Children's Approaches to Area Measurement through Different Contexts

Maria Kordaki and Despina Potari

University of Patras

261 10 Patras, Greece.

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ABSTRACT

This study focuses on 12 years old children's approaches to area measurement in a project environment. These approaches are not explored through a specific set of mathematical tasks. The tasks, here, are defined through researchers' and children's interactions in a classroom. The children by working in small groups are asked to make a proposal about the location and the form of an area which would be given to them for their leisure activities. This environment defines different contexts where the children act and consider different aspects of the area measurement. These aspects are identified and compared among the three groups of children. The study has shown that the concept of area measurement carries different cultural dimensions for the children. Moreover, the children use those elements of the concept which fit in with their personal experience and the tasks they have to face.

"Measurement is the third "universal" and significant activity for the development of mathematical ideas, and is concerned with comparing, with ordering, and with quantifying qualities which are of value and importance" (Bishop, 1988). In particular measurement of area is part of our culture, of science and technology but also of our everyday life (Hirstein, Lamb and Osborn, 1978; Sanders, 1976). It is also closely related to the number concept (Steffe & Glasersfeld, 1985; Skemp, 1986) and is used as an embodiment to introduce other mathematical concepts. The importance of area measurement also appears in the understanding of multiplicative structures (Douady and Perrin, 1986) and in the link between the abstract world of numbers and the concrete world of physical objects (Hiebert, 1981). Moreover, it allows children to relate spatial concepts to numerical ones.

In this study, we attempt to study children's thinking on area measurement and its development through different contexts. These contexts are defined through researcher's and children's interactions while the children work in small groups in a project-environment. By "project- environment" we mean a situation which is concerned with the relationship between mathematics and society (Bishop, 1988) in which the concept of area measurement is embedded. In particular, in this setting we wanted to investigate:

- the measurement tools that children construct and use in order to measure areas and the meaning they attribute to those
- the ways in which children imagine real areas and approximate their size
- the ways in which children represent real areas on the paper
- the effect of the everyday and school knowledge on children's decisions and actions

Research on children's thinking about area measurement emphasises the various aspects of this concept. The basic aspects that have been identified are the selection of the unit of measurement, the unit iteration, the counting of units of measurement and the use of area formulae. Piaget, Inhelder and Szeminska (1981) consider the invariance property of area as a prerequisite before measurement. Children need to develop an understanding of area as a space inside a figure for the above aspects to be meaningful (Hirstein, et al., 1978; Maher and Beattys, 1986). Large amount of research has dealt with the aspect of unit of measurement (Carpenter, 1975; Driscoll, 1981; Hiebert, 1981; Mayer and Beattys, 1986; Reynolds and Wheatley, 1996; Steffe and Hirstein, 1976) while some of them have focused on the inverse relationship between the size of the unit and the number of units (Carpenter and Lewis, 1976). Douady and Perrin (1986) study children's conceptions concerning area measurement by using three interacting settings: the setting of surfaces (geometry without measure), that of areas as products of lengths and that of numbers operating on areas.

The above studies have investigated children's thinking on specific aspects of area measurement and in most cases in a mathematical context. Moreover, most research tasks have used surfaces drawn on paper. Our decision to use a more wide setting and to encourage children to act in different contexts was based on our goal to investigate the complexity of children's thinking about area measurement. As Mayer (1987) claims, asking a pupil's opinion about a single fact, or a single isolated concept, could hide the complex network, the conceptual map which lies behind, or around, every concept. Tirosh and Stavy (in press) also argue that scientific frameworks are not rich enough to explain the ways students think and act. Patronis (1996) emphasises the "scene - setting" as an alternative to the use of a particular "embodiment" as a way of providing children the opportunity to develop both common sense arguments and mathematical concepts. Bishop (1983) and Potari and Spiliotopoulou (1996) acknowledge the need to use in research and in teaching geometry not only small-scale models but also the real, large-scale space. Nunes, Light and Mason (1993) have

attempted to investigate the effect of the cultural framework, especially of the conventional measurement tools, on children's reasoning about area. A more recent study (Baturo and Nason, 1996) concerns the type of knowledge which student teachers use in the domain of area measurement. But even in the last two examples of research that consider the problem of area measurement in an integrated way, the tasks were mathematical and the surfaces were presented to the children on the paper.

In most studies children's thinking and its development has been explored through clinical interviews or tests which are mainly applied to individual children. In our work we study groups of children and we try to see the effect of the co-operation of the individuals on their conceptual development. Cobb, Wood and Yackel (1992) suggest that individual and group development are interdependent and that they are related reflexively. They also propose classroom settings as research environments for studying children's constructions.

THE PROJECT-ENVIRONMENT

Children were asked to make a proposal to the mayor of their town about the location and the form of an area which would be given to them for their leisure activities. They were encouraged to co-operate with each other and with the teacher by working in groups and as a whole class. The activity has been developed in different phases . These were the results of our initial research goals, of children's choices and decisions and of our interpretations of children's actions. First, children attempted to make the problem their own by identifying the constraints of the project. In the next phase, children expressed their cultural values and interests to determine the activities that would take place in this area. Then, they were asked about the shape and the size both of the whole area and of the areas that corresponded to the chosen activities. After this, a whole class discussion took place where children's plans were discussed and negotiated. In the next phase, children were asked to express the meaning that they

were giving to the standard units of measurement that they used in the previous phase. This phase had not been planned from the beginning but it arose from the whole class discussion. In this phase the children were asked to write a description about the size of two rectangular areas with known dimensions expressed in standard units that they used in their proposal, one of a large and the other of a medium size. The different size of the areas was chosen because children had already exhibited different approaches in their group discussions. Another question was about the meaning of a square metre and how this standard area unit was related to the smaller of the above proposed areas. Up to then, children's measurement approaches had been expressed verbally in an imaginary situation. To investigate further how they transformed their approaches in another context, we asked them to find a way to place their chosen areas on an irregular shape of paper considering it as a plot of land. We used the irregular shape of paper as a challenge for children also to express other than the conventional school measurement procedures. Without proposing the use of any measurement tools, we asked children to measure the formed areas and discuss their methods. The children's tendency to use the measurement area formulas to calculate areas prevented us from studying some qualitative aspects related to the concept of area measurement. This led us to give the children an irregular shape and ask them to measure it without allowing them to use the area formulas and other conventional measurement tools. This created environment is of the same nature as the one described by Athanasopoulos, Patronis, Potari, Spanos and Spiliotopoulou (1993).

METHOD

A classroom teaching experiment was organised in a primary school of Patras, in Greece. Fifteen children of 6th grade (12 years old) worked in groups of five for 8 teaching hours (45 minutes each) over a period of one week. The children's school experience on the area and its measurement was limited on the use of a square as a unit, of standard units of measurement, and of area formulas for calculating areas.

There was almost no experience at all with activities based on the conservation of area. The main work concerned the calculation of the area of regular shapes in a mathematical context or in mathematical problems using everyday life situations.

Groups were formed on the basis of children's personal relations and are referred to the paper as group A, group B and group C. A teacher participated in each group, co-ordinating the group discussion and asking children to clarify their approaches. These teachers co-operated with the authors before and after each teaching session. Through this co-operation , the actions of the children were discussed and they determined our decisions about the process of the project. One of the authors was a teacher of one group who also presented the tasks and organised the whole class discussion. The other two teachers had been working together with the authors for the previous two years. One of them was a mathematics teacher and the other a primary school teacher who was also a postgraduate student. Small group work was audio recorded while the whole class discussion was video recorded. The transcribed recordings, together with the children's written work constitute our data source for our analysis.

To investigate children's thinking and its development in area measurement we used a methodological approach similar to that described by Cobb et al (1990). It is an extension of the constructivist teaching experiment proposed by Steffe (1983) in the classroom. In this methodology, teachers act as researchers who try to interpret children's actions in the process of constructing mathematical knowledge. These interpretations are subjective and depend on the researchers' mathematical knowledge and personal views and theories about teaching and learning. Children's mathematical knowledge is personal and depends on the ways that children make sense of their experiences. Children, through communication with the teachers and the other pupils, express, negotiate and develop mathematical meanings. The small group and the whole class discussion according to our perspective can provide, as Cobb et al (1990)

suggest learning opportunities for children and researchers and can give us some evidence concerning the development of children's thinking.

RESULTS

In the analysis we looked for children's meanings about aspects of area measurement and how they developed through the communication in the groups and in the classroom. This development is the result of the contribution of the individual child's thinking in the group and of the way that children have interpreted the teacher's intervention. The group's decisions were the result of the negotiation of the meanings of the group members.

Our analysis is based on the work of the three different groups during the different phases of the experiment. We tried to identify children's thinking as it was expressed through communication in the groups. Children's performed actions, their expressed conceptions, their reasoning, beliefs and values are aspects of children's thinking that we explored in this study. An attempt to make some comparisons between the different cases is also made.

Connecting cultural and social values with the area concept

Familiarisation with the activity

Although the activity was an imaginary problem from our point of view, the children conceived it as a real situation which they had to face. They initially tried to define some constraints on the implementation of such a project. All the groups emphasised the time needed to complete the construction as they were interested to see the impact

of this construction on their life and on the life of other children. Group A and group C wondered about the funding and the finding of the location for the realisation of this establishment. Another point that group B and group C considered was the financial constraints in relation to their choices. All these social factors that the children considered were crucial in their decision to undertake the study of this project. The teachers' intervention helped the children to overcome their uncertainty and tackle the problem. This initiation into the problem shows how the social reality reflects on children's views (Lave, 1988; Bishop, 1988; Evans, 1991).

Making decisions about the kind of leisure activities

The group's proposals about the kind of leisure activities were the composition of individual desires which were accepted by the members of the group. The whole communication did not reveal any disagreement between the children as the project did not impose on them financial or location constraints.

All the groups wanted to have a sport centre where children could play their favourite games like basketball, volleyball and gymnastics. Their choices reflect the overall current preferences of entertainment in their society. Moreover, group C proposed places for other cultural activities like a library, an art room, a computer room and a theatre.

In all the groups , a rectangular shape was decided upon as the area to build their proposed construction, a shape which is commonly used for this purpose. The children interpreted the question about the kind of area that they had to choose in two dimensions. The first was the kind of leisure activities while the other was the shape of the proposed area. Group C was the only group that decided on the rectangular shape after rejecting the square and the trapezium. The square was rejected as occupying a bigger area compared to the rectangle, while the trapezium was not a

familiar shape for them. At this point, the children's comparisons indicate an intuitive feeling about the relation of area and the shape in which this area is enclosed.

Imagining and discussing the measurement of real areas

Children started to consider the size of the areas after the teacher's question "How big do you want this sport centre to be?". Up to then children were happy to discuss the kind of leisure activities as this helped them to express all their desires and they did not feel any need to "mathematize" the situation. In this phase, children imagined the real areas that they had proposed and discussed their size. This verbal description of the quantity of real areas showed us children's measurement approaches in this context. The strategies of each group and their development are summarised in figure 1.

[Insert figure 1 here]

Group A starts with a qualitative approach to the size of the area , moving towards the use of standard units of measurement of the dimensions of the area. At the beginning they used as a unit an area equivalent to the real area they had to measure. This large unit of measurement was familiar to the children. This was described as a whole area "it is almost the same as our school's playground", as an area enclosed by its boundaries "it starts from the end of the school building and ends at the end of the school grounds", and as a commonly accepted area used for this purpose in the society "it is the same as the usual basketball court". The last description arose from the moment that the teacher emphasised the need to discuss their proposal with the mayor of their town. This need probably led this group to define and iterate a unit smaller than the area they had to measure. They described this smaller unit as their classroom which they had to use twice to get the proposed area. As the children claimed, the

classroom could stand for a unit because all the classrooms in the schools of their town are "almost the same". Moreover, they believe that the size of the classrooms is known to the mayor. The doubts that the teacher expressed regarding this belief, seemed to make children reconsider their approaches and use standard units of measurement. This use was not obvious in all the children of this group. The view that the use of standard units was a necessity dominated because of the need to communicate with others. On the other hand, children's previous qualitative approaches came again to light during the discussion. In their attempts to give a numerical answer, they described the already chosen units by using standard units of length to express their dimensions. They also wanted to give meaning to these units of length by comparing them with familiar lengths " 1 m is the length of this desk and a little smaller". Children seemed to apply the standard units of length to measure only one dimension of the rectangle to describe areas. This approach is probably influenced by school knowledge where the areas are usually rectangles whose area measurement is defined by the multiplication of their dimensions.

Group B starts using a standard large unit of area measurement, (which is widely used in Greek culture) 1 "stremma" (1000 m^2). They proposed areas with measure multiples of this unit. The meaning that the children gave to this unit arose from their own environment as is indicated in the following discussion:

Teacher: How do you imagine the "stremma"
Pupil: We have as a base the house of Vassilis.
Teacher: Why?
Pupil: We know that is a stremma so we can measure the plot of land in our proposal.

Although, in the beginning, they discussed about areas using a standard area measure, they changed their holistic approach when they wanted to relate this area to the

dimensions of its rectangular shape. Then they considered areas as rectangles which were defined by their dimensions .This reveals the children's difficulties in giving a quantitative meaning of stremma.

The children of group C attempt to determine the whole area by finding the particular areas where each leisure activity would take place. This group appreciates the need to describe quantitatively the areas but also tries to give a meaning to those areas familiar to them. For example, they create a large familiar equivalent unit of area and then they try to describe it by using standard units of measurement. This description was made either by considering the areas as rectangles with dimensions expressed in standard units of length, or by using standard units of area. The following extract indicates this approach:

Pupil 1: The library to be the same as the classroom

Pupil 2: How many metres is the classroom?

Pupil 1: Its length is 10 metres and its breadth is 5 metres.

In a subsequent discussion another pupil described the area of the classroom as 50 m^2.

In the case of a football ground, children started with the use of the standard units of area measurement and then they tried to familiarise these units. This familiarisation was encouraged by the teacher's question: "Can you imagine and tell me how much 2000 m^2 is?" The children's meaning was expressed in various ways. First they corresponded the 2000 m^2 to a familiar to them equivalent area: "all the school playground". Then they compared with a widely known unit, the standard football ground with known dimensions, and identified the 2000 m^2 as half of that unit: "The standard football is about 90 metres long and 55 metres wide. It is about 4000 m^2". Another way was the use of a smaller area as a unit which was multiplied to find the area that they proposed:" The football area will be 40 times this classroom. This classroom is 50 m^2". Children decided on the size of the classroom by approximating

its measure, as they said, "by eye". Some children used a familiar area defined by its boundaries emphasising one dimension to approximate their chosen area.

From the above, it seems that qualitative units could be transferred to standard units and vice versa.

Investigating further children's meanings on the standard units of area measurement that they used.

The children expressed mainly the final approaches that they had developed previously in the group work and extended them further in this part of the activity. In particular, the children of the group A , compared the area to be measured with an area which they used as a unit. This unit was a rectangle with dimensions expressed in standard units. These dimensions had the same ratio with the dimensions of the area to be measured. The result of the comparison was the ratio of the corresponding dimensions: "the area of 40 m length and 20 m breadth is the area of four classrooms, each of them of 10 m length and 5 m breadth". In this case, children compared similar rectangular areas by comparing their corresponding dimensions . The children of group B tried to familiarise the dimensions of the rectangle by using analogies from their own environment while the last group showed a different appreciation of a small area from a large one as it had also been expressed in their previous work

Although all the children defined the square metre as the area of a square with sides of 1 m by recalling it from their school knowledge, only one child from group B and the children of group C managed to use it as a unit to measure areas. These children filled the area they had to measure with squares of side of 1m . Children of group A divided the area of 450 m^2 by 4 to find a relation with the square metre. This is probably

because of their tendency to consider the relation of areas as the relation of their dimensions. The use of "4" as a divisor could probably be due to children's dependence on linguistic aspects as the word " square" which in Greek is the word "tetragonikon" where "tetra" has its root in the number four. The rest of the children could not see any relationship between the square metre and the area they studied.

Representing real areas on paper

Children were asked to represent their chosen areas on paper to study measurement relations between the individual areas that they proposed. In this case the children's measurement procedures are studied in a different context. Looking into the children's ways of relating the area of the real surfaces with the drawn one , we identified groups' different strategies. The teachers emphasised the need to apply some rules in this transformation so that the children's results from the comparison of the simulated areas could be extended to the real areas. The need to use a scale for this representation was apparent to the children of groups A and C. On the other hand the children of group B initially used an arbitrary drawing of the areas. The teacher's emphasis on rethinking their methods led the children to a 1-1 correspondence of the lengths which, as they admitted, was impossible. This conflict led them to use a scale to make a smaller drawing to fit on the paper. Group A initially determines the desired relationship of these two areas and attempts unconsciously to draw a shape that satisfies this relationship. The teacher could not appreciate the children's mathematics at this point and did not encourage them to continue their attempts. Instead, he led them to consider the dimensions of their chosen area. So, finally, the children followed this alternative way by scaling the dimensions but from the discussion an appreciation of the impact of this scaling on the area did not appear. Both group A and B used the ratio of the dimensions to describe the ratio of the areas. The children in group C, initially, try to give a rectangular shape to the chosen real areas of which they

know the dimensions. They appreciate that this rectangular shape is not uniquely defined so they chose those which they consider realistic in terms of the project's requirements. They used a scale to calculate the dimensions of the represented areas. In their attempts to represent these rectangular areas on paper they used different scales for different areas. In this way, the relationship of the size of the real areas was not kept the same for the represented ones. In extreme cases, children realised this incompatibility " the kiosk cannot be larger than the classroom" which led them to recognise the need for a common scale to allow them to make comparisons.

The groups' strategies are summarised in figure 2.

[Insert figure 2]

Measuring areas of irregular shapes

At this point children were asked to measure the area of the land which was left after placing the chosen areas to accommodate the leisure activities. Children divided this irregular area into known geometrical shapes of whose area they calculated by using the area formula. They chose to use the "ruler" as a tool to measure the lengths used in the formula. The children of groups A and B could not extend this method to cover areas with curved parts. As a result, they did not get a good approximation of the area. The work of the children in group C was more systematic as they chose only one shape, the triangle, to cover the area with the least waste. They justified their choice by claiming that the triangle is the most flexible shape to fit in the curved parts of the area and that they need to know only two measurements to apply the formula. As Miller (1984) states the existence of a quantitative rule which is applied systematically, implies the basic concept that is found in the measurement. This method has also a social value as the civil engineers use the same one to calculate areas.

The tools that the children used to measure these irregular areas did not allow us to explore other aspects of measurement. In the last part of the activity, by asking children not to use area formulas, we aimed to deepen our understanding about their thinking in this context. Children had not faced similar problems at school, so, at first, they did not have any idea how to begin. The teacher encouraged them to construct their own tools. The context posed by the teacher's question: "how could you explain to your young sister or brother how big this area is by using personal things " led the children to use basic area measurement strategies. Children in all groups introduced a unit of measurement. They iterated this unit and split the irregular area into the formed units. The chosen unit was a rectangle for groups B and C, while for the children of group A it was an isosceles triangle. The use of the rectangle as a unit has also been reported as a dominant shape by Heraud (1987). An explanation could be that this shape is common in the culture. The iscoceles triangle could probably have been selected as the most appropriate to fit in the irregular area. After splitting the area into units, the groups faced the area pieces which did not form whole units differently. Group A could not reconstruct whole units from their parts. Group B formed rectangular units by joining two triangles. Group C used different pieces to recombine whole units. The reconstruction of a unit as a way of approximating area is a complicated process that children performed in this context.

DISCUSSION

A large amount of research deals with the relationship between the context and the children's thinking (Evans, 1991; Lave, 1988; Saxe, 1990; Walkerdine, 1988). They emphasise that the context, the problem situation and the individual are closely related. This holistic and systemic view is also supported by the results of this study. Throughout the activity, different aspects of the concept of area measurement appeared. The different nature of areas, the different size and shape of the areas and the constraints on the availability of measurement tools defined the contexts where the

children acted and influenced their actions. Moreover, the whole cultural environment influenced children's choices and formed the overall framework in which the children worked. In this framework, the different contexts were not independent and children showed certain tendencies that appeared throughout these contexts. We summarise below the main findings that seem to arise from this study according to our research questions.

The context of measuring real areas : In this context, children used knowledge coming both from their everyday and school practices. The children tended to use initially personal measuring tools which through the communication were replaced by culturally accepted ones. Some of the latter ones were also met in their school curriculum. For example, initially, they used equivalent familiar units such as the playground of their school or smaller units as their classroom. The need to communicate with others led them to use a commonly accepted area, the basketball ground, for comparison. The kind of familiar units that they used was based on children's interests, values and believes relevant to the concept of area. The use of standard units of measurement and the application of the area formulae to calculate rectangular areas were practices which the children developed probably from their school experience. The use of these practices did not necessarily mean understanding of the concept of area. For example, most children could not connect standard area units with standard length units or could not appreciate the size of a rectangular area with known dimensions. Similar difficulties have also been found by Baturo and Nason (1996) to student teachers. The fact that activities concerning the concept of area itself are often neglected in school while the emphasis is given on its calculation is probably an explanation for these difficulties.

The context of representing real areas on paper: The aspects of area measurement which emerged in this context were the search of a scale to make the area fit on the paper, the effect of the scaling of the dimensions to the represented area. Children's appreciation of the need to use a common scale to represent different parts of an area also appeared. Almost all the children believed that the ratio between areas is the same

as the ratio between their corresponding dimensions. The conflicts that the children faced in their attempts to implement on paper their initial strategies led them to reconsider and improve their approaches.

The different size and shape of areas : In considering large areas, children used mainly everyday practices while with small areas they recalled school methods. The shape of the areas also had an effect on the children's approaches to measurement. For example, multiplication approach was used in rectangular areas while in irregular areas triangulation or splitting into known geometrical shapes were the dominant approaches.

The availability of tools : While working on the real area context the children used measurement tools either from their school experience or from their personal environment. For example, these tools could be areas from their everyday life, standard tools commonly used in society or formula from their school experience. When the context was more mathematical then the tools used from their school experience dominated. The influence of the tools on the children's actions was very apparent when the children were restricted in the use of tools. For example, the tools that they developed when they could not use formula and standard measurement tools were more related to the concept of area itself. Moreover, these tools exposed children's thinking about fundamental aspects of the area concept like conservation, unit and unit iteration. In particular, the measurement of the area of irregular shapes revealed children's construction of units and co-ordination an important aspect of area measurement as supported by Reynolds and Wheatley (1996).

The communication process : The development of children's strategies was influenced by the teacher's intervention, children's interactions in the groups and the need to communicate with others. Despite the fact that our whole work with the children was not of a long time period, we observed a conceptual development in the groups. As Cobb (1989) supports through his research, the individuals in the groups have a variety of qualitatively distinct meanings from those that appear in the groups, therefore further research is needed to see what the development of the individuals

was concerning the area measurement. The communication in the groups during the project influenced children's actions. Each child in the group expressed his/her opinion on facing the problem. This was negotiated in the group either verbally or not until a final agreement was reached. We cannot claim that at that phase all the children had equivalent implicitly interpretations. During the whole communication different conflicts arose mainly from the results of their actions. These conflicts led children to reconsider their choices. In the whole class discussion, the communication process was different as the groups had to present their work and develop justifications to defend their proposal. In some cases when the children came to a conflict after the teacher's intervention they adopted the approach implicitly suggested by him. This indicates that in the negotiation of meanings, children accepted the teacher's opinion as more appropriate. Moreover, the need to communicate with others outside the classroom led to the development of more socially accepted practices.

CONCLUSIONS

This study has demonstrated the complexity of the concept of area and its measurement in the children's mind. This concept is not only a mathematical one which is taught in schools but one which carries different cultural dimensions. From this point of view, children use those elements which fit in with their personal experience and the tasks they have to face. This implies that school practices need to broaden towards a more integrated and cultural orientation of the concept of area. The results of this study suggest that by providing children the opportunity to face the area measurement through different settings they may form connections between the different aspects of the concept and construct a mature concept of the area measurement.

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Acknowledgements

We would like to thank all the teachers, children and researchers who participated in the experiment. Our many thanks also to Dr Jeff Evans and Dr Anna Tsatsaroni who read and commented on a first draft of this paper.

The context of measuring real areas

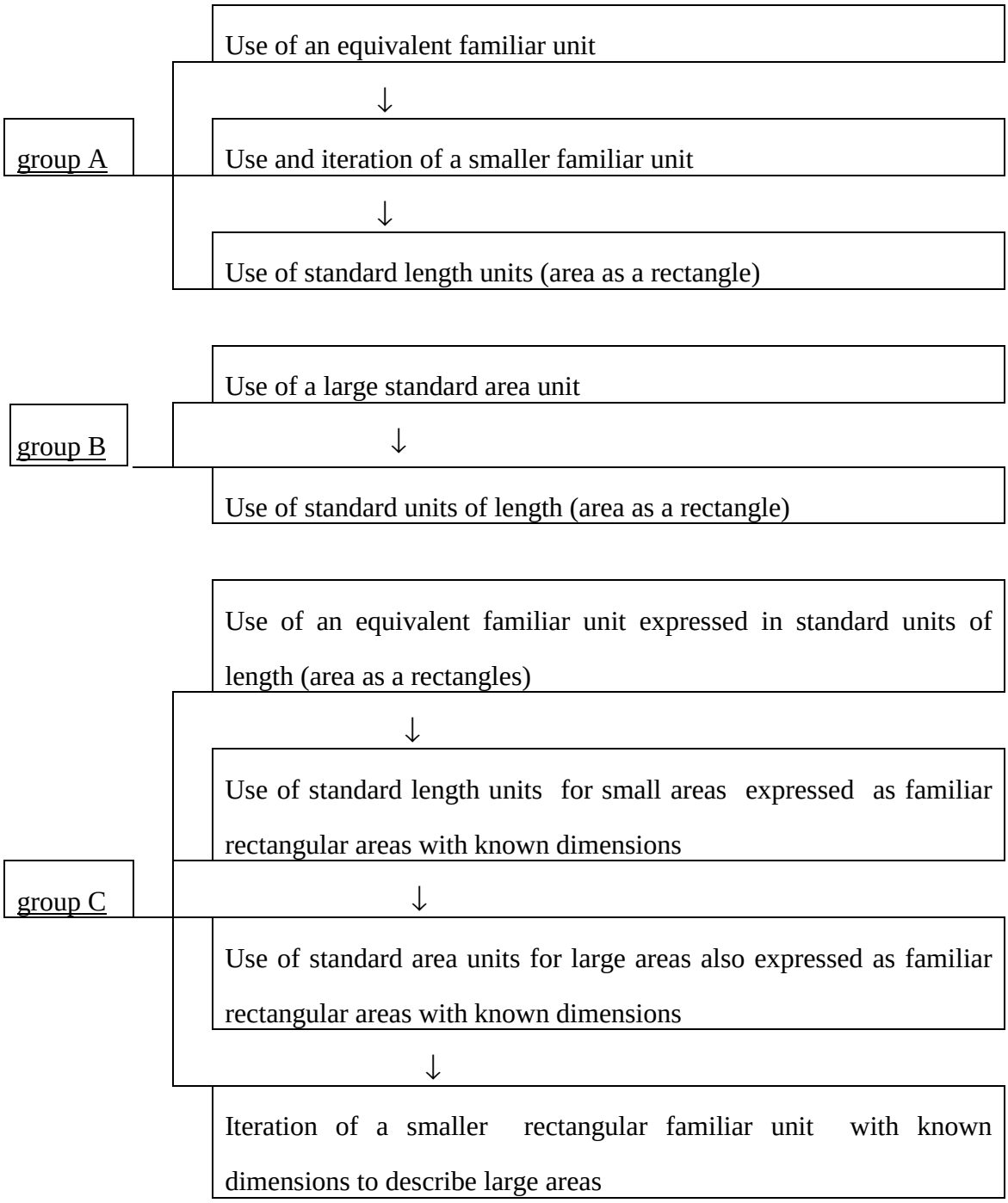

fig.1 : Children's strategies for measuring real areas

The context of representing real areas on paper

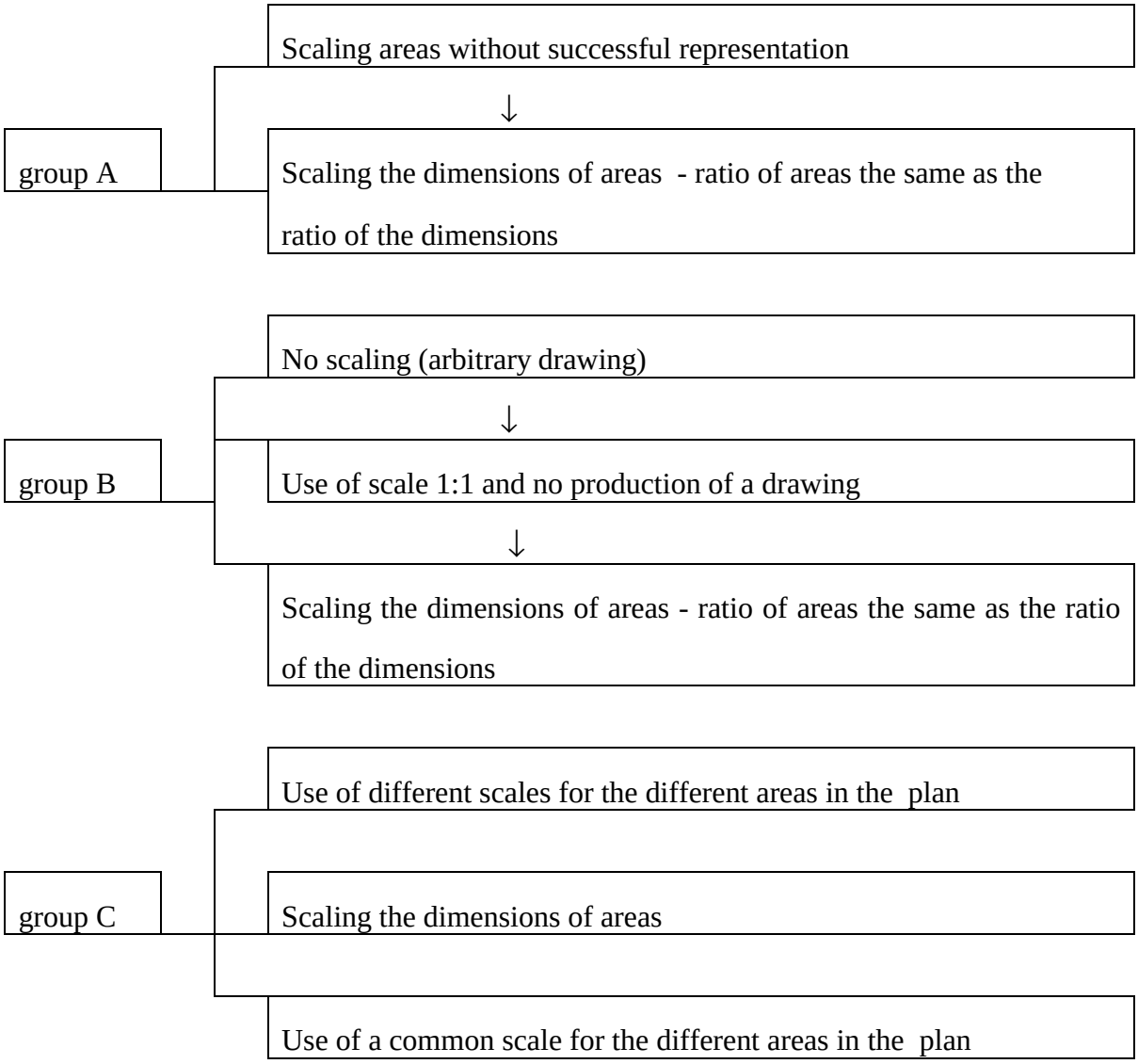

fig.2 : Children's strategies for representing real areas on paper