming its dimensions
c) comparison of volume for a variety of parallelepipeds,
d) investigation of the basic properties of a parallelepiped, e.g. the length of its edges, the area of its sides, its angles etc., and
e) investigation of equivalence in the volume of parallelepipeds.

FUTURE WORK

It is clear that more research is needed, using the VOLUME microworld in the field with real students in order to test both its learnability and its usability. The findings emerging from such a study would be of great assistance in the re-designing of this environment so as to better meet students' cognitive needs in terms of their understanding of the concept of volume.

Acknowledgements: Many thanks to Stephen Taylor at TES, Patras for proof reading and assisting with our English.

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