EMPHASIZING MULTIPLE REPRESENTATION SYSTEMS FOR THE DESIGN OF LEARNING ACTIVITIES WITHIN DYNAMIC GEOMETRY SYSTEMS

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Abstract:
This paper presents a teaching experiment emphasizing the role of Multiple Representation Systems (MRS) within the context of Dynamic Geometry Systems such as the well-known educational software Cabri-Geometry II [1]. Eighteen 11th grade students (17 years old) participated in a teaching experiment aiming the learning of the notion of the power of a point with respect to a circle. Within the context of this experiment, a learning activity was proposed in which students would interact and which emphasized the use of interlinked MRS in terms of provision of: a) interactive, dynamic, visual geometrical constructions regarding the concepts in focus; namely, a circle (O, R) and a secant of this circle from point P intersecting this circle on points A and B , b) numerical representations of key aspects that constitute these concepts, c) visual area-representations of the multiplication of the segments that a secant of a circle intersects, d) a visual graphical representation of the curve drawn by the point where coordinates are the length of the segments PA, PB in a Cartesian system of coordinates when point B is dynamically dragged on the circle (O, R). The analysis of the data shows that all students were impressed by the dynamic and interlinked MRS provided and were actively and passionately involved in the activities set. All students also exploited the dynamic character of the MRS provided and expressed/verified conjectures regarding the concepts in focus. On the whole, different RS provided students with complementary ways to conceptualize the concepts in focus and also supplied them with opportunities to extend alternative meanings to these concepts.

Keywords: Dynamic Geometry Systems; secants of a circle; learning activities; secondary education; teachers

1. Introduction

Student inequality in learning and achievement at school has been linked to their inter-individual learning differences [2-3]. In fact, most learner difficulties are found in the gap

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between their intuitive knowledge and the knowledge they need to express in the proposed RS for use [4]. For example, prepositional, symbolic and abstract RS prevent some learners (usually beginners) from expressing their knowledge, the same systems being intended for use by advanced learners. Contrariwise, metaphors of everyday life and visual RS are more suitable for beginners.
To this end, the role of MRS is acknowledged as crucial in encouraging the expression of learners' individual variety as well as in enabling each learner to acquire a broad view of the concepts in focus [5-8]. Furthermore, MRS can provide students with opportunities to make connections between the different kinds of knowledge they possess, such as intuitive, visual and symbolic knowledge. In this way, each learner can make connections between different aspects of a learning subject [8-9], [4]. In addition, multiple and linked RS provide learners with opportunities to study how variation in one system can affect another and realize the basic aspects and structure that constitute these concepts. The role of translation and transformation among different RS as well as within the same RS has been viewed as crucial [10], [4], especially in the learning of mathematical concepts. Students can also actively construct their own knowledge by experimenting with cognitive transparent RS while they can explore the knowledge of others by interacting with opaque RS. The understanding of a concept can be also viewed as a process of giving meaning to its different representations as well as making connections between these different meanings. On the whole, the role of MRS in students gaining knowledge is acknowledged as significant by many researchers [5], [11-14], [7].

Computers are an ideal medium, providing wide opportunities for the construction of a variety of different, linked and dynamic RS such as; texts, images, equations, variables, tables, graphs, animations, simulations of a variety of situations, programming languages and computational objects [15]. To this end, computer learning environments providing opportunities for dynamic interlinked MRS have been acknowledged as essential for the learning of concepts from various domains of knowledge and especially for concepts of Euclidean geometry [16-18]. Well-known examples of such environments are: the Logo programming language environment, Dynamic Geometry Systems (DGS) such as Cabri-Geometry II [1], the Geometers' Sketchpad [19] etc. At this point, it is worth noting that DGS also provide possibilities for direct manipulation of the geometrical constructions represented on the screen of the computer, using the 'drag mode' operation. This means that the form of these constructions can be dynamically altered while conserving their properties.

Taking into account all the above, a learning activity based on the formation of MRS-based interactive constructions for the learning of the notion of the power of a point with respect to a circle has been constructed by exploiting the tools and the operations provided by the well-known educational software Cabri-Geometry II. It is worth noting here that students have difficulties in conceptualizing this notion and usually memorize it as a meaningless formula. The design of this learning activity is presented in the following section, and subsequently the results emerging from its implementation using real students are reported. Finally, there is a discussion of these results and conclusions are drawn.

2. The notion of the power of a point with respect to a circle: the design of the proposed learning activity and its implementation in the classroom

2.1. Definition of the basic concept

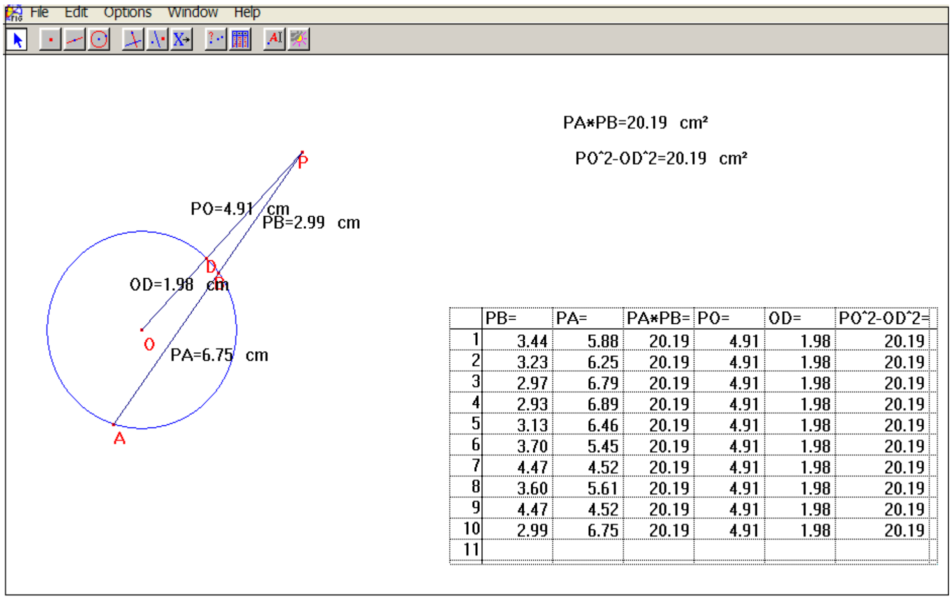
The power of a point P with respect to a circle (O, R) is the constant multiplication of two varying quantities; namely, the length of the segments PA and PB, where point B is on the circle (O, R) and A is the intersection point of the secant PB and this circle. Here. it is worth noting that the power of a point P with respect to a circle (O,R) is constant, independent of points A and B, and equal to PO^2-OD^2 where OD=R, and D is the intersection point of the segment PO to the circle (O, R). This definition reminds us of both the reversely proportional quantities and the graphical representation of a hyperbola as well as rectangular figures of equal areas with dimensions of the said quantities PA and PB. Based on all the above, and exploiting the dynamic character of DGS, especially Cabri Geometry II, specific geometrical constructions were formed to provide students with opportunities to form both algebraic and geometrical meanings to the power of a point P with respect to a circle (O, R).

The proposed activity was tested in the field using real students at the mathematics laboratory of the 3rd High School of Nafpaktos. The 18 students that took part were from the 2nd grade of the 1st Lyceum of Nafpaktos. They visited this lab in order to take apart in learning activities using the educational software Cabri-Geometry II. The said activity was carried out in two hours. Students taking part had no previous experience in using computers for the learning of mathematical concepts. Students worked in groups of two while each group worked with one computer. The researcher participated as a teacher who encouraged the students not to give up but to focus on the tasks at hand and on the questions posed as well as to reflect on the results of their experimentation within Cabri and the feedback provided. Students were also provided with a lesson sheet where they had to answer specific questions. The aim of these questions was to provide valuable data about the impact of the proposed activity on student learning.

In the following section, the proposed activity is presented, followed by a report illustrating its effect on student learning. This presentation is organized in such a way as to provide evidence for both the algebraic and the geometrical meanings given by the students to the notion of the power of a point P with respect to a circle (O, R).

2.2. Algebraic meanings of the power of a point P with respect to a circle (O, R).

Here, it was considered essential to provide students with chances to: a) realize that the power of a point P with respect to a circle (O, R) is the constant multiplication of the two varying quantities PA, PB and also that it is constant and equal to PO^2-OD^2 (see the interactive construction illustrated in Figure 1), b) understand that a mathematical formula can describe a constant value or relationship and c) connect the power of a point with reversely proportional quantities (also see the interactive construction illustrated in Figure 1).



	PB=	PA=	PA*PB=	PO=	OD=	PO^2-OD^2=
1	3.44	5.88	20.19	4.91	1.98	20.19
2	3.23	6.25	20.19	4.91	1.98	20.19
3	2.97	6.79	20.19	4.91	1.98	20.19
4	2.93	6.89	20.19	4.91	1.98	20.19
5	3.13	6.46	20.19	4.91	1.98	20.19
6	3.70	5.45	20.19	4.91	1.98	20.19
7	4.47	4.52	20.19	4.91	1.98	20.19
8	3.60	5.61	20.19	4.91	1.98	20.19
9	4.47	4.52	20.19	4.91	1.98	20.19
10	2.99	6.75	20.19	4.91	1.98	20.19
11						

Figure 1. *Experimenting with algebraic meanings of the secants of a circle*

Here, students had the chance to experiment with the interactive geometrical construction presented in Figure 1 and automatically tabulate the numerical data illustrating the measures of PA, PB, PA*PB, PO^2-OD^2. At this point, students were asked the following questions: 1) Observe the geometrical construction on the screen and drag point B on the circle (O,R). Also focus on the numerical data tabulated in the related Table. Which quantities are constant, which vary?; 2) What are the extreme values of the varying quantities and when can they be observed?; 3) Observe the values of the varying quantities; 4) Is there any relationship that connects them?

Upon dragging point B and observing the moving segment PB in relation to the numerical data presented in the accompanying Table, some students expressed their opinions in mathematical terms while other students used no mathematical language at all. In terms of the former, some of the answers given were of the sort "*One segment gets smaller and the other one gets proportionately bigger*". Concerning the latter, there were answers of the sort "*Some points change position, some don't*." After suggestions from the teacher to concentrate on the numerical data presented in the said Table, all students realized that the data contained in its 3rd and 6th columns were equal. At this point the teacher introduced these students to the concept of the power of a point P with respect to a circle (O, R) as the constant multiplication of the two varying quantities PA, PB. Regarding this specific geometrical construction, students also expressed the conjecture that the result of the multiplication of the two segments would be constant and equal to 20.19. By observing the numerical data in the previously-mentioned Table, students also realized that the power of a point P with respect to a circle (O, R) is constant and equal to PO^2-OD^2.

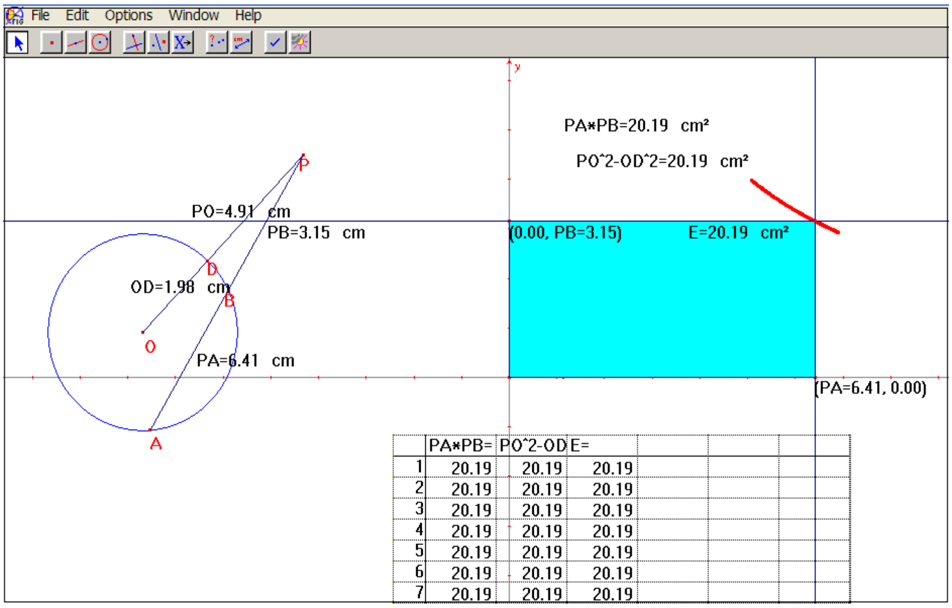
With regard to extreme values, students had no difficulty recognizing the positions where the quantities attain maximum or equal values. Some students were also able to observe that when one quantity attains maximum value, the other attains the minimum.

With regard to the question of whether there is some relationship between the two quantities, there was animated discussion in the classroom, because it was initially difficult for the students to figure out an obvious relationship. Some students, upon observing integer values, expressed the notion that "*the second segment takes the values of the first segment after the middle*" and one student also observed that '*the value of the segments when they become equal is the division of their maximum and minimum values'*.

2.3. Geometrical meanings of the power of a point P with respect to a circle (O, R)

Here, as well, it was viewed as critical to provide students with chances to: a) give spatial meanings to the formulae PA*PB in terms of the area of a rectangle with dimensions PA and PB (see the interactive construction presented in Figure 2), and b) realize that the graphical representation of the point that has as coordinates the length of PA and PB is a hyperbola (see also the interactive construction demonstrated in Figure 2). At this point, students were given the following tasks: 1) Observe the geometrical construction on the screen and drag point B on the circle (O, R). Also focus on the area of the light blue-rectangle and on the numerical data tabulated in the related Table. Can you form some hypotheses? 2) Based on the curve that describes the dynamic point on the screen, draw some hypotheses about this kind of this curve and also about the possible relationships between the varying quantities. 3) Based on your experience, express a proposal that describes your hypotheses regarding the power of a point P with respect to a circle (O, R).

As regards the first question, all students easily realized that all columns of the Table illustrated in Figure 2 have equal numbers. However, all of them were impressed by the fact that the power of a point P with respect to a circle (O, R) can be represented as a rectangular area. Concerning the second question, students debated the significance of the dynamic point and of the curve that could possibly create its representation. A high percentage of the students were of the opinion that the curve created is part of a parabola and two students expressed this. At this point, students expressed the conjecture that the result of the multiplication of the two segments is constant. With regard to the third question, students formed correct hypotheses regarding the formula of the power of a point P with respect to a circle (O, R), that is: PA*PB= PO^2-OD^2. However, students encountered difficulties in expressing this formula in their natural language and, at this point, they received continuous guidance and encouragement from their teacher in the use of mathematical terms.



	PA*PB=	PO^2-OD	E=
1	20.19	20.19	20.19
2	20.19	20.19	20.19
3	20.19	20.19	20.19
4	20.19	20.19	20.19
5	20.19	20.19	20.19
6	20.19	20.19	20.19
7	20.19	20.19	20.19

Figure2. *Experimenting with geometrical meanings of the secants of a circle*

Finally, two secants from point P and intersecting the circle (O, R) were drawn on the screen of the computer so that students would be aided visually in the procedure of proving the proposal: if we have a point P and two secants that intersect a circle (O, R) on points A, B and C, E, correspondingly, then the following formula is true: PA*PB=PC*PE. At this point, students implemented the formula they had realized in the previous activity for the secant PB: PA*PB= PO^2-OD^2 and also for the secant PE: PE*PC= PO^2-OD^2. Consequently, all of them realized the truth of the previously-mentioned proposal.

3. Discussion and conclusions

This paper presented the design and the implementation in a real classroom of a learning experiment consisting of learning activities based on the use of Multiple Representation Systems within the context of Dynamic Geometry Systems, and especially Cabri-Geometry II. Specifically, a teaching experiment was designed for the learning of the mathematical notion of the power of a point P with respect to a circle (O, R) using visual, numerical and dynamic interlinked representation systems. By using appropriate interactive constructions interlinked with numerical data, students seemed to understand that the power of a spontaneous point P with respect to a circle (O, R) is the constant multiplication of the two varying quantities PA, PB and also that it is constant and equal to PO^2-OD^2, where: a) point B is on the circle (O, R) and A is the intersection point of the secant PB and this circle, and b) point D is the intersection point of the segment PO to the circle (O, R) and OD=R. By using dynamic visual constructions in combination with a Cartesian system of coordinates, students also realized that the curve created by the point with coordinates the lengths of PA, PB is part of a parabola and also that PA and PB are reversely proportional quantities. Students also helped to give spatial meanings to the notion of the power of a point P with respect to a circle (O, R) in terms of rectangular area representations of the varying quantities PA, PB by exploring visual spatial representations of a rectangular area with dimensions PA and PB. Finally, it is worth noting that all students were impressed by the dynamic and interlinked MRS provided and were actively and passionately involved in the activities set. All students also expressed and verified conjectures based on their concentration on the dynamical change of the constructions in combination with the numerical data. On the whole, different representation systems provided students with complementary ways to conceptualize the concepts in focus and also supplied them with opportunities to extend alternative meanings to these concepts.

Acknowledgements This work was funded through Project 128989-CP-1-2006-1-RO-Comenius-C21 from the European Commission, Education and Training, School Education: Socrates: Comenius. We thank all the partners and teachers for all their cooperation and work and especially to the mathematics teacher, Mr. Dimitris Kalogeras, for his help in the implementation of the activity reported in this paper.

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