A LEARNING ENVIRONMENT FOR THE CONSERVATION OF AREA AND ITS MEASUREMENT : A COMPUTER MICROWORLD

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Introduction

Measurement is a universal and fundamental activity in all cultures as it has its roots in financial and commercial life, so it has social power [2]. Area measurement in particular is a part of the culture of all societies, a part of science and technology, and a part of people's every day life [27], [11]. Areas are also used as embodiments for teaching other concepts in mathematics [11]. Moreover the multiplication structures are related to this activity [7]. Area measurement connects the concrete world of natural objects with the abstract world of numbers [10].

The concept of area measurement consists of the concept of unit, the concept of unit iteration, the counting of units, and the calculation formulas [24], [19], [11]. The area as a space inside a figure and the conservation of area are preliminary concepts for the operation of measurement [24], [1], [19], [11]. By conservation we mean that modification in form cannot produce change in area [24]. The part-whole aspect and vice versa, and the concept of compensation are basic concepts of the above aspect [28], [24], [4]. Children can master these concepts through *cut* and *paste* in order to recompose areas [1],[11].

A large amount of research focuses on illuminating children's difficulties in area measurement. In comparison tasks, children use perceptual justifications, focus only on the dominant dimension of the shape, or compare the areas of the shapes by their perimeters [7].

They often have difficulty in conceiving the inverse relationship between the size of a unit and the number of units required to cover the same area [4]. Children also have problems in finding a strategy to measure irregular shapes [7]. Moreover children cannot conserve units from their parts [11].

In primary school, children are introduced to the concept of area measurement by using grids and counting the squares inside a geometrical figure. Gradually, they use a given square unit to cover regular shapes to measure them. Finally, they learn the calculation formulas. The absence of activities for area manipulations, especially those which initiate the sensory- motor actions of the children, the skipping of the concept of conservation of area and the premature use of area formulas in school mathematics causes most of the children's difficulties in this topic [14], [24]. Moreover children do not have the opportunity to create their subjective tools for measurement, for example units or grids, due to the introduction of a proposed unit by the teacher. The quick transition from areas to numbers leads children to develop in any way, numerical answers [10], [7]. So a gap between the concepts of measurement operation and its results can be created. This instrumental approach, which is common in school mathematics, has been criticized [25].

When area's quantitative value remains unaltered, its shape can be qualitatively new. So we can view this new shape as a new representation of this area. Moreover when an area is measured using one or more different units then a new representation appears as a combination of two representation systems, the system of numbers and the representation system of units of area . The multiple representations of a concept and the translation from one to another representation has an impact on children's thinking [12]. The computer allows the integration of a variety of representation systems in a powerful way [12]. These systems can interact, so by changing one variable in one system it can have an effect on another, and these changes can be visualised.

Children, by interacting within different representation systems, can see the results of their actions on these systems and after reflection may form abstract concepts [8]. Computer environments can help students to investigate geometrical concepts [6], [17]. In the case of area, when children have the opportunity to see and manipulate areas in different representation systems they can distinguish areas from their shapes and so they can form the concept of conservation. Moreover, when they use a variety of measurement tools, they can appreciate the concept of unit [23].

In this study a microworld has been developed which attempts to integrate all the different aspects which make up the conservation of area and its measurement. The part which concerns the conservation of area has been further discussed in [15].

The rationale of the 'Conservation of Area and its Measurement (C.AR.ME)' microworld.

By using the term microworld we mean a conceptual world that consists of a set of objects, a set of rules about their relations, and a set of operations that can act on them [18], [26]. In our case, the objects are the polygons constructed by the student while interacting with the microworld. The relations between the objects are the transformations of polygons into other geometrically equivalent shapes. The operations which are provided by the system are the simulations of children's sensory-motor actions, and subjective, informal or standard tools for measurement.

From another point of view the C.AR.ME microworld can be viewed as a model that consists of the synthesis of three models. Computer modelling is used in education to support learning in artificial words. It is about providing children with computer tools to enable them to create their own worlds, to express their own representations of the world, and also to explore other people's representations [20]. The design of the microworld was based on the following models: First, we constructed the model about the subject-matter. It consists of the conservation of area and its measurement concepts. This model is based on the psychological foundations of the above concepts as comes from past research in these areas. Moreover, research findings on children's difficulties in area measurement helped us to enrich the model. Secondly, we designed the model of children's sensory-motor actions during measurement tasks. These actions introduced new dimensions to the subject matter model. Thirdly, we attempted to express in electronic words our view about the construction of knowledge, so we designed the learning model.

Moreover, our design aimed to support children's work on specific tasks in the above conceptual areas. An attempt has been made to integrate the microworld into a variety of contexts. The features of electronic media, also had a potential impact on each model's construction. In particular, possibilities such as the use of multiple representation systems, the graphical representation of children's sensory-motor actions, and the digital representation of children's informal or intuitive knowledge during their interactions with the microworld, gave multiple views of the above concepts [8], [12].

The models are designed as hierarchical trees. In the model of the subject-matter each level of a concept includes its sub concepts until finding elementary ones. In the learning model each process includes its sub processes. The process of reduction from the highest level entries of the trees to lower level ones succeding in satisfying two main objectives. The first one was to break down the knowledge into its parts to allow children to develop their own strategies by using these elementary parts. The second one was to make it possible to transform these elements into computer algorithms. The elements of the lowest level entries of each model characterize the educational requirements which were the base for the design of the software. These requirements were expressed by the basic operations of the user interface.

The model of the subject matter

In Figure 1, the analysis of the concept of conservation of area is shown. The conservation is viewed : after the change of a shape's position without the modification of the shape; after modification of the shape by splitting it and recombining its parts; and finally, after modification to other known geometrical shapes. The changes are realized by geometrical transformations of translation, rotation, symmetry and their combinations.

The concept of area measurement is broken down into its subconcepts in Figure 2. These subconcepts are the unit of area, the unit iteration and the counting of units. Units can be subjective, informal or standard. Subjective units are units which are constructed by the student. Informal units are square or rectangular offered by the system, while units of 1 cm^2 are provided as standard units.

Children's difficulties led us in addition to consider other aspects of the concepts such as:

* area manipulations without the use of numbers.

* area manipulations using the simulations of children's sensory-motor actions.

*area measurement by the construction of subjective measurement tools, or by the use of informal or standard ones.

Moreover, the microworld provides operations to transform any polygon automatically to a series of equivalent geometrical shapes such as a square, a rectangle, a parallelogram, a triangle or a family of them. These transformations form a dynamic mode of area representation that can help students to separate areas from their shapes. This feature that which was implemented by the use of electronic media, is very difficult to create in the paper and pencil environment.

The model of children's sensory-motor actions

Children's sensory-motor actions develop through their attempts to adapt in their environment. This process may lead to the acquisition of some knowledge. Especially, on area measurement or conservation problems, these actions play a potential role in cognitive development of these concepts [24].

Modelling children's sensory-motor actions, while they are involved in conservation of area and measurement problems, led us to implement the *cut, translation, paste, rotation, and the rotation about the z axis in 180 degrees (symmetry)* operations. These actions are mainly used in tasks of transformation or comparison. Children using the simulations of their sensory-motor actions have first to specify the properties of these actions, second to transform continuous or analogic actions into a set of discrete commands, and third they have to know if their procedures are an adequate description of intended action as soon as they "run" the program [9].

The model of the learning process

From the constructivist perspective learning is viewed as an active, subjective and constructive process [30]. The features of electronic media can support this perspective and offer new possibilities for learning. For example, multiple representations of the same concept may have a potential impact on children's cognitive development [12]. The visualisation of children's actions also gives children an opportunity for reflection, a basic operation for the development of the cognitive structures [8]. Moreover, the knowledge which is embodied in the software is an expert's knowledge that also gives children the opportunity to interact and reflect, enriching their experience and their cognitive schemes [21], [3]. From a Papertian point of view, embodying children's sensory-motor actions in a microworld can also move the children away from their egocentricity to a more object-centred perspective [9]. The objects in the sense of the C.AR.ME-environment are areas.

In Figure 3, an attempt has been made to express the principles of active, subjective and constructive, in terms of computer words. We tried to transform all these above cognitive requirements into software requirements. A set of tools is given to the children to draw, measure and transform areas. In this environmnent, children can develop their own approaches while they solve problems. This is possible because the operations of the microworld are independent, students can also define their personal objects and there is not an evaluation system in the form of correct answers. The learning process can be constructive as the environment supports interactive, problem solving experimental situations which can be expressed in multiple representation systems or contexts. Moreover, children can reflect on their actions when they visualise the results of these actions on the computer screen.

Features of the C.AR.ME environment

In this section we describe the features and components of the Conservation of Area and its Measurement (C.AR.ME) environment. These are:

*drawing (segments and polygons)

* manipulation of areas using the simulations of children's sensory-motor actions

* measuring areas using a variety of tools

* transforming areas automatically into equivalent ones

* measuring lengths and angles automatically

* providing material for off-line study

* comprehensive, on-line help

Invoking C.AR.ME brings up a dialogue box which appears to save students' interactions with the microworld operations in a log-file, for off- line study. Then, the main menu is shown (Figure 4).

Drawing segments and polygons

Under the Drawing menu as demonstrated in Figure 4, there is a set of tools to draw. In particular, children have at their disposal two operations , to draw segments or polygons. When children select the "draw polygons" operation and hold down the mouse, they draw a point that is a vertex of a polygon. This point is joined automatically by a segment to the next point that the student will draw in the same way. Children have to close their polygons, but the system automatically closes the polygon by the selection of the "End draw polygons" operation. The closing is necessary for the automatic area measurement. Moreover children can use the two dot grids (a square and a triangle), which are available from the system, to draw specific types of polygons. These can have segments which can be proportional and angles which are multiples or certain divisors of 90 or 60 degrees in each case of dot grids respectively. These grids can also help the students to approach length in a qualitative way by avoiding the early use of numbers and the standard units of length measurement. The system also offers an opportunity to draw segments in the same way as polygons .

Manipulation of areas using the simulations of children's sensory-motor actions

Under the "Edit" menu, as demonstrated in Figure 4, there are nine operations to manipulate areas. Polygons, which children have drawn using the drawing tools of the system, can be manipulated with the above operations after the "Select All" selection. This menu command creates a copy of the polygon. So, the prototype shape does not alter after manipulations. In the paper and pencil environment, this feature is not possible. Polygons in particular can be translated and pasted at another point on the screen of the computer by moving. These can also be rotated or animated at an angle drawn by the student. To draw the angle of rotation, the student has to draw three points which are joined automatically by segments to give a graphical representation of the angle. In this way the concept of angle gets a qualitative meaning. This is important for young children because they do not have a good estimation of an angle in terms of degrees. The point of the rotation of the polygon is the vertex of the angle. After rotation, the previous shape is erased. Continuing to click on the "rotate" command animates the polygon by the angle of rotation.

Moreover, polygons can be transformed into their symmetrical ones about an axis defined by the student. In Figure 5 examples of these transformations are shown. The axis of symmetry can be given by the student at "run" time by drawing two points which define the axis direction. These points are joined automatically by a segment to give the graphical representation of the axis. Continuing to click on the "Symmetry about axis" command, the symmetrical polygon is drawn automatically while the previous form of the polygon remains drawn in dashed line mode. With the above operations and their combinations we implement the concept of conservation of area by changing the shape's position and not its form .

Students can also transform areas by changing their shapes. This is possible in many ways by using the same operations used for transforming the whole shape. In this case instead of using the " Select All" operation the "Select Part" is used. The selection of a part is done by redrawing the chosen part, a way which is closer to children's actions. The "cut" operation creates a shape congruent to the selected one in dashed line mode and in the same position, so the previous form of the shape is connected to the next one. From an educational perspective, this connection which is realized with the "copy" and "cut" operations is important because it may help students to conceptualise the conservation of area.

Moreover, three erasers (small, medium, large) are offered by the system.

Tools for measuring areas

Students have at their disposal four kinds of tools for measuring areas. These are a set of units, the operations of unit iteration and the counting of units, and a set of grids. The set of units consists of a square and a rectangular unit which are offered by the system. These units are characterised as informal. Moreover, the system gives the children the opportunity to draw their subjective units of any shape they like. For completion, the system also offers the automatic operation of measuring polygons using the standard units of area measurement. Children can cover their shapes by units using the operation of "Unit iteration". So the unit can be moved and pasted by double click on a selected point of the polygon. The "Counting units" operation is an operation that the system automatically executes. Numerical results are given in the " Numerical results" window. Children have the opportunity to compare their results of counting units with the system's results. So, they can evaluate their solving procedures.

Finally, the system provides grids to complete the availability of tools for area measurement. Two grids, a square and a rectangular, are offered. The children can also design their own rectangular grids by defining their dimensions. In Figure 6 you can see examples of measuring areas using units or grids. When children use a variety of units and grids they interact with a different pair of representation systems, the system of numbers and the system of units. Working in this way, they move gradually through subjective and informal units to standard ones. So, they may conceptualise the concept of unit of area measurement better.

Transforming areas automatically into equivalent ones

Under the "Automatic Transformations" menu there is a set of operations which can be used to transform automatically every polygon drawn by the student into a variety of equivalent geometrical shapes. These shapes are a square, a rectangle with dimensions of ratio 1:2, a right-angled triangle with the perpendicular sides of ratio 1:2, a family of rectangles, a family of parallelograms and a family of triangles. These families of shapes have a side of the same size which is drawn by the student at "run" time. By clicking the mouse on the relevant menu command for having a specific family, a new shape of this family appears on the computer screen. All these shapes have a common side, the one given by the student. New families can be drawn every time the student defines a different side. In Figure 7, the transformation of a polygon into a number of eqivalent shapes is shown. Students can also see in the Numerical Results list, the lengths of the height and the base of the transformed shapes by clicking on the "Show numerical elements" sub menu command .

The above operations allow the teacher to help children reflect on the feedback they take from the program and realise the mathematical knowledge which is implied in these transformations. For example, the teacher can pose comparison tasks by drawing equivalent shapes which is not possible in the paper and pencil environment. Moreover, questions can be asked about the possibility of conservation as expressed in the above transformations. Children can also be asked to find the rules or procedures which characterise these transformations, for example multiplication structures or formulas. In addition children can develop new ways of solving comparison tasks using the concept of transitivity.

When shapes are transformed into their equivalent ones, a different representation of area appears on the screen. In particular, when shapes are transformed into families of equivalent shapes a dynamic mode of representation is realised. So, shapes can be expressed in terms of squares, families of triangles or parallelograms. These representations give the children the opportunity to separate areas from their shapes and to create more abstract concepts. Moreover, it is impossible tomake these transformations in a static medium such as the paper and pencil environment.

Tools for automatic measurement

The system can measure segments in centimetres, angles in degrees, and areas in square centimetres automatically . Students can see these numerical results in the Numerical Results list. Summarizing, in this list the student can see :

* The length of the side of the square which is equivalent to a polygon in centimetres.
* The lengths of the bases and the heights of the families of rectangles, parallelograms, and triangles in centimetres.
* The lengths of the dimensions of the student's grids in centimetres.
* The number of units that are needed to cover a shape.

By using the automatic measuring commands children can *evaluate* the methods which they developed when they deal with comparison or transformation tasks.

<u>Providing material for off-line study</u>

The environment includes the facility of saving, recalling and printing all of the student's drawings using the sub menu commands of "Save as", "Open" and "Print" respectively. Such output can be used by the student or the teacher for further study. Moreover, as the student enters this environment, a log file is created which is saved automatically and can be recalled with all children's interactions with this package of software. It is a very valuable source for the teacher and the researcher to investigate children's paths through the software, the preferred operations by the student, and the children's difficulties in the use of the tools of the interface.

<u>Comprehensive, on-line help</u>

A Help component is an important part of a microworld especially in preparing the students to interact with it. The information that is included in this component is broken down into small pieces and each piece has its own submenu in the interface of the Help window. The interface of the "Help" form is the same interface as the main form of the developed microworld. By clicking on the submenu "Show contents" under the "Help menu" on the main form the "Help" form pops up in another window and is available to the student.

The microworld is implemented in Visual Basic running under the Windows 3.1 or Windows 95 on a PC (386 or newer) with appropriate memory.

The Pilot Study : The initial face of the evaluation process of the microworld

The theoretical framework

The theoretical perspective of the design of the microworld defines also its evaluation. So, in our approach we focus on children's mathematical constructions when they interact with educational materials [29]. Moreover, we investigate the role of the context of the educational tasks in the development of these constructions[22], [23], [14]. In terms of the methodology, this evaluation study has its roots on phenomenology and is characterised as a qualitative study[5]. Especially, in this initial phase, the evaluation of the microworld is needed to be formative. Here, we present an extension of the pilot study on the evaluation of a part of the microworld, which consists only of the simulations of the children's sensory-motor actions [16].

The questions

This work is focused on two different kinds of questions. The first is concerned with the operational aspects of the microworld and children's familiarization with them. The second was about the evaluation of the microworld as a possible learning environment. Two eleven and two-twelve years old children participated in the study to help us select the appropriate age for the main evaluation study. Children worked individually with the microworld while one of the researchers acted as a teacher. In this interaction between the teacher, the children and the microworld, the role of the softwarre on children's thinking was mainly investigated. To achieve this, the researcher limited her teaching interventions.

Two tasks assigned to the children were considered. One was the transformation of an irregular polygon to any other shape with the same quantity of area and the second was the comparison of areas of an irregular polygon with a rectangular shape, not easy to be compared by "eye". The nature of the tasks allows the children to consider different processes in the conservation and measurement of area and develop different thinking strategies [14], [30]. The familiarization of the operations of the microworld was done through childrens' involvement with the tasks.

The sources of data were automatically created log.files wich contained the history of children's actions, the electronically saved pictures of their results and the audio recorded of every discussion between children and the researcher during the study.

Data analysis and results

Difficulties oncerning the interface : Children explored unsystematically the microworld operations. They could not relate the microworld operations with their corresponding explanations in the "help" menu while some of them did not use at all the help component. Sometimes the "navigation" through the software statements prevented children to complete the tasks of the study. Some children used only the first item from each menu of the interface. For all the above reasons we decided to separate the training process with the microworld's statements from the process of children's involvement with the tasks. The titles of the interface items were well understood from the students, except from the title "graphics" and "edit". In addition, the communication of the children with the computer appears to have some problems as the children expected the computer to understand what they wanted to do without some special instructions.

Concerning the time needed to complete the tasks the older children needed less time than the younger ones.

Learning outcomes: The tasks motivated the children to get really involved. They also seemed to be appropriate to children's cognitive development. It was realised during the pilot phase of evaluation study that the children could enrich their understanding of the concepts of area and its measurement by realizing different representations of them. To provide children this opportunity, we decided to ask the students to construct all the possible solutions of the tasks. This was investigated during the pilot phase. By analysing the data from the study we can identify the follow learning outcomes:

Each child constructed a variety of solution strategies for each task. The children exploited the multiple representations which were available in the microworld and managed to develop these strategies.

Children expressed their intuitive knowledge through their solution strategies. These were the transformations of shapes after "cut" and "paste", after "rotations", after "symmetry about axis" and after some combinations of them. Moreover, children gave solutions by using these transformations on some parts of the shapes. So, they conserved areas as wholes and also through a synthesis of the conservation of their parts. In the case of the comparison task the children superimposed the shapes in order to compare them. This kind of experience is something unusual in the school practices. Examples from children's work are given in figures 8, 5, 11.

C*hildren explored other mathematical concepts*. By the above transformations children considered new concepts such as the rotation of a shape about a point by a chosen angle and the construction of the symmetrical shape about an axis of symmetry. Moreover when children realised automatic transformations, they considered different shapes and their properties.

Children’s reversible thinking about conservation and measurement was enabled. Children through the automatic transformations could easily see a series of equivalent shapes. This is probably not easy to be done in the traditional paper and pencil environment. By asking the children to justify the equivalence of the shapes, they are led to discover the underlied relationchips about conservation of area. The same happened in children’s attempts to justify the comparisons which were realized by using the operation of automatic measurement. The automatic features of the microworld were the most preferable by the children. To take the most advantage from these features, children need to justify the results of the automatic operations. Examples from children's work are given in figure 7.

Children can give meaning to the measurement concept by using the measurement operation. In the environment of the microworld, children used a variety of units to measure areas. In school practices children usually do not use units to cover a shape without gaps or overlaps and they also do not have in their disposal a variety of units or grids. Children never construct personal units or grids. In the context of the microworld, children also used the operation of measurement in a combination with automatic transformations. They realised the unit iteration process with gaps and overlaps. By equivalence of shapes they changed their corrected solution procedures. Examples from children's work are given in figures 6, 9, 10.

On the whole the children liked to work with the microworld. The following coment of a girl illustrates this positive attitude “I would like to work every day by this way”.

Conclusions

The C.AR.ME. microworld is an environment which supports *multiple representations* of the concept of area. Areas can be represented in different regular or irregular shapes, and they can also be expressed in terms of numbers and a variety of units. The set of tools which are offered in this environment provide children with the opportunity to express their personal knowledge using *the digital graphical representations*. So, by reflecting on these representations they may move *from intuitive to formal knowledge*.

Children, using as tools the simulations of their sensory-motor actions have to *move from their egocentricity to an area-object-centred approach* and become more conscious of their actions. By interacting with the knowledge that is embodied in these simulations, they can possibly learn the relevant mathematical concepts. When areas are transformed by using the above functions, *the previous and the next form of the shapes are connected*, so children can have a better understanding of the concept of conservation of area. In the paper and pencil environment the previous form is substituted by the next. When areas are automatically transformed to their equivalent geometrical ones, children have the opportunity to *reflect on the knowledge which is implied in these functions* and possibly improve their understanding or construct new knowledge.

Another characteristic of this enviroment is its flexibility; it can be used in different educational settings varying from individualised to whole class. It can also support teaching based on children's needs as the teacher can have a picture of the children's knowledge and its development through their experimentation in the microworld. The teacher can also pose problems which are not possible in the traditional paper and pencil environment such as the generation of a series of different shapes with the same area.

The variety of tools for transforming and measuring areas provided in the C.AR.ME. microworld can help children to *move from their subjective tools to standard ones*, in this way constructing an understanding of the concept of area measurement. The main phase of the evaluation study of this environment with the children can help us to investigate further the possibilities of this environment and show us future extensions of it. It can also provide us with information about possible ways for integrating it into the whole classroom environment.

References

1. Beattys, B. C. & Maher A.C. (1985). Approaches to learn area measurement and its relation to spatial skill. *Proceedings of the* 7th *PME N.A* (pp. 2-7). Colombus Ohio.

2. Bishop, J. A. (1988). *Mathematical Enculturation*. Dordrecht: Kluwer Academic Publishers.

3. Bliss, J. (1994). *From Mental Models to Modelling.* In H. Mellar, J. Bliss, R. Booham, J. Ogborn, C. Tompsett (Eds.), *Learning with Artificial Words: Computer Based Modelling in the Curriculum*. London: The Falmer Press

4. Carpenter, P. & Lewis, R. (1976). The development of the concept of a standard unit of measure in young children. *Journal for Research in Mathematics Education*, *7,* 53-58.

5. Cohen L. & Manion L. (1989). *Research Methods in Education*. In Routledge (eds). London.

6. Dorfler, W. (1993). Computer use and views of the mind. In C. Keitel & K. Ruthven (Eds), *Learning from computers: Mathematics Education and Technology,* Berlin: Springer - Verlag.

7. Douady, R & Perrin, M-J (1986). Concerning conceptions of area (pupils aged 9 to 11). *Proceedings of PME* 10, London, 253-258.

8. Forman, G. (1988). Making Intuitive Knowledge Explicit Through Future Technology. In G. Forman, B. P. Pufall (Eds), *Constructivism in the Computer Age*, Hillsdale, NJ: Lawrence Erlbaum Associates.

9. Forman, G., Pufall, B. P. (1988). Constructivism in the Computer Age: A Reconstructive Epilogue. In G. Forman, B. P. Pufall (Eds), *Constructivism in the Computer Age*, Hillsdale, NJ: Lawrence Erlbaum Associates.

10. Hiebert, J. (1981). Units of measure:Results and implications from National Assesment. *Arithmetic Teacher, 28* (*6),* 38-43.

11. Hirstein, J., Lamb, E. C, & Osborn, A. (1978). Students' Misconceptions about area measure. *Arithmetic Teacher, 25 (6).*

12. Kaput, J. J. (1992). Technology and Mathematics Education. In D. A. Grouws (Eds). *Handbook of Research on Mathematics Teaching and Learning*, N.C.T.M.

13. Kouba, L. V., Brown, A. C., Carpenter, P. T., Lindquist, M. M., Silver A. E., & Swafford O. J. (1988). Results of the Fourth NAEP Assesment of Mathematics: Measurement, Geometry, Data Interpretation, Attitudes, and other topics. *Arithmetic Teacher*, *35 (9),* 10-16.

14. Kordaki, M. & Potari, D. (1997). *Children's Approaches to Area Measurement through Different Contexts*. Manuscript submitted for publication.

15. Kordaki, M. & Potari, D. (1997). The concept of the conservation of area in a computer environment. *Proceedings of the Third Panhellenic Conference on Mathematics and Computers in Education,* Patras, Greece.

16. Kordaki, M. & Potari, D. (1998). The pilot study of a microwrld about the conservation of area. *Proceedings of Conference on Mathematics Education,* Rethymnon, Greece.

17. Laborde, C. (1993). The computer as part of the learning environment: the case of geometry. In C. Keitel & K. Ruthven (Eds), *Learning from computers: Mathematics Education and Technology*, Berlin: Springer - Verlag.

18. Laborde, J-M. (1990). Cabri-Geometre: A microworld of geometry for guided discovery learning. *ZDM., 5*, 171-177.

19. Maher, A. C.& Beattys, B. C. (1986). Examining the Construction of area and its Measurement by Ten to Fourteen Year old Children. *8th PME N.A.*, 163-168.

20. Mellar, H. & Bliss, J (1994). Modelling and Education. In *Learning with Artificial Words: Computer Based Modelling in the Curriculum*. In H. Mellar, J. Bliss, R. Booham, J. Ogborn, C. Tompsett (Eds.), *Learning with Artificial Words: Computer Based Modelling in the Curriculum*. London: The Falmer Press.

21. Murray, B. F. (1988). The child - Computer Dyad and cognitive development. In G. Forman, B. P. Pufall (Eds), *Constructivism in the Computer Age*, Hillsdale, NJ: Lawrence Erlbaum Associates.

22. Noss, R. & Hoyles, C (1996). The visibility of meanings: modelling the mathematics of banking. *International Journal of Computers for Mathematical Learning* 1, 3-31.

23. Nunes, T., Light, P., Mason, J., (1993). Tools for thought: the measurement of lenght and area. *Learning and instuction*, *3*, 39-54.

24. Piaget J. , Inhelder B., Sheminska A.(1981). *The childs conception of geometry,* N.Y: Norton & Company.

25. Patronis, T. & Thomaidis, Y. (to appear). On Arithmetization of School Geometry in the Setting of Modern Axiomatics, *Science and Education.*

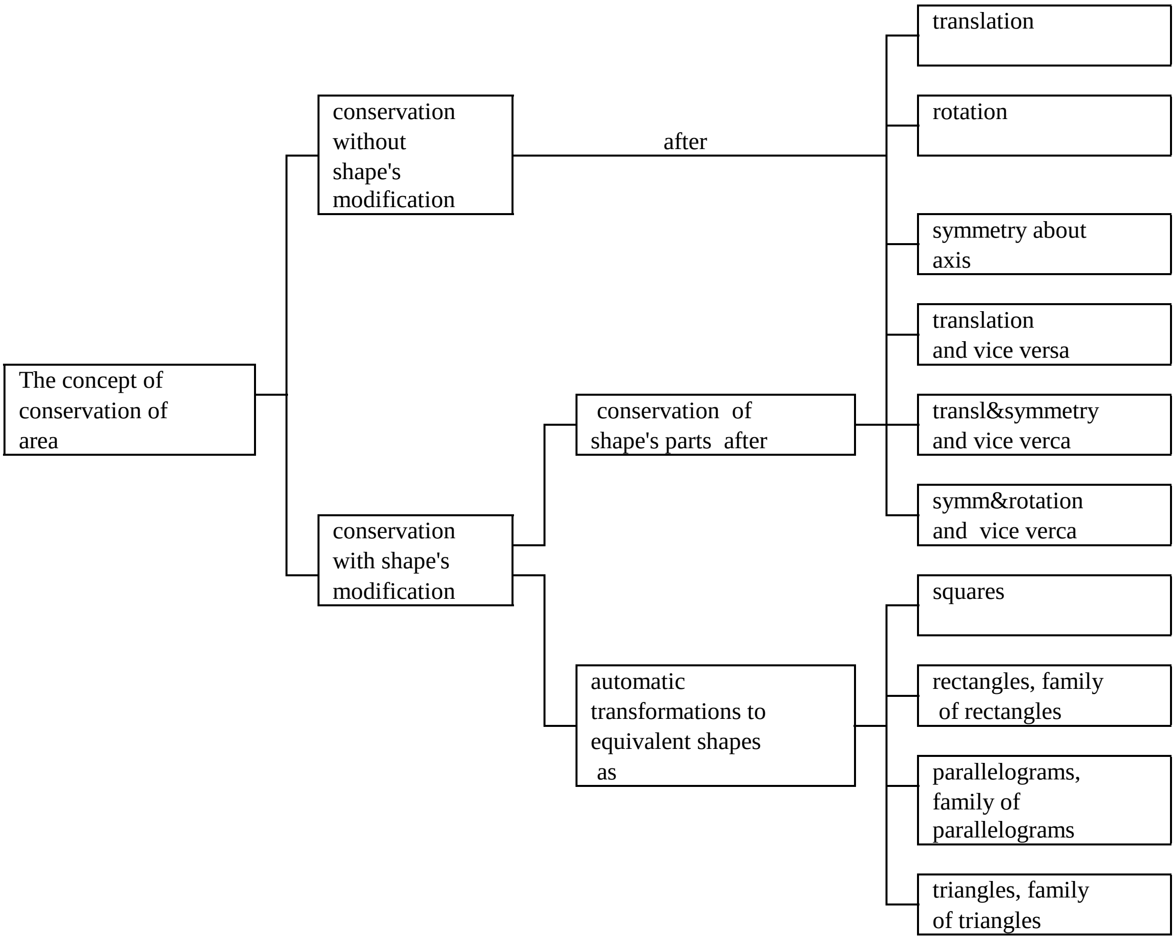
26. Pufall, B.P.(1988). *Function in Piaget's System: Some Notes for Constructors of Microworlds.* In G. Forman, B. P. Pufall (Eds), *Constructivism in the Computer Age*, Hillsdale, NJ: Lawrence Erlbaum Associates.

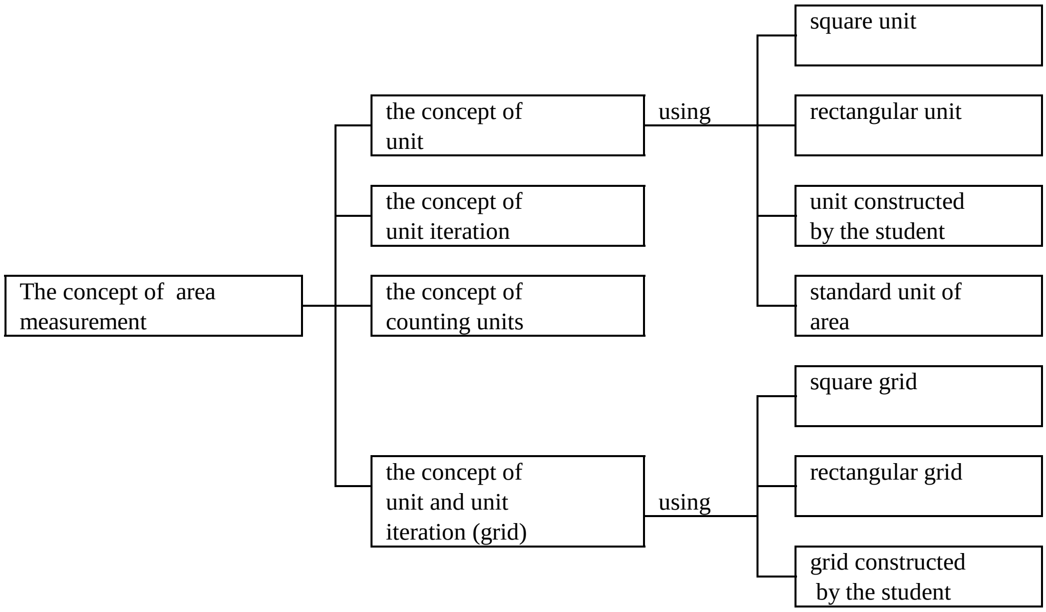
27. Sanders, J.W.(1976). Why measure?. In D. Nelson, R. Reys (Eds). *Measurement in school Mathematics*. Reston, VA: N.C.T.M.

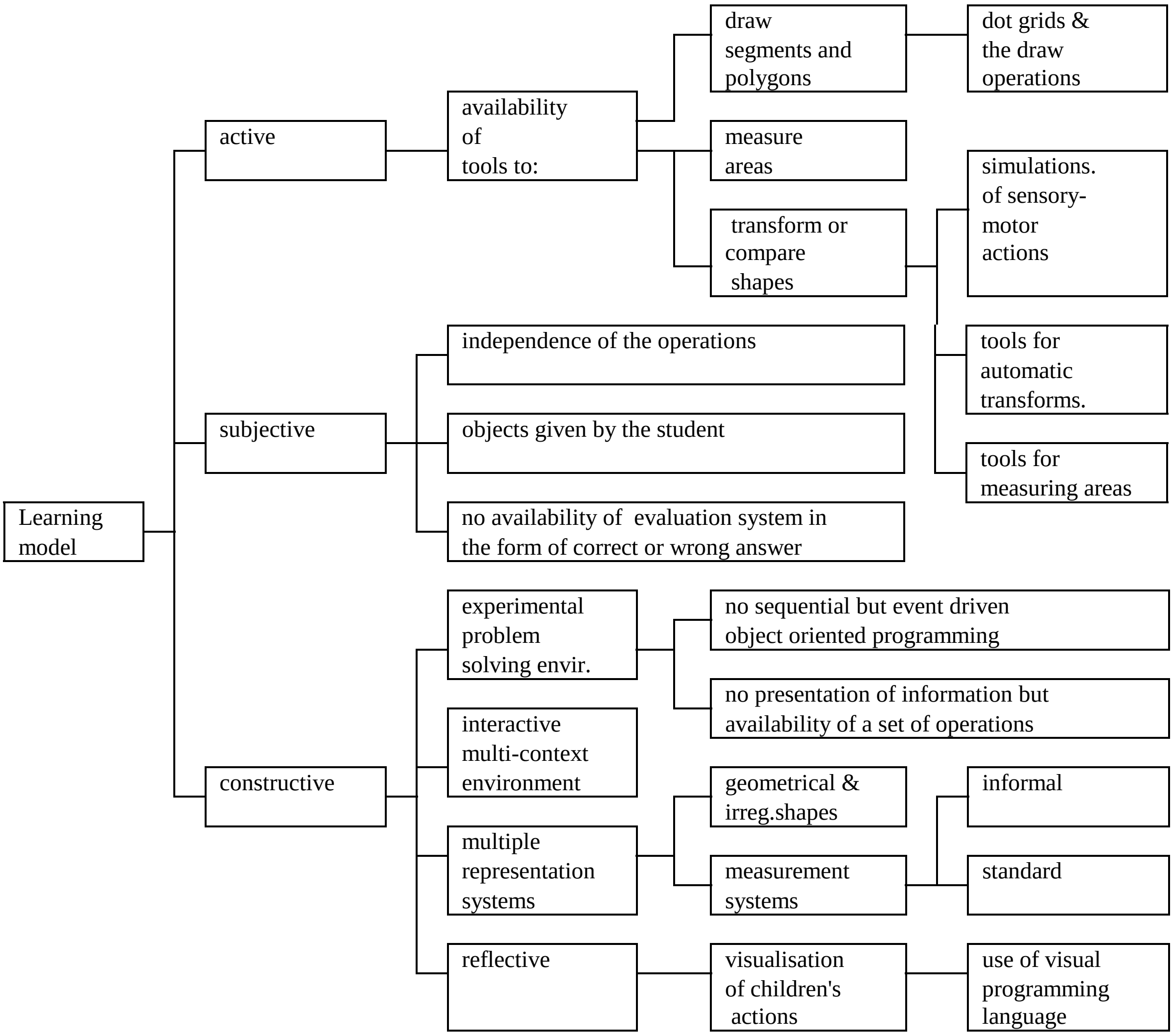
28. Steffe, P. L., Hirstein, J.J. (1976). Childrens' thinking in measurement situations. In D. Nelson, R. Reys (Eds). *Measurement in school Mathematics*. Reston,VA: N.C.T.M.

29. Stenhouse L. (1989). *An introduction to curriculum research and development*. In Heinemman Educational Books Ltd. Oxford.

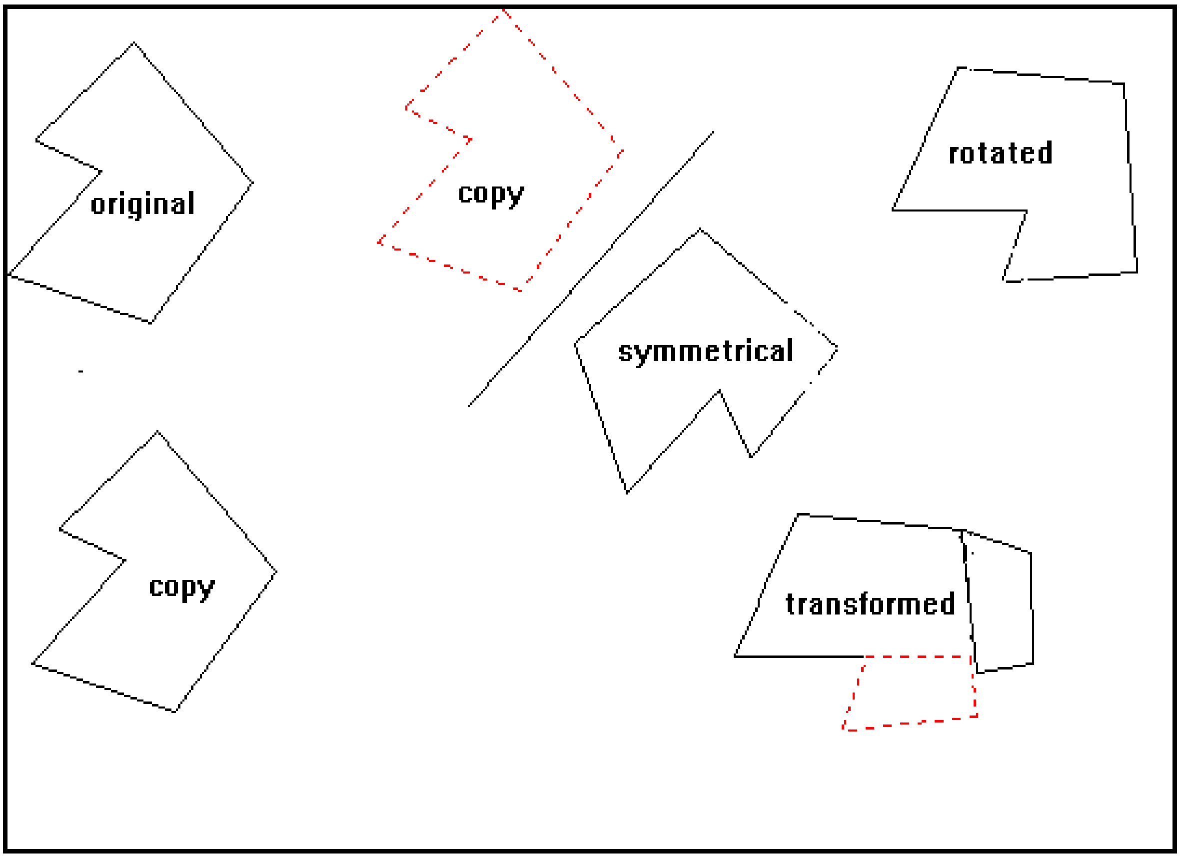
30. von Glasersfeld, E. (1990). An Exposition of Constructivism: Why Some Like It Radical. In R. B. Davis, C. A. Maher, and N. Noddings (Eds). *Constuctivist views on the teaching and Learning of Mathematics*, Journal for Research in Mathematics Education Monograph Number 4. Reston VA: N.C.T.M.

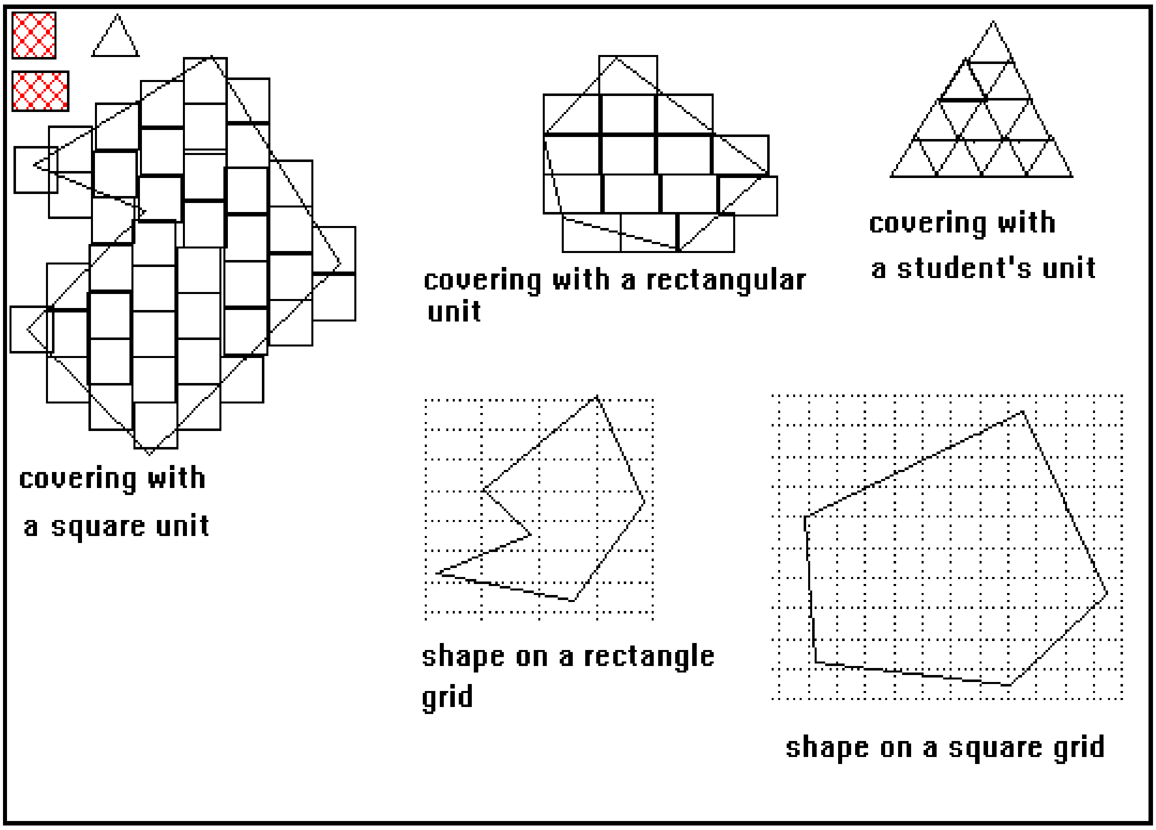

The concept of conservation of area
conservation without shape's modification
after
conservation with shape's modification
conservation of shape's parts after
automatic transformations to equivalent shapes as
translation
rotation
symmetry about axis
translation and vice versa
transl&symmetry and vice verca
symm&rotation and vice verca
squares
rectangles, family of rectangles
parallelograms, family of parallelograms
triangles, family of triangles

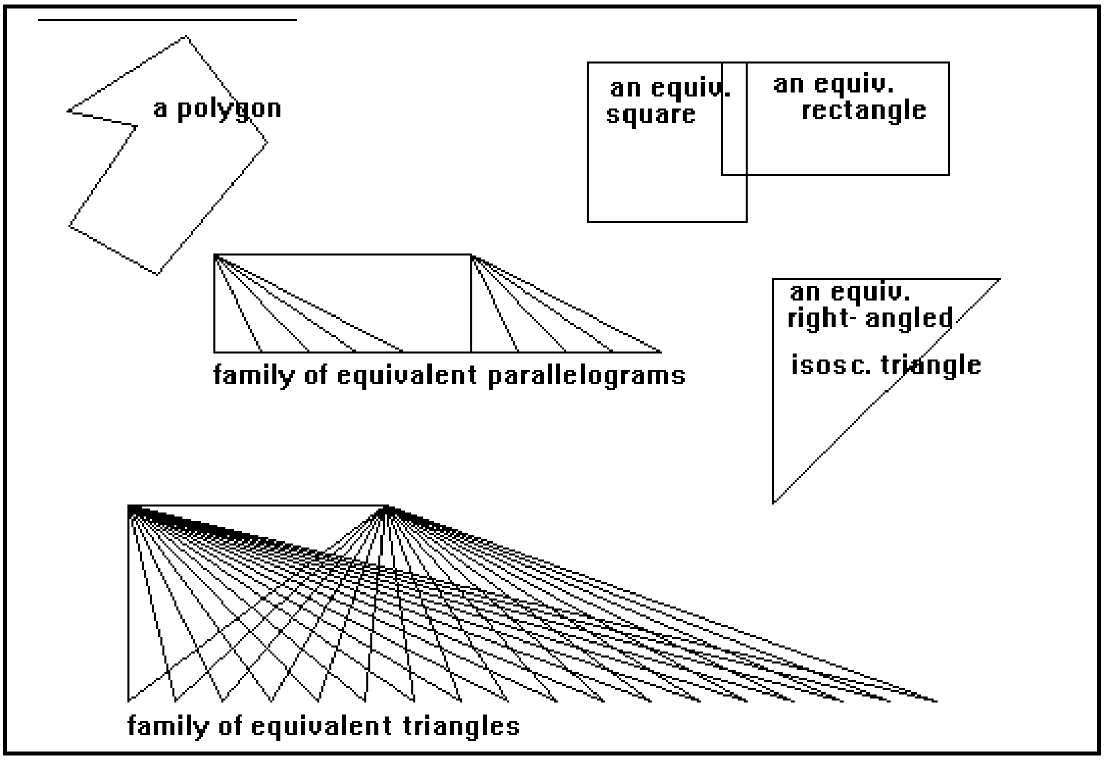

The concept of area measurement
the concept of unit
the concept of unit iteration
the concept of counting units
the concept of unit and unit iteration (grid)
using
square unit
rectangular unit
unit constructed by the student
standard unit of area
using
square grid
rectangular grid
grid constructed by the student

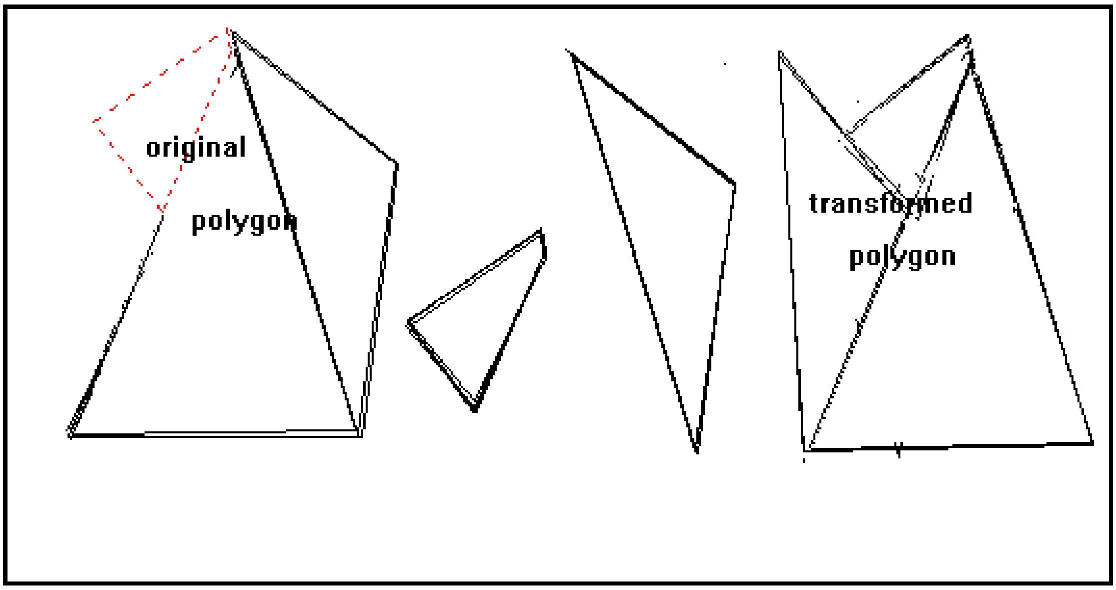

Learning model
active
subjective
constructive
availability of tools to:
draw segments and polygons
dot grids & the draw operations
measure areas
transform or compare shapes
simulations. of sensory-motor actions
tools for automatic transforms.
tools for measuring areas
independence of the operations
objects given by the student
no availability of evaluation system in the form of correct or wrong answer
experimental problem solving envir.
no sequential but event driven object oriented programming
no presentation of information but availability of a set of operations
interactive multi-context environment
multiple representation systems
geometrical & irreg.shapes
measurement systems
informal
standard
reflective
visualisation of children's actions
use of visual programming language

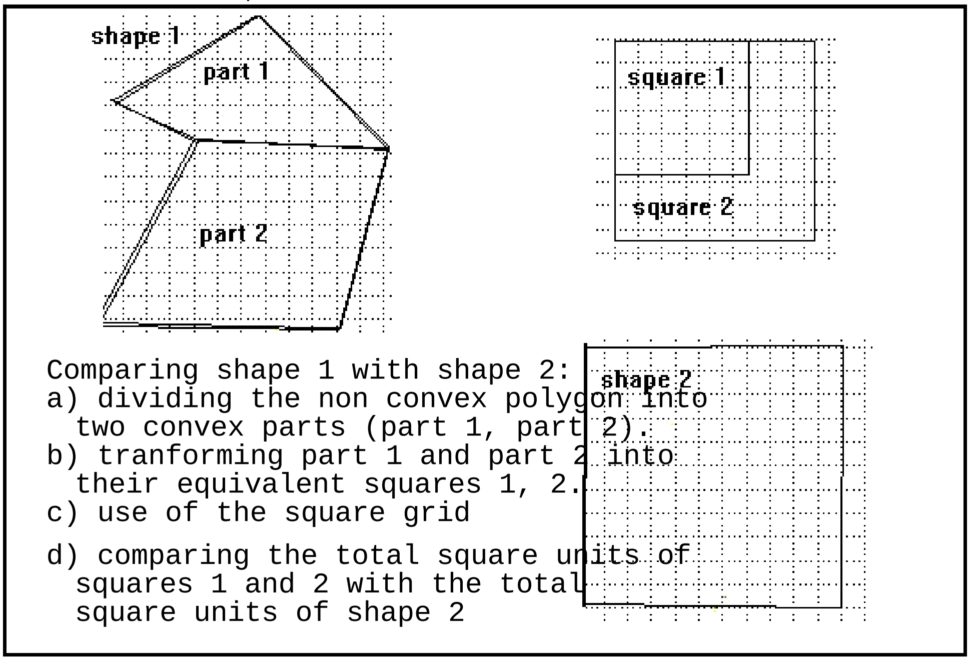
File	**Draw**	**Edit**	**Measuring**	**Automatic Transformations**	**Tools**	**Help**
Open	Dot Square Grid	Select Part	Areas	Square	Square unit	
Open Last	Dot Triangle Grid	Select All	Angles	Rectangles and Family of Rectangles	Rectangle unit	
Save Last	Draw Polygons	Cut	Segments	Parallelograms and Family of Parallelograms	Student unit	
Save As	Draw Segments	Paste		Triangles and Family of Triangles	Unit iteration	
Print	End Draw Polygons	Draw the angle of Rotation			Counting of units	
Exit	Clear	Rotate			Square Grid	
		Draw the axis of Symmetry		Show numerical elements	Rectangle Grid	
		Symmetry about axis			Student Grid	
		Erasers				

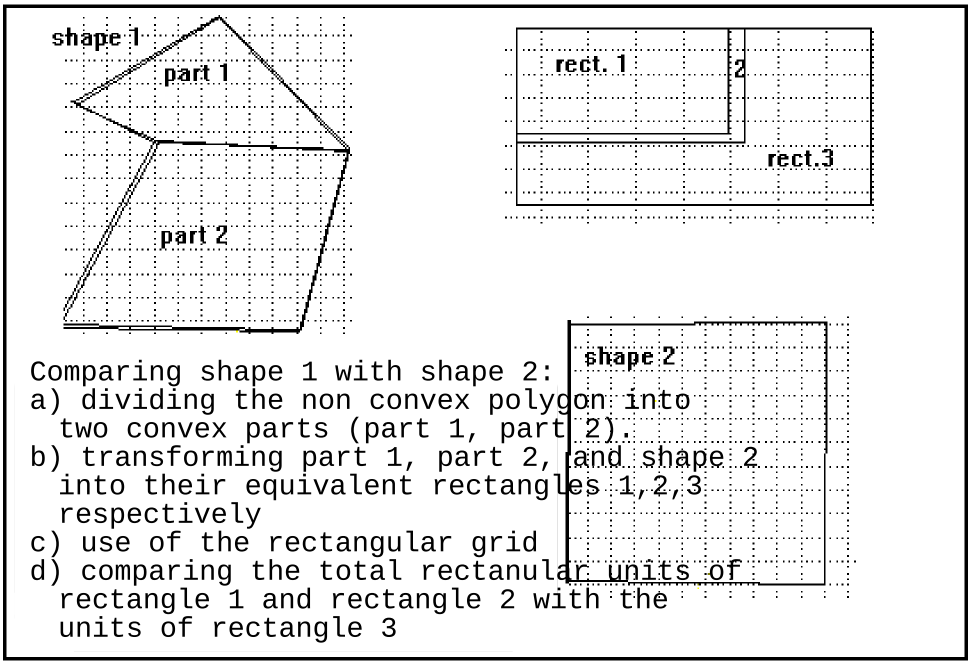

original
copy
symmetrical
rotated
copy
transformed

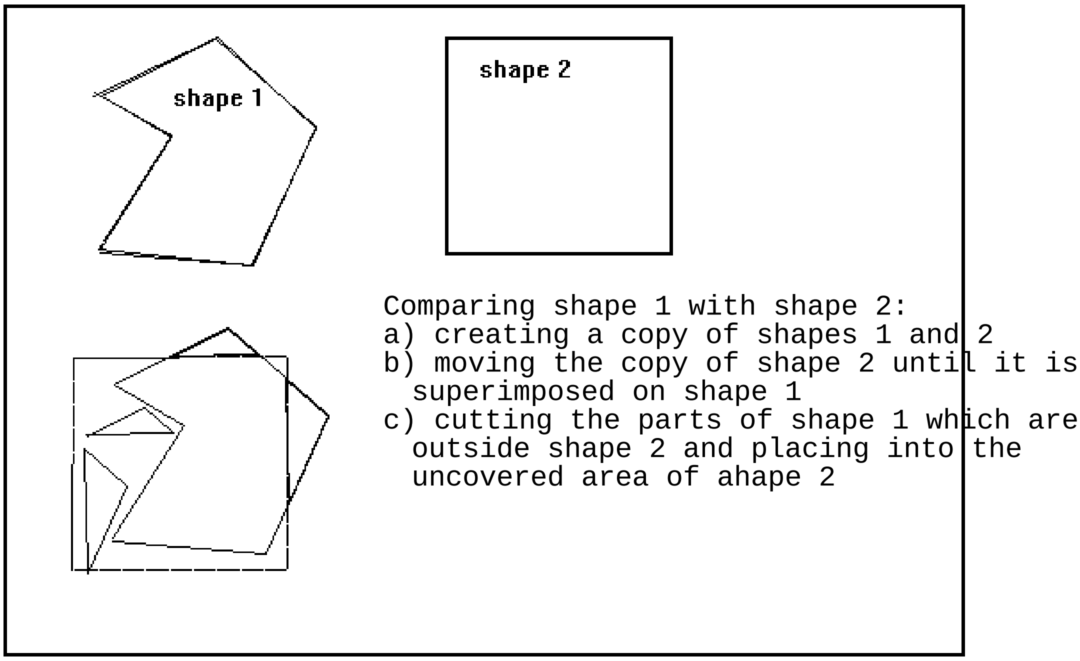

covering with
a square unit
covering with a rectangular
unit
covering with
a student's unit
shape on a rectangle
grid
shape on a square grid


a polygon
an equiv.
square
an equiv.
rectangle
family of equivalent parallelograms
an equiv.
right- angled
isosc. triangle
family of equivalent triangles


original
polygon
transformed
polygon


shape 1
part 1
part 2
square 1
square 2
shape 2
Comparing shape 1 with shape 2:
a) dividing the non convex polygon into two convex parts (part 1, part 2).
b) tranforming part 1 and part 2 into their equivalent squares 1, 2.
c) use of the square grid
d) comparing the total square units of squares 1 and 2 with the total square units of shape 2


shape 1
part 1
part 2
rect. 1
2
rect.3
shape 2
Comparing shape 1 with shape 2:
a) dividing the non convex polygon into two convex parts (part 1, part 2).
b) transforming part 1, part 2, and shape 2 into their equivalent rectangles 1,2,3 respectively
c) use of the rectangular grid
d) comparing the total rectanular units of rectangle 1 and rectangle 2 with the units of rectangle 3


shape 1
shape 2
Comparing shape 1 with shape 2:
a) creating a copy of shapes 1 and 2
b) moving the copy of shape 2 until it is
superimposed on shape 1
c) cutting the parts of shape 1 which are
outside shape 2 and placing into the
uncovered area of ahape 2