MULTIPLE SOLUTION TASKS WITHIN DYNAMIC GEOMETRY SYSTEMS

A. BALOMENOU[1] and **M. KORDAKI**[*,2]

[1] Secondary Education, Patras, Greece
[2] Department of Computer Engineering and Informatics, Patras University, 26500, Rion Patras, Greece

Abstract:
This paper presents a learning experiment using the tools provided by the well-known educational software Cabri-Geometry II [1] within the context of a Multiple Solution-based Task (MST) regarding the mathematical notion of area. Specifically, this paper presents the diversity of students' approaches to the concepts of area and perimeter in triangles, with special reference to the conservation of area in triangles enrolled in a rectangle. 25 students (12-15 years-old) participated in a learning experiment where they were asked to face 'in as many ways as possible' a specifically-designed learning activity from daily life, using the tools of the previously-mentioned educational software. The analysis of the data shows that students successfully used the tools of Cabri-Geometry II in order to study the relations between: a) the area of both a rectangle and the equivalent triangles enrolled within this rectangle, b) the area of each of these triangles and c) the area in relation to the perimeter in a class of equivalent triangles enrolled within this rectangle. Students also developed various strategies to the said task, exploiting the diversity of tools provided by Cabri, and helped to move from the understanding of conservation of area in a limited number of triangles to a class of triangles with common bases and equal heights.

Keywords: Multiple Solution Tasks; geometry; Dynamic Geometry Systems; secondary education; experimental study

1. Introduction

The role of learning activities has been acknowledged as crucial in the whole learning process in terms of motivating/de-motivating students to be actively involved in their learning as well as in shaping the kind of knowledge they construct [2-3]. To this end, the role of Multiple Solution-based Tasks in the development of multiple perspectives regarding the concepts in focus is essential. In the context of MST, learners can develop their creativity as well as their problem-solving skills, their reasoning capabilities and also their Advance Mathematical Thinking [4-8]. To this end, the type of activities proposed for

* Corresponding author: e-mail: kordaki@cti.gr

mathematical learning that both allow for and demand the construction of diverse solution strategies by each individual student are also of special interest [9-11]. However, when students are asked to perform MST within e-learning contexts providing Multiple Tools they are presented with great opportunities for both constructing various solution strategies to the task at hand as well as relating the concepts in focus with diverse concepts provided as tools by the e-learning context within which they interact [11].

Dynamic Geometry Systems and specifically the well-known educational software Cabri-Geometry II is an e-learning context that provides *diverse tools* to support a *plethora of concepts of Euclidian geometry*. In addition, Cabri provides tools to construct *multiple representations*, both numerical and visual, such as geometrical figures, tables, equations, graphs and calculations which can also be *interlinked*. To this end, there is also the possibility of collecting *large amounts of numerical data* which can be used by the students to form and verify conjectures regarding the geometrical concepts in focus. Furthermore, the geometrical *constructions within Cabri are dynamic* in that they can be directly manipulated by using the '*drag mode'* operation, thus conserving their basic properties while altering their form. Moreover, Cabri is a *highly interactive* environment that also provides learners with *multiple types of both information* (text, figures, numbers, equations) *and feedback* (intrinsic visual and extrinsic numerical) to facilitate the formation and verification of conjectures as well as self-correction of their constructions. Finally, the *history of student actions* is captured and possibilities for the *extension* of the environment are also available to teachers and learners through the addition of specific macros on the Cabri-interface.

In this study, student approaches to the geometrical notion of area, a significant mathematical concept at all levels of education, in mathematics, in society and in everyday life [12], are investigated through the performance of a MST within the context of Cabri-Geometry II. Despite its significance, students face difficulties in understanding this concept. Specifically, students encounter difficulties in understanding conservation of area in non-congruent shapes and also in discriminating between area and perimeter [12-13]. In the following section, the context and results of this study are presented, followed by discussion of these results and consequent conclusions drawn.

2. The context of the study

The experiment presented in this study took place in a typical, provincial, state secondary school in Patras, Greece. Three complete classes of students participated in this experiment, totalling 25 students: eight 1st grade students (13-years-old), nine 2nd grade students (14-years-old) and eight 3rd grade students (15-years-old). These students were separated into 9 groups in order to perform a MST. They were asked to encounter 'in as many ways as possible' a specifically-designed learning activity from daily life, using the tools of the educational software Cabri-Geometry II. More specifically, they were asked to 'construct, in an unused part of the school yard, rectangular in shape, 10 meters in length and 4 meters in width, as many triangular flower beds as possible, each of them using one of the rectangle's sides as the base and a random point of the opposite side of the rectangle as the vertex. After that, students were to choose which one of these triangular flower beds would be most profitable to construct, so that when fenced in with wire netting, they would have the minimum cost'. It is worth noting that such a task has not yet been reported. This task was chosen in order to give students the chance to develop their own approaches to the concept of area and the conservation of area in triangles enrolled in a rectangle as well as to the discrimination between area and perimeter in equivalent triangles. By being asked to create as many possible solutions as they can, students have the chance to express different perceptions of the concepts they study and to develop a broader view of these concepts [10]. In terms of methodology, this is a qualitative study [14].

3. Results

Group solution strategies to the said task were classified into three categories (Ci, i=1, 2, 3). These categories are presented in Table 1. In this Table, the specific groups and the number of students who performed strategies falling into each category are also presented. By using the symbols A, B, G we mean students of 1st , 2nd and 3rd grades correspondingly, while the symbols Ai, Bi, Gi (i=1, 2, 3) which are used in column 2 are assigned to each specific group of students. The numbers used before capital letters A, B, G represent the number of students from each specific grade that performed the corresponding strategy presented in Column 1 of this Table.

Table 1. Categories of group strategies for the construction of equivalent triangles in the context of Cabri-Geometry II

Categories of group strategies within the context of Cabri-Geometry II	Number of groups	Number of students
Categories performed by		
C1: choice of significant points of the rectangle as vertexes of the equivalent triangles constructed	A1, A2, A3, B1, B2, G1, G2, G3	8A, 6B, 8G
C2: choice of arbitrary, random points on the sides of the rectangle as vertexes of the triangles	B1, G1, G2	3B, 5G
C3: using the 'drag' mode in combination with automatic area measurement and length	B3, G2	3B, 3G

Strategy C1

Students who performed this strategy chose some significant points of the rectangle (middle points of its sides as well as its vertexes) as vertexes of the triangles they constructed. An example of a student strategy that falls into this category is presented in Figure 1. By using this kind of strategy, students realized that it is possible to construct triangles of equal areas but of different form. It is worth noting that, before this experiment, these students did not realize this possibility. By automatically measuring the area and the perimeter of each triangle, these students also clarified that area and perimeter are different entities.

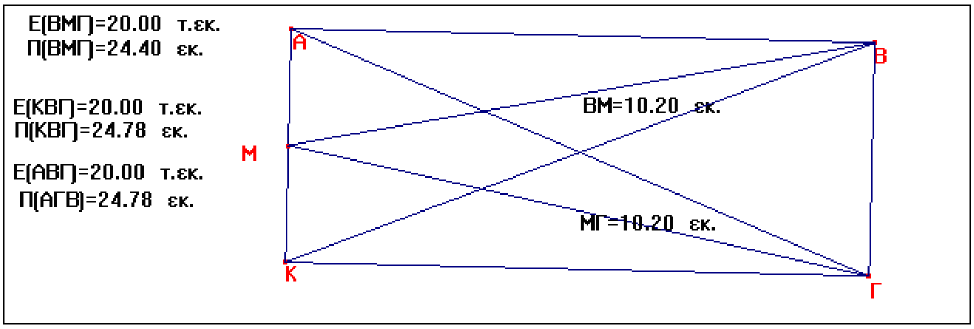


Figure 1. Example of student strategies falling into category C1

Strategy C2

Here as well, students chose some arbitrary or random points on the sides of the rectangle as vertexes of the triangles they constructed. An example of a student strategy falling into this category is demonstrated in Figure 2.

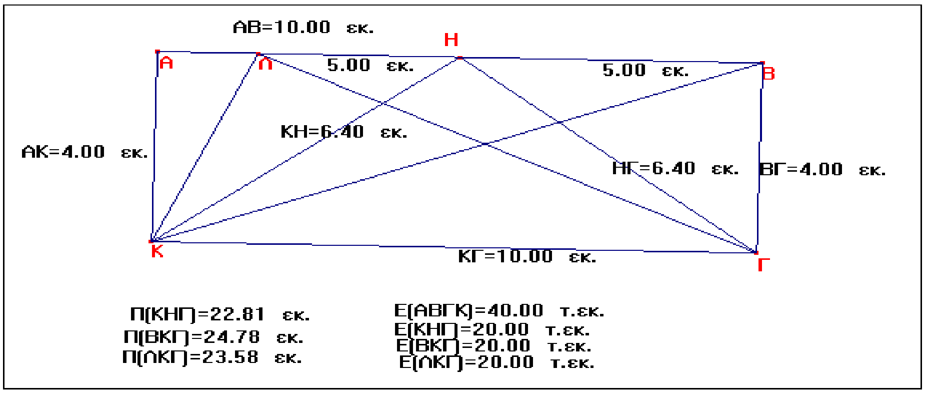
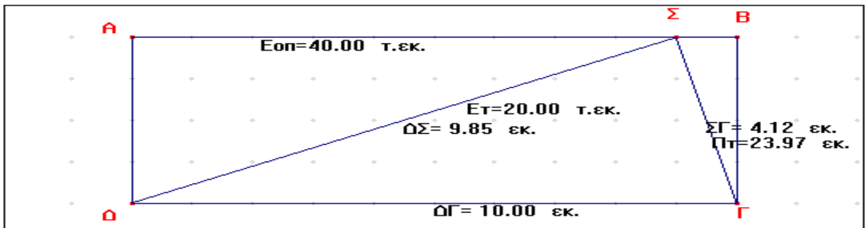


Figure 2. Example of student strategies falling into category C2

Students who performed this strategy moved on from the construction of equivalent typical isosceles and right-angled triangles (as was the case in Strategy 1) to the construction of equivalent scalene triangles. Students also studied the relationship between the area of these triangles and the area of the rectangle. In addition, these students seemed to realize that the isosceles triangle that has a vertex at the mid point of a side of this rectangle (point H) and base on its opposite side is the triangle of the minimum perimeter.

Strategy C3

Here, students used the 'drag mode' in combination with automatic area measurement and length. An example of a student strategy falling into this category is shown in Figure 3.



	ΔΣ=	ΣΓ=	ΔΓ=	Πτ=	Ετ=	Εοπ=
1	6.40	6.40	10.00	22.81	20.00	40.00
2	5.00	8.06	10.00	23.06	20.00	40.00
3	4.47	8.94	10.00	23.42	20.00	40.00
4	4.47	8.94	10.00	23.42	20.00	40.00
5	4.47	8.94	10.00	23.42	20.00	40.00
6	4.47	8.94	10.00	23.42	20.00	40.00
7	4.47	8.94	10.00	23.42	20.00	40.00
8	4.47	8.94	10.00	23.42	20.00	40.00
9	4.47	8.94	10.00	23.42	20.00	40.00
10	4.47	8.94	10.00	23.42	20.00	40.00
11	4.47	8.94	10.00	23.42	20.00	40.00
12	4.12	9.85	10.00	23.97	20.00	40.00
13	4.12	9.85	10.00	23.97	20.00	40.00
14	4.12	9.85	10.00	23.97	20.00	40.00
15	4.12	9.85	10.00	23.97	20.00	40.00
16	4.12	9.85	10.00	23.97	20.00	40.00
17	4.12	9.85	10.00	23.97	20.00	40.00
18	4.12	9.85	10.00	23.97	20.00	40.00

Figure 3. Example of student strategies falling into category C3

By reflecting on the numerical data collected in the Table presented in Figure 3, students who performed this strategy were helped to move from the construction of a limited number of equivalent scalene triangles to the construction of a class of equivalent triangles and they formed appropriate hypotheses. Students also studied the relationship between the area of these triangles and that of the rectangle and made appropriate generalizations. They also experimented with the area and the perimeter of these triangles and found the triangle with the minimum perimeter as well as forming specific generalizations.

Strategies across categories

Each group's solution strategies across categories and the path of their solution strategies are presented in Table 2. As is shown in this Table, the total number of strategies performed by the groups across each category were: C1: 15 strategies, C2: 3 strategies, C3: 3 strategies.

Table 2. Group strategies across categories

Groups' strategies across categories									
Group	**C1**	**Group**	**C1**	**C2**	**C3**	**Group**	**C1**	**C2**	**C3**
A1	1, 2	B1	1, 2	3	-	G1	2, 3	1	-
A2	1, 2	B2	1	-	-	G2	1, 2	-	3
A3	1, 2	B3	-	-	1, 2	G3	2, 3	1	-

Table 3 presents the strategies across categories and the path of solution strategies across categories as well as the total number of strategies performed by each individual student who participated in this MST. Furthermore, Table 3 contains the number of strategies across categories (C1:41 str, C2: 8 str, C3: 9 str), a total of 58 strategies performed by all students.

Table 3. Strategies across categories per student

Strategies across categories per student													
Grade A (Stud)	Cat. Strat		Grade B (Stud)	Categories of strategies				Grade G (Stud)	Categories of strategies			
	C1	Total		C1	C2	C3	Total		C1	C2	C3	Total
M1	2	2	M1	2	1	-	3	M1	2	1		3
M2	2	2	M2	2	1	-	3	M2	2	1		3
M3	2	2	M3	2	1	-	3	M3	2	1		3
M4	2	2	M4	1	-	-	1	M4	2		1	3
M5	2	2	M5	1	-	-	1	M5	2		1	3
M6	2	2	M6	1	-	-	1	M6	2		1	3
M7	2	2	M7	-	-	2	2	M7	2	1		3
M8	2	2	M8	-	-	2	2	M8	2	1		3
			M9	-	-	2	2					
Total	16	16	Total	9	3	6	18	Total	16	5	3	24

As we can see in Table 3, the most common student strategy is C1 (41 strategies out of 58 in total). This can be interpreted as normal, because when students are new to a concept and try to explore it, they use shapes that are familiar to them (in this case, isosceles and right-angled triangles) whose properties they know well and can exploit to reach conclusions for the concepts they are dealing with. From the path of student solution strategies, we can see their mathematical evolution in terms of the concept of area and the conservation of area as well as in discrimination between area and perimeter of equivalent triangles.

On the whole, it is worth noting that: a) all students realized that triangular areas could be conserved while altering their forms. To this end, all students made appropriate generalizations, b) all students realized that the area of the enrolled triangles within a rectangle - as described in the previous section - is half of the area of the said rectangle, c) 23 out of 25 students realized that area and perimeter are different entities, d) 1 out of 4 students formed classes of equivalent triangles with common bases and equal heights. To this end, the role of MST in asking for 'as many solutions as possible' and the role of the 'drag mode' operation became crucial, e) more that half of the students realized conservation of area in scalene triangles. Here as well, the role of MST was essential because students did not remain at their first attempt using right-angled and isosceles triangles but tried to form other solution strategies using arbitrary points of the sides of the given rectangle as their vertexes, f) all students were actively and passionately involved in the task at hand.

4. Discussion and Conclusions

This study presented student approaches to a Multiple Solution-based Task asking each of them to form 'as many solutions as possible' - regarding the mathematical notion of area - and exploiting the variety of tools provided by the educational software Cabri Geometry II. The Dynamic character of the said software in combination with the demand of the MST to be solved 'in as many ways as possible' encouraged students to realize more than one correct solution strategy and to explore conservation of area in various triangles of different form. With the completion of the given task, all students had recognized that triangles of the same area are not always equal and that triangles with a common base and equal corresponding heights are equivalent. To this end, the MST and the diverse tools provided by Cabri had an essential role to play. Furthermore, while performing their solution strategies, all students managed to distinguish and discriminate between the concepts of area and perimeter, as well as find out that equivalent triangles do not necessarily have the same perimeter. Most of the students (23 out of 25) also managed to arrive at a conclusion about which of all the equivalent triangles has the minimum perimeter. These students concluded that, among equivalent triangles enrolled in a rectangle, the isosceles have the minimum perimeter and, among all the equivalent isosceles triangles, the one with the bigger base has the minimum perimeter. Student conclusions regarding area in relation to the perimeter of equivalent triangles were reached through observation during experimentation with the dynamic performance of several solution strategies.

Acknowledgements This work was funded through Project 128989-CP-1-2006-1-RO-Comenius-C21 from European Commission, Education and Training, School Education: Socrates: Comenius. We thank all the partners and teachers for their cooperation and work.

References

[1] J-M. Laborde, Cabri-Geometry [Software]. France: Universite de Grenoble (1990).
[2] B. A. Nardi, Studying context: A comparison of activity theory, situated action models, and distributed cognition. In B. A. Nardi (Ed.), Context and consciousness: Activity theory and human-computer interaction. Cambridge: MIT Press (1996).
[3] H.D. Jonassen, Revisiting Activity Theory as a Framework for Designing Student-Centered Learning Environments. In D.H. Jonassen & S.M. Land (Eds), Theoretical Foundations of Learning Environments, London: Lawrence Erlbaum Associates, 89-121 (2000).
[4] A. Schoenfeld, Mathematical Problem Solving. New York: Academic Press (1985).
[5] G. Ervynck, Mathematical creativity. In Tall, D. (Ed), Advanced Mathematical Thinking (pp. 42-53). The Netherlands: Kluwer, Academic Publishers (1991).
[6] E. A. Silver, H. Ghousseini, D. Gosen, C. Charalambous, & B. Strawhun, Moving from Rhetoric to Praxis: Issues Faced by Teachers in Having Students Consider Multiple Solutions for Problems in the Mathematics Classroom. Journal of Mathematial Behavior, 24, 287-301 (2005).
[7] R. Leikin, A. Levav-Waynberg, I. Gurevich, & L. Mednikov, Implementation of Multiple Solution Connecting Tasks: Do Students' Attitudes Support Teachers' Reluctance? Focus on learning problems in mathematics, **28**(1), pp. 1-22 (2006).
[8] R. Leikin, & M. Lev, Multiple solution tasks as a magnifying glass for observation of mathematical creativity. In J.H. Woo, H.C. Lew, K.S. Park, & D.Y. Seo (Eds.), Proceedings of the 3th Conference of the International Group for the Psychology of Mathematics Education Seoul: PME (2007), Vol. **3**, pp. 161-168.
[9] E. A. Silver, S.S. Leung, & J. Cai, Generating multiple solutions for a problem: a comparison of the responses of U.S. and Japanese students. Educational Studies in Mathematics, 28, pp. 35-54 (1995).
[10] M. Kordaki, The effect of tools of a computer microworld on students' strategies regarding the concept of conservation of area. Educational Studies in Mathematics, 52, 177-209 (2003).
[11] M. Kordaki, 'MULTIPLES': a challenging learning framework for the generation of multiple perspectives within e-collaboration settings. In T. Daradoumis, S. Caballe, J.-M. Marques and F. Xhafa (Eds) 'Intelligent Collaborative e-Learning Systems and Applications', Springer-Verlag (2009,in press).
[12] J. Piaget, B. Inhelder, and A. Sheminska, The child's conception of geometry. N.Y: Norton & Company(1981).
[13] M. Kordaki, & D. Potari, The effect of tools of area measurement on students strategies : The case of a computer microworld. International Journal of Computers for Mathematical Learning, **7**(1), pp. 65-100, (2002).
[14] L. Cohen, & L. Manion, Research Methods in Education. London: Routledge, (1989).