DIVERSE TYPES OF LEARNING ACTIVITIES DESIGNED BY MATHEMATICS TEACHERS WITHIN A VIRTUAL COMMUNITY COLLABORATIVE SPACE FOR SCIENCE EDUCATION

Maria KORDAKI
Dept of Computer Engineering and Informatics, Patras University, 26500, Rion, Patras, Greece
E-mail: kordaki@cti.gr

Abstract ***This study presents the variety of learning activities performed by mathematics teachers using the tools of the well-known educational software Cabri-Geometry II, within the context of a Virtual Community Collaborative Space for Science Education (VccSSe; European Commission Project number:128989-CP-1-2006-1-RO-COMENIUS-C21). Specifically, eight essential and innovative approaches for the design of learning activities were formed by these teachers, namely: a) Forming/verifying conjectures by focusing on the alteration of a geometrical construction using the drag-mode operation, b) Forming/verifying conjectures by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation, c) Verifying a formula by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation, d) Multiple Representation-based activities, e) Constructions simulating real-life problems, e) Black-box activities, g) a scenario-based approach emphasizing the formation of networks of learning concepts and h) Multiple-solution activities.***

Keywords: Cabri-Geometry II, e-learning, teachers, learning activities

1. INTRODUCTION

Cognition is not an event that takes place inside one's head in isolation, but can be looked at as a distributed phenomenon that is more global in nature - one that goes beyond the boundaries of a person to include environment, artifacts, social interactions, and culture (Rogers, [1]). Subsequently, it is acknowledged that individuals learn from activity and the tools supporting it to extend their cognitive capabilities (Nardi, [2]). Constructivism and social views of learning stipulate that meaning is a function of how the individual creates meaning from his or her experiences and interactions within authentic contexts (Jonassen,[3]; [4]; Noss and Hoyles, [5]). In fact, promoting authentic learning activities is a central aspect of constructivist learning that is also viewed as an active, subjective and constructive process. To this end, it is worth noting that the context or activity that frames the construction of knowledge is of equal importance to the learner as the knowledge itself (Dabbagh, [6]).

Authentic learning tasks emphasize the encouragement of learners to explore different solution strategies while simultaneously forming various hypotheses and observing their effects (Collins, [7], p. 135). Promoting the development of multiple perspectives during collaborative learning settings is a learning approach that provides opportunities for the construction of flexible and meaningful knowledge structures (Duffy & Cunningham, [8], p. 178), at the same time encouraging the expression of learners' inter- and intra-individual differences (Kordaki, [9]). Being exposed to multiple points of view, students also have the chance to develop broad views regarding the learning concepts in focus (Kordaki, [10]). With the promotion of multiple perspectives, learners also have the chance to become aware that there are multiple approaches to an issue, which is the case in real-life situations. In addition, learners have the chance to explore each perspective to seek a meaningful resolution to the issue at hand, constructing new meaning within the context of their own experiences and knowledge (Dabbagh [6]).

However, emphasizing multiple perspectives involves not only presenting information in various ways but also encouraging learners to use multiple learning tools and representation systems to construct their own multiple solution strategies for the tasks at hand and to document their own explanations. To improve their problem-solving skills, students also need to be encouraged by their teachers to share different solution strategies when solving a specific problem (Silver, Ghousseini, Gosen, Charalambous, & Strawhun, [11]; Leikin, Levav-Waynberg, Gurevich, & Mednikov, [12]). To this end, the type of activities proposed for mathematical learning that both allow for and demand the construction of diverse solution strategies by each individual student are of special interest (Silver, Leung & Cai, [13]; Kordaki and Potari, [14]; Kordaki, [10]). Multiple solution-based learning activities are acknowledged as essential for the development of mathematical reasoning, creativity and advanced mathematical thinking (Polya, [15], Schoenfeld, [16]; Leikin, Levav-Waynberg, Gurevich, & Mednikov, [12]). Hypothesis generation and exploration also go hand by hand with the acquisition of problem-solving and decision-making skills that are potentially essential in both the personal and professional life of a learner (Dabbagh, [6]).

The use of Multiple Representation Systems (MRS) is acknowledged as crucial in encouraging the expression of learners' inter/intra-individual variety regarding the subject to be learned (Dyfour-Janvier, Bednarz & Belanger, [17; Janvier, [18]). In addition, multiple and linked RS provide learners with opportunities to study how variation in one system can affect the other. In this way, each learner can make connections between different aspects of a learning subject (Lesh, Mehr & Post, [19]; Janvier, [18]. The role of translation and transformation among different RS as well as within the same RS has been also viewed as crucial (Janvier, [18]), especially in the learning of mathematical concepts.

Appropriately designed educational software can catalytically affect the changes in the whole learning context in terms of learning content, learning activities and the roles of both teachers and learners (Solloway, 20; Noss and Hoyles, [21]). In fact, in the context of Information and Communication Technologies (ICT), modern social and constructivist perspectives of teaching and learning can be realized (Papert, [22]; Balacheff and Kaput, [23], Vygotsky, [24]). From these perspectives, the need for training primary and secondary level education teachers in the use of ICT in education is of vital importance not only for their integration into the modern social and educational context created, but also for the integration of ICT into education (European Commission, [25]). The necessity of training teachers in ICT concerns the acquisition of basic technical and pedagogical skills related to the use of ICT so that they will be capable of integrating it into the teaching and learning of the subject-matter they teach (Davis and Tearle, [26]).

Dynamic Geometry Systems, and specifically the well known educational software *Cabri Geometry II* (Laborde, [27]), offer a context where constructivist mathematical learning settings can be supported. In fact, Cabri is highly capable of facilitating the design of learning activities that encourage learners to take an investigative and exploratory perspective, to express their inter/intra-individual learning variety and make self-corrections, as well as formulate and verify conjectures (Straesser, [28]; Kordaki & Balomenou, [29]). In addition, authentic, meaningful, real-life learning activities can be integrated within the context of this software. In particular, Cabri provides students with potential opportunities in terms of: i) *A rich set of tools* to perform diverse geometrical constructions according to various concepts in Euclidean Geometry, *ii) Tools to construct a variety of representations, both numerical and visual*, such as geometrical figures, tables, equations, graphs and calculations. These representations are of different cognitive transparency; consequently, students can select the most appropriate to express their knowledge, iii) *Linking representations*, by exploiting the interconnection of the different representation modes provided, iv) *Dynamic, direct manipulation* of geometrical constructions, by using the 'drag mode' operation, enhancing their knowledge about the issue at hand by dynamically exploring the invariance of their constructions, e) *The possibility of collecting large amounts of numerical data*. These data can be used by the students to form and verify conjectures regarding the geometrical concepts in focus, f) *Interactivity and multiple types of feedback* (intrinsic visual and extrinsic numerical), providing learners with opportunities to form and verify conjectures as well as self-correct their constructions, g) *Presenting information to the students in various forms, h) capturing the history of student actions* to provide teachers and researchers with a valuable amount of data for further studies, and i*) Extension*. Certain operations could be added as buttons on the Cabri interface following the formation of specific macros.

2. THE CONTEXT OF THE STUDY

This study focuses on the diversity of the learning activities designed by the mathematics teachers (28 teachers) who participated in a virtual community aiming at: a) the learning of basic pedagogical features provided by Dynamic Geometry Systems, b) the familiarization of basic features of the well-known educational software Cabri-Geometry II, c) the design of appropriate interactive learning constructions using Cabri-Geometry II, d) the design of lesson plans using ICT, e) the design of specific lesson plans using Cabri-Geometry II and the implementation of these lesson plans in real classrooms. To successfully grasp these aims, various training materials and virtual laboratories were prepared and offered –online - during a blended learning course in order for these teachers to interact. The teachers' lesson plans were analyzed and the learning activities they had designed and given to their students were classified into categories which are presented in the following section of this paper. In terms of methodology, this is a qualitative work (Cohen and Manion, [30]) and can be characterized as a case study.

3. RESULTS

Teachers were satisfied with the tools provided by Cabri-Geometry II and designed a diversity of learning activities. Analysis of the teachers' lesson plans reveals that eight categories of learning activities were formed. These categories are described below:

a) *Forming/verifying conjectures by focusing on the alteration of a geometrical construction using the drag-mode operation.* For example, some teachers formed a learning activity representing an interactive dynamic construction consisting of a triangle and its heights. By dynamically transforming this construction, students can conjecture that 'when this triangle is oblique, its heights are intersected internally at a single point, whereas, when a triangle has an obtuse angle, its heights are intersected externally and, when a triangle is right-angled, then its heights are intersected at the right angle'.

b) *Forming/verifying conjectures by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation:* A case in point is when a teacher formed an interactive construction consisting of a triangle ABC illuminating also the sizes of its angles A, B, C, then automatically calculating their sum A+B+C, while at the same time dragging the triangle from its vertexes and tabulating the data automatically produced by the aforementioned measurement operations. By focusing on these numerical data, students can conjecture that 'the sum of the size of the angles of a triangle is always 180 degrees'.

c) *Verifying a formula by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation.* For instance, students can verify the truth of the formula expressing the mathematical congruence α^2+b^2+2*a*b= (a+b)^2 using geometrical embodiments such as: three squares of sides (a+b), a and b correspondingly and a rectangle of sides a and b. By measuring the areas of the previously mentioned figures and automatically calculating the results of α^2+b^2+2*a*b and (a+b)^2 while at the same time altering a and b -by dragging -and automatically tabulating these results, students can verify the said mathematical congruence.

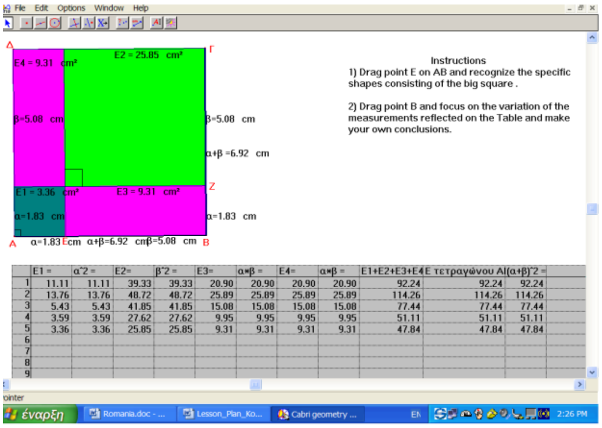

Figure 1. Verifying mathematical conjectures within DGS

d) *Black-box activities.* Here, for example a teacher used a specifically-designed pentagon A1A2A3A4A5A1 where A1A2=A4A5=a and A2A3=A3A4=b and asked his students to transform it by dragging, so as to investigate this construction and discover relationships among its sides and its angles as well as to investigate cases where the perpendicular line on the mid point of A1A5 passes through point A3 (Fig. 2a).

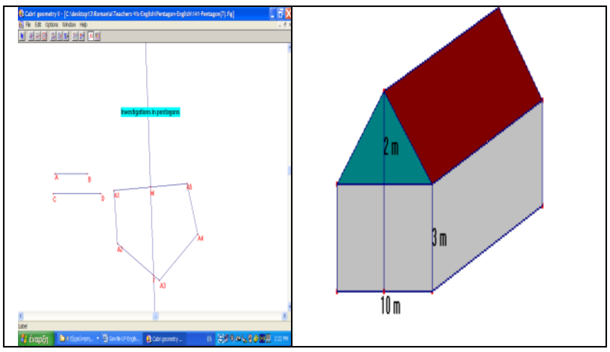

Figure2a. Black-box	Figure 2b. Real-life activity

e) *Constructions simulating real life problems*. Here as well, a teacher tried to help his students to learn about the area formula in trapeziums by managing to calculate the cost of painting a house, some of the edges of which are in the form of a trapezium (see Figure 2b).

f) *Multiple Representation-based activities*: Teachers also seemed to understand the significance of providing their students with opportunities to study a concept in Multiple Representation Systems and especially how the variation of the representation of a concept in one system can effect its representations in other systems. For example, a teacher formed an interactive construction for the study of 'Power of a point with respect of a circle'. Specifically, this construction was formed in the following way: a) construction of a circle (O, R), a point P outside this circle and the segment KB where point B is on the said circle, b) construction of point A that is the intersection point of KB with this circle, c) measurement of the segments KA, KB, KO and R, d) calculation of KA*KB and KO^2-R^2, e) representation of KA and KB on the axes of coordinates X'X and Y'Y correspondingly, f) construction of the rectangle (KADBK) with sides KA and KB using these axes, g) measurement of the area of KADBK, h) tracing point D and i) tabulation of KA, KA, KB, KO, R and KA*KB and KO^2-R^2, KADBK and i) dragging point B on the circle (O, R) at the same time tracing point D and automatically tabulating the previously mentioned data (See Figure 3).

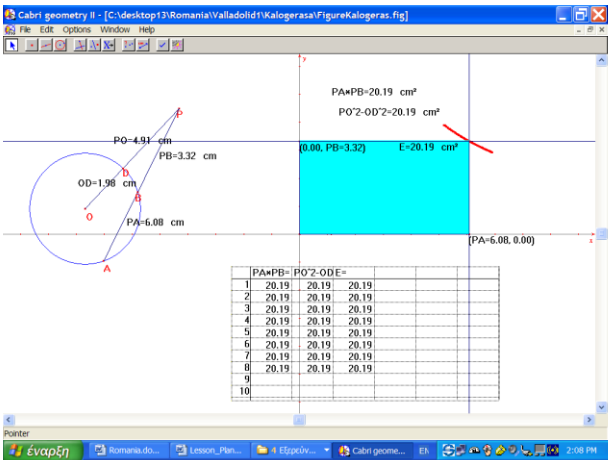

Figure 3. MRS-based learning activities

g) *Emphasizing the formation of inter-relationships within a network of concepts*. Here, as well, some teachers tried to design learning activities emphasizing student learning of a main concept/topic and how this relates to many other concept/topics. To design this kind of activity, teachers formed a network of related concepts/topics and designed a number of lesson plans to help their students learn all of these. For example, one teacher selected as a topic the "Thales theorem" and designed lesson plans for the implementation of this theorem in triangles, trapezia, ratio of segments, analogies and their properties, equal triangles, ratio of equality, similar shapes, the segment that connects two midpoints of two sides of a triangle, and properties of the median of a trapezium as well as in real-life activities.

h) *Multiple-solution activities:* As Cabri provides a variety of tools and operations; these can be effectively combined to support the performance of multiple-solution activities. As a paradigm, a teacher asked her students to perform the following activity: 'construct, in an unused part of the school yard, rectangular in shape 10 meters in length and 4 meters in width, as many triangular flower beds as possible, each of them using one of the rectangle's sides as the base and a random point of the opposite side of the rectangle as the vertex. After that, students should choose which one of these triangular flower beds would be most profitable to construct, so that when fenced in with wire netting, they would have the minimum cost'.

4. DISCUSSION

The analysis of the data presented in this paper shows that mathematics teachers can effectively design innovative learning activities using ICT and use them in their classrooms. In fact, eight categories of learning activities were developed by the mathematics teachers who participated in the VccSSe virtual environment, encouraging them to learn design and implement lesson plans in their classrooms using ICT. Each type of the reported categories has its own significance for student learning and this is highlighted in the following section.

a) Forming/verifying conjectures by focusing on the alteration of a geometrical construction using the drag-mode operation. By performing activities falling into this category, students have the opportunity to experiment with the interactive constructions provided by teachers using the tools of Cabri-Geometry II. In this way, students can focus on the alteration of a geometrical construction which conserves its basic properties and can form/verify hypotheses and conjectures. The hypotheses formed in this way are very stable as they are based on a plethora of visual data. To this end, it is worth noting that hypotheses formation is a basic skill of critical thinking.

b) Forming/verifying conjectures by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation. By performing this kind of activity, students can also experiment with dynamic geometrical constructions, focusing on the plethora of numerical data produced during the alteration of these constructions using the 'drag mode' operation. In this way, students can form/verify hypotheses and conjectures which are also very stable, as they are based on an abundance of numerical data.

c) Verifying a formula by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation. Cabri-Geometry II provides possibilities for the formation of dynamic formulae. The variables within these formulae are interlinked with the measurements of the entities they represent which are essential elements of a geometrical construction. By dynamically altering a specific geometrical construction, students have the chance to verify the truth of a specific mathematical congruence. This kind of verification, being based on a large amount of numerical data, is sufficiently solid .

d) Black-box activities. By experimenting with these activities, students have the chance to explore geometrical constructions with some of their properties hidden, which they then have to discover.

e) Constructions simulating real-life problems. In fact, real-life problems can help students to develop strong motivation in their learning and approach mathematics as a human activity as well as to put mathematical concepts into an interdisciplinary context (Bishop, [31]).

f) Multiple Representation-based activities. By performing activities in this category, students have the chance to understand how the variation of a concept in one representation system can affect its variation in other representation systems. Students also can select the representation system appropriate to their knowledge to express themselves. Finally, students can acquire a broad view of the concept in focus by studying it in multiple representation systems.

g) *Emphasizing the formation of inter-relationships within a network of concepts.* By studying a concept within a network of related concepts, students have the opportunity to make connections among these concepts. This is very useful in problem solving and in professional life where solutions to problems require interconnections among different kinds of knowledge as well as the use of various concepts and skills.

h) Multiple-solution activities. By trying to solve problems 'in as many ways as possible' students have the chance to form alternative solutions to the tasks at hand, to select those that are optimum, to make connections between different kinds of knowledge they possess as well as to develop their creative and critical thinking skills. When students, are provided with opportunities to face tasks which could be solved in various ways, they can express their inter/intra individual learning variety. In addition, students can form broad views of the concepts in focus.

On the whole, all these type of learning activities described above put the learner at the centre of the learning process and provide them with opportunities to experiment and to actively construct their knowledge.

5. CONCLUSIONS

This paper presented the diversity of learning activities designed by the mathematics teachers who participated in a Virtual Community Collaborative Space for Science Education (VccSSe). At first glance, the results emerging from this study show that teachers designed a variety of types of learning activities, namely; a) Forming/verifying conjectures by focusing on the alteration of a geometrical construction using the drag-mode operation, b) Forming/verifying conjectures by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation, c) Verifying a formula by focusing on the numerical data automatically collected during the alteration of a geometrical construction using the drag-mode operation, d) Multiple Representation-based activities, e) Constructions simulating real-life problems, e) Black-box activities, g) a scenario-based approach emphasizing the formation of networks of learning concepts and h) Multiple-solution activities. Most importantly, these teachers used these activities in their actual classroom practices. By performing these learning activities, students can experiment, express their inter/intra-individual learning variety, make interconnections with various concepts, develop multiple perspectives regarding the learning concepts in focus and can also be motivated to be actively involved in their learning process. On the whole, the results of this study suggest that teachers are able to design pedagogically sound learning activities using ICT and especially Cabri-Geometry II and also use this activities in their teaching practices. More effort is needed to ensure support in such communities for teacher professional development and lifelong learning as well as for the use of ICT in everyday classroom practices.

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