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THE EFFECT OF AREA MEASUREMENT TOOLS ON PUPILS' STRATEGIES: THE ROLE OF A COMPUTER MICROWORLD

RUNNING HEAD : AREA MEASUREMENT TOOLS AND PUPILS' STRATEGIES

ABSTRACT

This study focuses on the role of tools, provided by a computer microworld, (C.AR.ME) on the strategies developed by 14-year-old pupils for the area measurement of a non-convex polygon. Pupils' strategies on a transformation and a comparison task were interpreted and classified into categories in terms of the tools used for their development. The analysis of the data shows that an environment providing the pupils with the opportunity to select various tools and asking them to produce solutions 'in any possible way' can stimulate them to construct a plurality of solution strategies. The pupils selected tools appropriate for their cognitive development and expressed their own individual approaches regarding the concept of area measurement. The nature of tools used affected the nature of solution strategies that the pupils constructed. Moreover, all pupils were involved in the tasks and succeeded in completing them with more than one correct solution strategy thereby developing a broader view of the concept, although not all of them realized the same strategies. Three different approaches to area measurement emerged from the strategies which were constructed by the pupils in this microworld: automatic area measurement, provided by the environment, the operation of area measurement using spatial units and the use of area formulae. Almost all the pupils experienced qualitative aspects of area measurement through being involved in the process of covering areas using spatial units. Pupils also managed to use the area formulae meaningfully by studying it in relation to automatic area measurement and to area measurement using spatial units. Through these strategies, the concepts of conservation of area and its measurement as well as area formulae were viewed by the pupils as interrelated. Finally, some basic difficulties regarding area measurement were overcome in this computer environment.

KEY WORDS : area measurement, computer microworld, computer tools, secondary education, educational research, geometry, problem solving

INTRODUCTION

The concept of area measurement is a rather complicated one for pupils to grasp as it consists of a network of concomitant related concepts : the conservation of area, the unit and its iteration and the counting of units (Piaget, Inhelder & Sheminska, 1981; Hirstein, Lamb & Osborn, 1978; Hart, 1984). These concepts can be expressed in various representation systems : for example numerical, visual and symbolic. Understanding these concepts is a process of giving meaning to their different representations. This meaning is also influenced by the context in which these representations are expressed.

Previous research has been based on Piaget's work (Piaget, et al.,1981) and has investigated children's thinking on different aspects of area measurement. Recently, the research has considered the role of context and the availability of tools in children's constructions (Kordaki & Potari, 1998a; Nunes, Light, & Mason, 1993). However, in many cases, the concept of area measurement has been considered in a rather fragmented way, and this is reflected in the tools that have been used. These tools focus on specific aspects of area measurement without taking into account its global consideration. At the same time children are not encouraged to develop their own personal tools for area measurement. Moreover, the tools that are used do not support the construction of a variety of representations of this concept. In the school context pupils are forced to express their mathematical knowledge exclusively in symbolic systems such as area formulae. Other representation systems emphasizing intuitive knowledge and area measurement by covering it with spatial units are often overlooked. As a result the pupils do not have the opportunity to make sense of the concept according to their cognitive development nor to express different pieces of knowledge they possess (Hoyles & Sutherland, 1989; Weir, 1992; Lemerise, 1992). Although the research literature has explored the possibility of the computer providing pupils with opportunities to experience multiple representations of a concept (Noss & Hoyles, 1996; Balacheff & Kaput, 1996; Borba & Confrey, 1996), the concept of area measurement has not been investigated.

In our attempt to create an environment for pupils to experience the different aspects of area measurement, we have developed the C.AR.ME microworld (Kordaki & Potari, 1998b). This microworld is an exploratory, open, problem solving environment that offers tools to help pupils construct multiple representations of the concept of area measurement. In this paper, we study the effect of the tools of this microworld on pupils' thinking. More specifically, we explore:

- pupils' area measurement strategies while interacting in the context of the computer microworld;
- the role of the tools that are offered in the computer environment in pupils' constructions.

THE CONCEPT OF AREA MEASUREMENT

The measurement of area is a process of attributing a real positive number to a certain surface. Although the unit of measurement is fundamental in this process, it is usually neglected in mathematics teaching, and the focus is on the numerical result only. So, the spatial properties of the area are not given significance while the measurement operation is reduced to a meaningless manipulation of numbers.

Area measurement is considered by Piaget, et al., (1981, p.3) as "to take out of a whole one element, taken as a unit, and to transpose this unit on the remainder of a whole: measurement is therefore a synthesis of sub-division and change of position." From this point of view, area is equivalent to the sum of its equal parts; in Piaget's words : "when measuring an area we assume, as we do for all measurement, that partial units are conserved and can be composed in a variety of ways to form invariant wholes." (Piaget et al, 1981, p.262). More specifically, the concept of area measurement consists of the concept of unit, the concept of unit iteration, the counting of units, and the calculation of formulae (Hirstein et al., 1978; Piaget et al., 1981; Maher & Beattys, 1986).

The concept of unit is a connective concept involving all the above (Driscoll, 1981). It is built on the unitizing operation which is a mental process of segmenting experience - isolating one aspect of experience while at the same time leaving it embedded in the whole (von Glasersfeld, 1991). This operation is not restricted to only area measurement but is considered central to the construction of other mathematical concepts such as whole numbers, decimals and fractions. The understanding of the spatial characteristics of the unit (Hirstein et al, 1978), its invariability during the measurement process (Carpenter, 1975), its conservation through partitioning and recombining (Hart, 1984) as well as the inverse relationship between the size and the number of units needed to cover the same area (Carpenter, 1975; Carpenter & Lewis, 1976; Cambell, 1990) is essential in the construction of a broader meaning of this concept.

The shape of the unit plays a crucial role in the measurement process. Children tend to use the square (Maher & Beattys, 1986) and the rectangle (Heraud, 1987) as non-standard units of measurement. Pupils usually relate the shape of the units to the shape of the surface to be measured (Heraud, 1987). Besides choosing the unit of measurement, pupils have to iterate units to cover areas without gaps and overlaps, and to count the units. To be successful in the counting process, pupils have to be able to divide units into parts and arrange these parts to make whole units (Hiebert, 1981).

The transition from the spatial consideration of area measurement by using units, to the multiplication approach using area formulae is a complex and non-transparent process (Nunes, et al., 1993; Tierney, Boyd & Davis, 1986). Research shows that children can be successful in using the spatial unit to cover areas while they have difficulties in the appropriate use of area formulae (Bell, Costello & Kuchermann, 1983; Nunes et al, 1993). To fill the gap between the use of units and the use of area formulae demands an ability to relate the visual to the symbolic representations of area measurement. In general, children are able to express their mathematical ideas better in visual representation systems than in symbolic ones (Sutherland, 1995).

A number of studies have attempted to identify and classify the variety of approaches pupils use in tackling the concept of area measurement. These studies show hierarchical levels of their understanding of this concept. In particular, pupils first explore the spatial characteristics of an area to be measured; second, they transform and compare areas by using their sensory-motor actions without the use of numbers; third, they use a variety of units to measure areas and finally they use area formulae (Inskeep, 1976; Driscoll, 1981.) In using units, the progression from the construction of personal units, to informal ones and finally to standard units and metric systems it is essential for pupils to achieve a broader view of the concept of unit (Inskeep, 1976). Through the above processes pupils have opportunities to construct different representations of area measurement, to form abstractions and to construct a global view of this concept.

Pupils face difficulties in measuring areas at primary school level, which may remain when they are in secondary school or university (Tierney et al., 1986; Menon, 1996). They understand area as a multiplication of the lengths of two sides of a polygon regardless of its shape. Moreover, they manipulate the numbers given as lengths or measurement of angles, just to provide a numerical answer (Tierney, et al., 1986; Douady & Perinn, 1986; Hart, 1989). Other difficulties arise when pupils use area formulae to calculate the area of a triangle or a parallelogram. Here pupils consider altitude as the length of one of the sides (Comiti & Moreira-Baltar, 1997). Pupils also reduce area to length by using area and perimeter alternately (Tierney, et al., 1986; Douady & Perinn, 1986). Moreover, pupils use visual perception as a criterion to compare areas (Carpenter, 1976).

Difficulties also arise with the concept of the unit of area. Children often accept the units as points, disregarding their spatial characteristics (Hirtsein, et al., 1978). They also have difficulties in iterating units without gaps and overlaps (Owens & Outhred, 1997). Furthermore, children cannot conserve units from their parts. In some cases, they ignore the fractions of units or count them as whole units (Hiebert, 1981) while, in others, they are known to face difficulties in matching quarter squares to recompose the square unit (Hart, 1989). The inverse relationship between size and the number of units needed to cover an area also presents another difficulty in children's understanding of area measurement (Carpenter & Lewis, 1976).

Difficulties also exist relating to the form of the shapes to be measured. Most of the research literature focuses on problems with the area measurement of typical geometrical shapes such as squares, rectangles, parallelograms and triangles (Johnson, 1986). However, in the study of irregular shapes, fundamental concepts such as the conservation of area and the unit of measurement cannot easily be appreciated and are difficult for pupils to grasp (Maher & Beattys, 1986; Liebeck, 1987).

Pupils' difficulties with area measurement are attributed to different causes. One reason is the gap in children's conceptual development between area measurement using spatial units and area formulae which is infused in the culture (Baturo & Nason, 1996). This gap becomes wider through the premature introduction of area formulae in schools, while the manipulation of area in a qualitative way is overlooked (Rahim & Sawada, 1990; Kidman & Cooper, 1997). Another reason is that the concept of area is not considered in a dynamic perspective which involves focusing on the relationship between the boundary of a shape and the amount of surface that it encloses (Baturo & Nason, 1996).

THE COMPUTER MICROWORLD

The 'Conservation of Area and its Measurement' (C.AR.ME.) microworld (Kordaki & Potari, 1998b) has been designed as a synthesis of three models: a model of learning, a model of subject matter and a model of children's possible actions when they are involved in the conservation and measurement of area. The model of learning is viewed from a constructivist perspective (von Glasersfeld, 1990) as an active, subjective and constructive process. Moreover, social considerations concerning the role of tools in the learning process have been taken into account in the construction of this model (Crawford, 1996a, 1996b; Noss and Hoyles, 1996; Vygotsky, 1978). The model of the subject matter is based on an analysis of the fundamental aspects that constitute the concepts of conservation of area and its measurement as has been reported in previous research. The model of children's possible actions is based on children's sensory-motor actions. These actions are reported in the literature as essential for the understanding of these concepts. The three models have been further discussed in Kordaki & Potari, (1998b).

The microworld has been designed as an interactive exploratory environment to support pupils in actively constructing their own approaches to area measurement. In this context, the teacher can pose a variety of tasks to the pupils, and they can realize a variety of representations of the conservation of area and its measurement. This environment does not provide the 'right' answers but offers visual feedback to the pupils' actions encouraging them to be responsible for their learning process. The microworld has also been designed to help pupils study the concepts of conservation of area and its measurement in a dynamic way as well as to give them the opportunity to bridge the gap between qualitative and quantitative approaches to these concepts.

Figure 1 about here.

The general interface of this microworld including all the provided tools is presented in Figure 1. Concerning area measurement, the tools provided emphasize the covering of areas by units. In this context, different units and grids are offered as tools for measurement. These tools are a *square unit*, a *square grid,* a *rectangular unit*, a *rectangular grid*, a *triangular grid* and a tool to create *personal units* and *personal grids* according to pupils' preferences. A tool to perform the *unit iteration* is also available. The tool for *automatic measurement of units* is also offered to the pupils as a way of verifying their measurement approaches. The *automatic measurement of area* by using the standard units of area is also included. This tool, in combination with the others, gives pupils the opportunity to develop their understanding by relating personal, informal and formal units of area measurement. By providing different units and grids as different representational systems, the pupils are encouraged to give different meanings to the operation of area measurement. Area formulae are not available as tools in this microworld in an attempt to encourage pupils to move from the established quantitative approaches of area measurement to qualitative ones.

Concerning the concept of conservation of area, two groups of tools are provided for the pupils. The tools in the first group are : *copy*, *cut*, *paste*, *rotate* and *symmetry*. By using these tools, pupils can conserve areas by changing their position only or by splitting them into parts and recomposing these parts to produce equivalent shapes. In this way pupils can express their intuitive approaches to the conservation of area in a qualitative and spatial way. This group of tools can be viewed as a way to simulate the pupils' sensory-motor actions in this computer environment. Examples of the use of these tools are presented in Figure 2.

Figure 2 about here

The second group of tools helps pupils automatically to produce a number of shapes equivalent to an area drawn with the tools of the microworld. These equivalent shapes are a *square*, a *rectangle with dimensions 1:2* and *other rectangles* with one dimension drawn by the pupils during their interaction with these tools. *Families of parallelograms* and *triangles* of different form with common bases and equal heights can also be constructed by using the appropriate tools (Figure 3). The pupils can draw the bases of the shapes included in each family using the drawing tools of the microworld. By altering these bases, a number of different families of equivalent shapes can be automatically constructed. By using the features for automatic transformations, the pupils have the opportunity to approach the concept of conservation of area in an intuitive and dynamic way. Such an approach is not possible in the paper and pencil environment.

Figure 3 about here

THE CONTEXT OF THE STUDY

The rationale and the methodology

This study aims to illuminate pupils' strategies to the concept of area measurement developed through their interactions within the context of the C.AR.ME. microworld. It is part of a wider evaluation research designed to investigate these strategies related to the concepts of conservation of area and its measurement in the above context (Kordaki, 1999). This investigation follows a pilot study (Kordaki & Potari, 1998c) which was designed to investigate the appropriateness of the tasks in relation to the age of the pupils and to alter any problematic operations of the software. It is a qualitative study (Cohen & Manion, 1989) as well as adopting a phenomenographic approach to evaluation (Marton, 1988). It takes a bottom-up approach which aims to construct knowledge by using inductive logic (Babbie, 1989).

The experiment

Thirty secondary school pupils from the 2nd grade (14 years old) participated in the study. A familiarization phase with the tools of the computer environment took place before the pupils commenced the main study. The need for this phase emerged from the pilot study, and was realized by asking the pupils to try consecutively all the operations in the menu bar of the microworld. For instance, "draw a polygon", "draw a segment", "clean the screen", "save your work", "select the dot square grid", "rotate a shape", "select a unit", "iterate a unit" were some typical examples of the tasks posed. The aim was to familiarize the pupils with the tools of the microworld and not to get them involved in the measurement process.

The pupils worked individually except in the familiarization phase, where they worked in pairs. They worked in a computer laboratory consisting of three computers where they worked in rotation. Overall, the pupils spent as much time as they needed to perform the given tasks. Each student spent on average about two hours per task. The familiarization phase lasted about 2 hours for each pair.

The researcher, one of the authors, participated as an observer with minimum intervention. All interventions that were realized by the researcher during this study are discussed in reference to the specific cases in the results section of this paper. The whole learning context consisted of the pupils who participated in the study, the researcher and the tasks faced by the pupils.

The data sources relate to the log files of pupils' interactions with the software, the electronic pictures of pupils' drawings, the audio taped discussions and the field notes of one of the researchers.

The tasks

Pupils faced two tasks: the transformation of a non-convex polygon to another polygon with equal area, and the comparison of a non-convex polygon with a square. All the shapes to be studied were drawn on the computer screen by the researcher, and the instructions were verbally presented to the pupils. In the transformation task, the pupils were asked to: ‘transform this polygon to another polygon with equal area in any possible way’. Where pupils used different units or grids to perform this task pupils were asked: ‘what is the relation between the size and the number of units you used ?’. In the comparison task, the area of the two shapes was not easily comparable by eye. In this task the pupils were asked to: ‘compare this polygon to that square in any possible way’. Where pupils used different units or grids to perform this task in different ways, they were asked the following question: ‘What do you think about the effect of the different units or grids on the comparison result ?’. The above tasks have been considered, by other researchers, as essential for pupils to build concepts regarding the conservation of area and its measurement (Carpenter, Coburn, Reys, & Wilson, 1975; Hiebert, 1981). However, pupils' thinking about area measurement of a non-convex polygon has not been investigated up to now. The tasks given encouraged each pupil to construct their own individual approaches to the related concepts as well as to express different views of them by allowing and asking for a variety of solutions (Weir, 1992; Lemerise, 1992).

The process of analyzing the data

The various types of data were organized according to the two different tasks. In each task individual pupils' multiple-solution strategies were identified and reported. These strategies were analyzed in terms of their conceptions of area measurement as these developed during the experiment. At the next stage of the analysis, where the focus was on the pupils as a group, the strategies were categorized in terms of the kind of tools used. Finally, the pupils’ area measurement strategies across categories were discussed as well as the role of the provided tools in the construction of these strategies.

RESULTS

In this computer environment, the pupils constructed a variety of solution strategies to the given tasks. To develop these strategies, they were encouraged by the nature of the tasks, which asked them to consider solutions 'in any possible way'. They were also supported in inventing and performing all these strategies using a variety of tools from different contexts. In particular, the pupils used independently or in combination, the tools provided by the C.AR.ME. microworld, as well as the area formulae from existing school knowledge. They also estimated visually the areas under study. In the following section we present and discuss: A) the categories of pupils' area measurement strategies B) pupils' strategies across categories.

A) Categories of pupils' strategies

Eleven categories of strategies were formed with the criterion being the tools used by the pupils. These categories are presented in Table 1. In this table the number of pupils who performed strategies in each category is also presented. The pupils' strategies are presented as they were performed by the pupils. The fact that most of these strategies were correct is possibly due to their having the opportunity to select from a variety of tools those that allowed them to express their own meaningful approaches. However, pupils' difficulties are discussed in reference to the specific cases that appeared. Moreover, the strategies that fall in each category were discussed in relation to those pupils who performed them.

Table I about here

C1: Comparing and transforming areas by 'eye'. In this category, pupils compared or transformed the given areas by using their visual perception. This approach, when accompanied by appropriate judgments, may illuminate pupils' abilities in estimating area. However, pupils who used this strategy could not give any explanation for their decisions. So, this approach can be interpreted as a 'primitive' approach to area measurement.

C2 : Comparing and transforming areas by using the perimeter of the shapes. By using this strategy, pupils (6) were possibly expressing an additive approach to area formulae or there was confusion between area and perimeter (Hart, 1984). To help pupils overcome this problem, the researcher drew two shapes on the screen with inverse relationships between their areas and their perimeters. Then pupils were asked to measure automatically the areas as well as the perimeters of the drawn shapes. This intervention led these pupils to a conflict, and finally to infer that : "*the area and the perimeter of a shape are independent*", '*the area and the perimeter of a shape are not the same*', '*even though this shape has a bigger perimeter than the other one it has a smaller area*', ' *I realized that the area and the perimeter of a shape are different*', '*this shape has a smaller area than the other but it has a bigger perimeter*'.

C3: Comparing areas by using the tool for automatic area measurement. Pupils' strategies in this category are presented in Figure 4 .

Figure 4 about here

Most pupils automatically measured the areas of both shapes, and compared them by using the numerical results of measurement. They used this strategy as their first attempt to compare areas, which probably signifies the pupils' preference for operations that were easily performed. During the measurement process, a number of pupils assumed that the units implied in the automatic measurement operation were the standard units of measurement ($1cm^2$). This appears to indicate a tendency to refer to school knowledge by relating the non-standard to standard units of measurement. Nevertheless, the pupils who split the areas under study into convex non-overlapping parts expressed an understanding of the areas as the sum of their subsets . Pupils often tended to split the non-convex polygon to common convex geometrical shapes to overcome its irregularity. Triangles were the most common shapes in which the initial shapes were split, as possibly their drawing on the screen did not require any use of specific rules or properties. Splitting the non-convex polygon also appeared in other categories of strategies that follow.

C4 : Comparing areas by using the tool for automatic area measurement in combination with specific simulations of pupils' sensory-motor actions. Pupils performed the strategy presented in Figure 5.

Figure 5 about here

In order to superimpose the non-convex polygon onto the square, the pupils constructed its symmetrical shape. They realized this by estimating visually the position of an axis of symmetry so that the symmetrical shape of this polygon would be superimposed onto the square. This strategy indicates that pupils can recognize the conservation of the area of a shape after its transformation by using the symmetry operation. Pupils who performed the above strategy (four pupils), also expressed an understanding of area as the sum of its non-overlapping sub-parts. They also seemed to appreciate the comparison of areas by comparing their non-common parts.

C5: Comparing areas by using the tool for automatic area measurement in combination with the tool for automatic transformations. In this category, the pupils adopted the strategies presented in Figure 6.

Figure 6 about here

By realizing the above first two strategies, pupils expressed a preference for comparing areas by measuring their equivalent shapes of the same form. In the third strategy, the pupils transformed the given shapes to different standard geometrical ones. Pupils' strategies in this category possibly indicate an understanding of the concept of conservation of area after having completed a number of transformations. In our view it also implies an understanding of the property of transitivity in relation to area measurement.

C6 : Transforming and comparing areas by using the tools that support the operation of area measurement using spatial units. Pupils' strategies which fall in this category are presented in Figure 7.

Figure 7 about here.

Pupils who performed strategies in this category were actively involved in the operation of area measurement by using the tool of *iterating units*. Most of them performed the iteration of units correctly. Moreover, those (5) pupils who performed this process with gaps and overlaps were helped to repeat it correctly by the researcher's prompt to '*think again'* about their approaches as well as by their reflecting on the visual feedback of their attempts. As they admitted to the researcher, by observing the drawings on the screen they realized that "*something was wrong*" with their approach. This realization helped them to reflect on their actions and improve their strategy. Pupils also faced difficulties in counting the units needed to cover the area of the shapes. All the pupils involved in these strategies correctly counted the whole units placed inside the shape; however, the difficulty of some (8) of the pupils was to estimate the parts of the units needed to cover the remaining shape. Some pupils ignored these parts, while others counted each part as a whole unit. This indicates pupils' difficulties in considering a unit as the sum of its parts and recomposing units from their parts. However, these difficulties were overcome by the pupils' attempt to evaluate their counting processes. Some pupils (5) measured both the area of the shape to be measured and the area of the chosen unit automatically, and by dividing the measurement results, these pupils evaluated their counting approaches. As a result the difference they found between their counting result and the one given by the division above forced them to re-consider the partial units. They subsequently started to recompose units from their parts and repeated the counting process. Other pupils (3) were encouraged to repeat this process more accurately by intervention of the researcher who asked them to justify their counting results.

The following table (Table II) shows the measurement units used by the pupils in relation to the number of strategies they developed. As shown in this table, the square unit or the square grid were used in most strategies while the rectangular units and grids were used in fewer strategies. Here, pupils experienced a deeper knowledge of area units. Six strategies involved the use of personal units while four used personal grids. Only in a few strategies were two units or two grids used by the pupils simultaneously.

Table II about here

Pupils who constructed personal units used the *student unit* tool to draw triangular, rectangular and square units. The triangular units were a right-angled isosceles and an arbitrary triangle. These units were used by the pupils as a more flexible means of covering the non-convex polygon but, after experiencing gaps and overlaps in the iteration process, they moved to rectangular or square units.

Figure 8 about here

As regards personal grids some pupils chose to construct these to measure the given areas while other pupils used these grids to enclose the given shapes in their cells (Figure 8). In this way, these latter pupils attempted to compare the areas of the shapes by comparing their complementary areas in relation to the cells in which they were enclosed. A few pupils also constructed two different personal grids, one over the other, since they were transparent. The size of the cells of these grids was in proportion so they were able to measure areas more accurately. Moreover, two different personal units were used simultaneously in order to achieve accuracy. The process of using two different units or two grids simultaneously is essential in the construction of the concept of a metric system. Some pupils also expressed the desire to construct a right triangular grid but this was not available in this computer environment.

Table III about here

Table III shows the pupils' strategies across the units related to the number of pupils who performed them. As indicated in this table, only one pupil did not use the operation of area measurement using area units or grids. Twenty-five pupils (the sum of the fourth and sixth rows) performed the operation of area measurement by covering areas with units while ten of these used different units. Furthermore, 26 pupils (the sum of the fifth and seventh rows) performed the measurement operation by using grids, thereby overcoming the problems of the iteration of units. Eight of these pupils used more than one different grid. Eleven pupils used the square grid exclusively, as it was a tool familiar to them from their school practices, while only four used the square unit exclusively. Twelve pupils used the square unit first but they moved to the use of the square grid as they probably considered it as a more convenient tool in the measurement process. Only three pupils, starting from the square grid were led to the square unit. In this way they started from a familiar tool and attempted to explore other units of area measurement.

Pupils who used more than one unit or grid were asked by the researcher about the relation between the size and the number of units required to cover the same area. From their responses, these pupils seemed to realize this inverse relationship. Pupils' typical responses were : "*when we use larger units, the number needed is smaller*", '*when we used the square unit the number needed is larger'*, '*when we used the rectangular unit the number needed is smaller'* (the square unit was smaller than the rectangular).

Finally, a number of pupils (17 pupils) used the automatic area measurement operation as a tool to evaluate the results of their transformation and comparison strategies based on the covering of areas by using units.

C7 : Comparing areas by using the tools that support the operation of area measurement using spatial units in combination with the tools for automatic transformations. Pupils performed the strategies indicated in Figure 9.

Figure 9 about here

Pupil's strategies in this category indicate an understanding of the concept of area measurement in relation to its conservation through a number of automatic transformations to equivalent areas. More specifically, the pupils who performed these strategies, seemed to appreciate the conservation of an area after changing its position. They also seemed to accept that different equivalent shapes produced automatically, conserve their area. Moreover, these pupils seemed to grasp the operation of area measurement by using area units. They also expressed an appreciation of the transitivity property by comparing the given shapes through their equivalent ones of the same form automatically produced by the computer. The comparison of these equivalent shapes was done by the use of units or grids of the same form as the transformed areas. For example, some pupils automatically transformed the initial shapes to their equivalent rectangles. To compare the areas of the initial shapes, the pupils compared the area of these rectangles measuring them using rectangular units. Some pupils also covered by area units both dimensions of the automatically produced equivalent squares or rectangles. They then counted the number of units needed to cover each dimension and multiplied them to calculate the total number of units needed for covering the area of these shapes.

Finally, when pupils used different units to compare areas they seemed to grasp that areas are not affected by the size of the units used : "*the comparison result does not depend on the size of the unit you used*", '*when we use square or rectangular units to compare these areas the result is the same'*, '*we used different units to measure these areas but the non convex-polygon looks larger than the square in all cases'* are examples of pupils answers to the researcher's question : 'What do you think about the effect of the different units or grids on the comparison result ?'

C8: Transforming areas by using area formulae. One pupil performed the strategy shown in Figure 10.

Figure 10 about here

The pupil attempted to apply his school knowledge in the computer environment. Here, he used the area formulae appropriately to calculate the areas of the square and the trapezium - the shapes in which the non-convex polygon was split. However, he faced difficulties in drawing the side of the equivalent square on the computer screen as he couldn't interpret visually the length of its side. This side had been calculated as the square root of the area of the non-convex polygon and it was expressed in the units used for automatic length measurement. The pupil could not make these length units correspond to a segment.

C9: Comparing areas by using area formulae in combination with the simulations of pupils' sensory-motor actions. One pupil performed the following strategy shown in Figure 11.

Figure 11 about here.

By performing this strategy, the pupil integrated analytic and intuitive approaches; area formulae and the simulations of pupils' sensory-motor actions, respectively. The intuitive component acted as a means of comparing areas. The pupil also demonstrated a need to compare areas of the same form. To draw the side of the square which was equivalent to the non-convex polygon, the pupil attempted to estimate the length of its side. This side had been calculated as the square root of the area of the non-convex polygon and it was expressed in the units used for automatic length measurement. The pupil needed to make these length units correspond to a segment. To realize this, he brought onto the screen the square grid and measured automatically the length of the side of its unit. Then, he divided the length of the side of the square by that of the unit of the square grid. This ratio expresses the measure of the side of the square when the unit of measurement is the side of the unit of the square grid. The pupil used this ratio to draw the transformed square by using the square grid. Through this experimentation, the pupil traced the relation between the units of different measurement tools, such as the side of the square grid, and the units used for automatic length measurement. By performing this strategy the pupil had the opportunity to get a visual sense of the length of the unit of the automatic measurement tool.

C10: Comparing areas by using area formulae in combination with the tool for automatic area measurement. One pupil performed the strategy shown in Figure 12.

Figure 12 about here

Here, the pupil correctly applied the area formulae to calculate the area of the square. This strategy indicates that this pupil understood the area as the union of its non-overlapping subsets and the concept of conservation of area after splitting and re-composition.

C11: Transforming areas by using area formulae in combination with the tools that support the operation of area measurement using spatial units. Figure 13 illustrates the strategies adopted by two pupils.

Figure 13 about here.

The pupils measured the area of the non-convex polygon by viewing it as a whole area but also as the union of its convex subsets. In the first case, these pupils used a variety of measurement tools such as the square unit and the square grid, as well as the rectangular and student's grid. In the second case, they used only the square grid. Besides the process of counting units, these pupils also attempted to transform the non-convex polygon to an equivalent shape. To do this they used the number of units that resulted from the counting process and the area formulae of the square to form an equation. By solving this equation, they calculated the length of the sides of the equivalent shapes. When they used the square unit, the produced shape was a square, and using as a length unit the side of the square unit they drew its side. In using a rectangular unit, the produced shape was an equivalent rectangle. To draw the width and the length of this rectangle, the pupils used as length units the width and the length of the rectangular unit respectively. In Figure 14, two examples are shown in respect of the above strategies.

Figure 14 about here.

On the left part of the figure, the area of the non-convex polygon is 25 square units. So, $25 = x^2$ is the equation formed where x is the size of the sides of the equivalent shapes and its solution is 5 units. The side of the equivalent square was produced by iterating five times the square unit. On the right part of the figure, the area of the non-convex polygon is about 36 rectangular units. So, $36 = x^2$ and $x = 6$ is the length of each dimension of the equivalent rectangle. The sides of this rectangle were six times the corresponding sides of the rectangular unit. This rectangle was produced by iterating the rectangular unit. Finally, to verify these strategies, these pupils used the automatic area measurement operation to measure the area of the shapes before and after the transformation.

By developing the above strategies, these pupils stayed in the context of area measurement by using area units although they used the area formulae as well. Moreover they were able to move from area to length units and vice versa through area formulae. In our view these strategies indicate that these pupils made a connection between the area measurement, by using units of area, and area formulae. They also extended the meaning of the area formulae from a manipulation of the linear elements of the shapes to a consideration of the area itself.

B) <u>Pupils' strategies across categories</u>

Pupils performed 328 strategies to complete the tasks of transformation and comparison. Of these strategies 154 refer to the concept of area measurement and have been presented and discussed in this article. The remainder refer to the concept of conservation of area and will be reported in a further paper (Kordaki, in preparation). In Table IV, the categories of the measurement strategies for each task are presented in terms of each individual pupil. The numbers in each cell indicate the order of the performance of the specific strategy. So, a sequence of numbers corresponds to each pupil. The missing numbers, which can be observed in some sequences, refer to strategies concerning the concept of conservation of area. For example, pupil 12 (P12) performed his first comparison strategy by using the automatic operation of area measurement, his second by using the perimeter of the shapes, the third by covering the areas of these shapes using area units and the fifth by using the area formulae. The third strategy in the comparison task and the fifth strategy in the transformation task were in the same category (C6). The last column of the table shows the total number of measurement strategies per pupil. The last row of the table shows the number of pupils who performed strategies that fall in each category. The row above the last presents the number of strategies for each category.

Table IV about here.

As it is shown in Table IV the most common strategies performed by the pupils were first, the automatic measurement of area (23 pupils performed 28 strategies in the comparison task), and second, the covering of areas by units (25 pupils performed 49 strategies for the comparison task and 19 pupils performed 40 strategies for the transformation). In our view the first strategy indicates a preference of the pupils for easily delivered strategies while the second one shows that the pupils still need to be engaged in the process of covering areas by using units, despite the fact that they have been introduced to area formulae from primary school. The number of strategies is different from the number of pupils as each pupil performed more than one strategy in each category.

In Table IV the categories C4, C5, C7, C9, C10, C11 consist of strategies developed using a combination of tools. Seventeen pupils performed 25 strategies of this type. These pupils managed to combine the variety of tools offered by this microworld and from their previous knowledge. To perform strategies by combining the tools, the pupils had to decide which tools to choose, also to develop a solution plan and not simply to use a specific tool. In this way, the pupils integrated different aspects of area measurement and showed an appreciation of their interrelationship.

Concerning the order of the appearance of the strategies during the comparison task, most pupils (19), solved this task by first using the automatic area measurement tool. The use of area units, was the second strategy used by a number of pupils (13) while for seven pupils it was their first. Looking closely at pupils' transition through their strategies to area measurement to complete the comparison task, one basic path emerged. This path started with the strategies that used the tool for automatic area measurement (Categories C3, C4, C5) and went on to the operation of area measurement using spatial units (Categories C6, C7). Seventeen pupils (P4, P5, P10, P11, P13, P14, P17, P19, P20, P21, P22, P23, P24, P25, P28, P29, P30) followed this path. For most of them (16), these were the only approaches to area measurement that they developed to complete the comparison task. Pupils' transition from the use of automatic area measurement to the use of spatial units to measure areas indicates pupils' pursuit for more than one solution to the task. It also indicates that the tools of the microworld were appropriate to meet the pupils' cognitive needs.

However, the above path was followed by pupil P1 except that he used his visual perception to compare areas as his first strategy. This path was also followed by pupils P12 and P18 except that it included the use of the perimeter of the shapes as a strategy to study their area. The behavior of these pupils indicates that correct or wrong approaches to area measurement can coexist in pupils' minds. In addition, it indicates that pupils can approach area measurement correctly in spatial representation systems while they can have problems in symbolic ones.

Moreover, there were a number of pupils (P2, P3, P9, P15, P16, P26) whose strategies remained stable in the categories (C6 and C7) that used the operation of area measurement using spatial units.

Combined strategies that used area formulae (strategies in categories C8, C9, C10 and C11) were developed latter on during the process of pupils' involvement in both of tasks. These strategies were followed by five pupils (P6, P8, P14, P26, P27) and indicate pupils' mathematical development concerning area measurement to a higher level.

DISCUSSION

The pupils in this computer environment developed a variety of strategies to the concept of area measurement. The nature of the given tasks, asking the pupils to consider them "in any possible way', encouraged this development. At the same time the variety of tools provided by the C.AR.ME. microworld, as well as the area formulae from pupils' existing school knowledge supported pupils' constructions used independently or in combination. In this section we discuss a) the pupils' approaches to area measurement constructed in the context of this microworld b) the role of tools on pupils constructions during this experiment.

a) Pupils' approaches to area measurement in the context of the C.AR.ME microworld

The reported approaches to area as they emerge from the literature start with the pupils' visual perception of areas, are followed by area manipulation with pupils' using sensory-motor actions, and are developed through the operation of area measurement. The latter is made by using units to cover areas and subsequently employ area formulae (Piaget, et al., 1981). These approaches are characterized, respectively, as primitive, intuitive, operational and analytical (Piaget, et al., 1981). The categories of pupils' strategies performed in the context of the C.AR.ME microworld express pupils' learning on area measurement in this context. These categories are discussed below in five groups, taking into account both these Piagetian approaches and the approach which uses the operation of automatic area measurement. The first group refers to those categories that express primitive approaches to area measurement, the second to the ones that use area and perimeter interchangeably, the third to those that emphasize the automatic operation of area measurement, the fourth to those categories that stress area measurement using spatial units, and the fifth to the ones that emphasize the use of area formulae.

The 'eye' approach to area. This approach (reported in category C1) is characterized as primitive by the literature (Piaget, et all. 1981) as it is exclusively based on pupils' visual estimation of the areas under study. Pupils are encouraged to move from this approach to other more sophisticated ones by taking advantage of the variety of the tools provided by this microworld.

The 'perimeter' approach to areas. This approach (reported in category C2) shows an additive approach to area or confusion between area and perimeter and is reported as one of the basic difficulties pupils have in area measurement (Tierney, et al., 1986; Douady & Perinn, 1986). Pupils were helped to overcome this difficulty in the context of this microworld by the intervention of the researcher who automatically measured the perimeter and the area of the shape leading pupils to infer that 'area and perimeters are different entities'.

The 'automatic area measurement' approach. Most pupils used the tool for automatic area measurement to construct their initial solution strategy to the comparison task (reported in category C3). In our view this approach illuminates pupils' preference for fast and easily delivered solutions. However, by asking pupils to justify the automatically produced results of measurement or to produce more than one solution strategy, we may encourage them to consider more deeply the process of area measurement. Although pupils automatically measured whole areas under study, they also used this operation to measure the area of the non-convex polygon in a process aiming at the overcoming of its irregularity. This process was performed by the pupils in three ways: by splitting it in convex shapes (reported in category C3), by automatically transforming it to other equivalent non irregular geometrical shapes (reported in category C5) and by superimposing it on to the square (reported in category C4). In the first two ways the produced shapes were measured automatically while in the third one the pupils automatically measured the non common parts of the superimposed shapes so they could compare them. All the previous ways regarding the non-convex polygon's irregularity indicate an understanding of the concepts of conservation of area and its measurement as interrelated as well as an understanding of the concept of transitivity in the process of area measurement. The use of automatic transformations also indicate a preference of the pupils for measuring areas of the same form. Although the automatic performance of area measurement gives a numerical and opaque character to the measurement process, the use of this tool in combination with the other tools provided by this microworld gives an active, spatial and qualitative character to the whole process of measurement.

The 'operation of area measurement using spatial units' approach. This approach expresses an active, spatial and qualitative approach to the concept of area measurement (reported in the category C6) and supports pupils' conceptual development of this concept (Piaget et all., 1981; Rahim & Sawada, 1990; Baturo & Nason, 1996; Kidman & Cooper, 1997). These strategies were the most common during this experiment. This tells us that while pupils have been introduced to area formulae in school, the majority of them did not use this method to measure areas but preferred to measure areas in a spatial way. Pupils used a variety of the provided units as well as constructing their own units to cover the areas under study. Moreover, pupils tried to identify relations between the non-standard units provided by this computer environment and the standard ones. By experimenting with a variety of units or grids, the pupils enriched their view about the concept of area measurement. They also verified the inverse relationship between the size and the number of units needed to cover a specific area which is reported as essential in the understanding of the concept of measurement (Carpenter & Lewis, 1976). By constructing their own units or grids pupils were helped to progress from their personal units to standard ones, a process that is reported by Inskeep (1976) as essential in the understanding of the concept of unit. Here, pupils also realised the constraints that the form of the unit imposes on the iteration process. Moreover, they seemed to appreciate the fact that the result of the comparison of two areas is not affected by the size of the unit they measured. Pupils also constructed the concept of sub-division of a unit or a grid in order to measure an area more accurately. This concept is important in the construction of the concept of a metric system and is performed easily in this environment as the various units or grids are transparent and can be easily superimposed. Pupils overcame the reported difficulties (Hiebert, 1981; Hart, 1989) regarding the re-composition of units from their parts in order to count them more accurately, by using the tool for automatic area measurement. This tool was used to measure both the whole areas and the used units. Here, pupils divided the results of these measurements and then, they used the result of this division as a means to verify their counting approaches. Pupils also used the iteration of spatial units to measure the automatically transformed equivalent areas of those given (reported in the category C7). Here, pupils' difficulties regarding the non-convex polygon's irregularity (Johnson, 1986; Maher & Beattys, 1986; Liebeck, 1987) were overcome by performing these transformations. Moreover, the above approach shows pupils' tendency to measure areas with units of the same form (Heraud, 1987). Pupils also viewed the concepts of conservation of area and its measurement as interrelated (Piaget et all., 1981; Rahim & Sawada, 1990; Kidman & Cooper, 1997) when they constructed the previously mentioned strategies.

The 'area formulae' approach. Here, pupils transferred their previous school knowledge to this computer environment. In particular, the pupils' solution strategies which used this approach can be seen as solution plans consisting of the following steps : first the overcoming of the non-convex polygon's irregularity, second the measurement of areas under study and third, the transformation and/or the comparison of these areas. Another step, the drawing of the transformed shape on the screen of the computer, emerged in the transformation task. This sequence of steps was rather typical of all the categories involving area formulae except in category C11 where the first step did not appear.

In the first step, pupils overcame the non-convex polygon's irregularity by splitting it in parts (categories C8, C9, C10). In the second, pupils measured these parts by using the automatic area measurement (category C10) and area formulae (categories C8, C9). Area formulae was also used for the measurement of the given square (category C10). Pupils also used the operation of area measurement using spatial units to measure the area of the non-convex polygon (category C11). In the third step, the produced results from the previous phase were used for the comparison of the given areas (category C10). In the same step the transformation of the non-convex polygon to its equivalent square was performed by using its area formulae (categories C8, C9, & C11). In this phase the area formulae of the square was equated to the area of the non-convex polygon and so the side of its equivalent square was calculated. To draw this side the pupils faced two different problems: to interpret, in appropriate length units, the numerical results produced by this area formulae, and to express the invisible units of automatic measurement of length in a visual way. In the first problem (category C11) the implied length unit was the side of the square unit used to measure the area of the non-convex polygon. With reference to this side pupils drew the side of the equivalent square on the computer screen. Here, pupils managed to construct relationships between spatial and analytic approaches to area measurement. In the second problem, some pupils (those who performed the strategies reported in category C9) managed to trace the relation between the units of different measurement systems while others did not (these pupils performed the strategies reported in category C8). These units were, the unit of automatic measurement of length and the length of the side of the square grid provided by this microworld. In the end, the pupils drew the equivalent square on the computer screen.

Although the pupils usually view area formulae as a meaningless calculation process (Douady & Perrin, 1986; Hart, 1989) this was not generally the case in this computer environment. Here, the strategies constructed by the pupils indicate an interrelation between analytical approaches such as the use of area formulae, intuitive approaches regarding the concept of conservation of area, spatial approaches using units to cover areas and the fast and easy automatic operation of area measurement provided by this microworld.

The variety of strategies pupils constructed illuminates a basic transition path. This started with the strategies that used the tool for automatic area measurement and went on to the operation of area measurement using spatial units. In our view it indicates pupils' attempt to construct more than one solution strategy as well as the fact that the tools of this microworld were appropriate to support this tendency. In some cases we also observed that correct and wrong approaches to area measurement can coexist in pupils' minds. The correct approaches relate to area measurement using spatial units while the wrong ones relate to the use of the perimeter. In our view it indicates that pupils can express their knowledge of area measurement better in spatial systems than in symbolic ones.

b) The role of the tools

The variety of tools, the area measurement meanings and the pupils' cognitive needs

The variety of different tools provided by this computer environment supported each individual pupil in constructing a number of solution strategies to area measurement giving different meanings to this concept. In this way each pupil had the opportunity to construct a broader view of the concept as well as to make connections between different pieces of knowledge they possess. Moreover, the variety of the provided tools gave pupils the opportunity to experience the concept of area measurement according to their cognitive development. In this way the pupils had the opportunity to select among different tools those that would enable them to construct their own approaches to area measurement and so they were not forced to use the area formulae exclusively. This seems to have been responsible for the majority of constructed solution strategies being correct. The availability of a variety of different tools also supported the pupils in progressing from primitive strategies (using 'eye') to more sophisticated ones, and in progressing from wrong strategies (using the 'perimeter' of the shapes) to correct ones.

Relations between tools and strategies

The nature of each tool affected the solution strategies performed by the pupils (Nunes et all, 1993; Vygotsky, 1978; Noss & Hoyles, 1996). For example, when pupils used spatial units to measure areas they gave spatial meanings to the concept of area measurement, while when they used the automatic area measurement they seemed to view it as an opaque "quick and easy" operation performed by the system and focused mainly on its numerical results. When pupils used qualitative and quantitative tools in combination, they constructed 'semi-qualitative' meanings to the concept of area measurement. Moreover, when pupils used a combination of tools that support the concepts of conservation of area and its measurement they viewed these concepts as interrelated.

The role of specific tools

The tool for automatic area measurement. This tool was used by the pupils to construct solution strategies to the given tasks but also as a tool for self-evaluation of their transformation strategies. Moreover, it helped pupils to improve their counting of unit approaches by taking into account the parts of the units and recomposing the whole units. Used in combination with the tool for automatic measurement of length it helped pupils to overcome the confusion between area and perimeter.

The tools for automatic transformations. These tools were used by the pupils to overcome the non-convex polygon's irregularity as well as to study areas of the same form. By using these tools pupils experienced the concept of conservation of area in a dynamic way.

The tools for measuring areas using spatial units. These tools helped the pupils to express their mathematical knowledge in spatial representation systems giving a qualitative character to the concept of area measurement.

The tools that simulate pupils' sensory-motor actions. Pupils used these tools to superimpose the shapes under comparison. In this way pupils compared the non-common parts of the given shapes overcoming the non-convex polygon's irregularity. Using these tools pupils experienced qualitative and spatial understanding of the concept of conservation of area after splitting it in parts and recomposing these parts to produce equivalent shapes.

The role of visual feedback. Most strategies constructed by the pupils were visually supported, as the majority of the tools in this environment provided intrinsic visual feedback. This visual feedback helped pupils to reflect on their strategies and overcome their difficulties concerning the iteration of units without gaps or overlaps (Owens & Outhred, 1997).

CONCLUSIONS

The present study demonstrates that a learning environment providing various tools that can support different representations of the concept of area measurement and asking pupils to solve tasks 'in any possible way' stimulated the pupils to construct a number of solution strategies to this concept. In this environment all pupils expressed a variety of approaches to area measurement and enriched their views concerning this concept. They also progressed from primitive approaches to more advanced and from wrong approaches to correct ones. Pupils also constructed representations concomitant with their cognitive development through the selection of the appropriate tools. As a result, most of the produced strategies were correct. Moreover, all pupils were engaged in the measurement process and constructed more than one solution strategy. Our observations demonstrate that the nature of tools used affected the nature of solution strategies that the pupils constructed.

In this learning environment three main approaches to area measurement were developed by the pupils and applied to the area of the non-convex polygon. To overcome its irregularity pupils developed specific procedures. In this way the concepts of conservation of area and its measurement were viewed as interrelated.

One of the measurement approaches stressed the use of automatic operation of area measurement and was the initial one used by the pupils. It shows a tendency to focus on the numerical results of measurement produced by the computer. The tool for automatic area measurement acted constructively as an evaluation tool despite its limitations in the process of developing meaningful solution strategies. As this tool was used in combination with the tool for automatic measurement of length, it helped the pupils to overcome the confusion between area and perimeter. This tool also helped pupils to construct semi-qualitative meanings to area measurement.

The second measurement approach emphasized the ‘operation of area measurement using spatial units’. Here, the pupils were actively involved in the process of area measurement, they focused on its spatial characteristics and attributed to it a personal, qualitative, visual and approximate meaning. Most pupils involved in this approach demonstrated that they could express their mathematical knowledge about area measurement better in spatial representation systems than in symbolic ones. Basic difficulties regarding area measurement, reported by the literature, did not present themselves, namely, the inverse relationship between the size and the number of units needed to cover a specific area. This is probably due to the fact that pupils experimented with the variety of the units provided. Other basic difficulties, namely, the iteration and the counting of units were overcome by taking advantage of the visual feedback and the automatic operation respectively. The concept of sub division of units, was also approached by the pupils in this context.

The third measurement approach emphasized the use of area formulae. Few pupils became involved in this approach as the constructed strategies were more advanced. Doing the process of using area formulae in this computer environment they looked more deeply into the relation between units of different measurement systems of length. Pupils also managed to bridge the gap between area measurement using spatial units and area formulae. In this way pupils gave qualitative and spatial meanings to area formulae connecting these symbolic representations to the spatial ones.

We conclude with a brief note on the implication of the study for teaching the concept of area measurement. The analysis of pupil's work illuminated the possibilities of the tools of the microworld providing pupils' with the opportunity of constructing a variety of approaches to area measurement, of building interrelationships between these various aspects of the concept and overcoming certain difficulties. It also shows the important role of the tasks that the teacher can plan to support pupils' involvement in this environment. Hence it may help teachers give answers to the following questions: a) how can we encourage pupils to construct their own approaches to the concept of area measurement, to rethink their conceptual map and to make connections between the pieces of knowledge they have b) how can we help pupils to overcome fundamental difficulties regarding this concept and c) how can we help pupils to learn area formulae not as a meaningless operation of numbers Further research is needed to investigate the role of the teacher and more specifically his or her interventions both at an individual level and in the classroom while exploiting this particular microworld.

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CAPTIONS

Figure 1. The general interface of the C.AR.ME microworld

Figure 2. Examples of the use of tools that simulated the pupils' sensory-motor actions

Figure 3. Examples of the use of tools for automatic transformations

Figure 4. Pupils' strategies based on the use of the tool for automatic area measurement

Figure 5. Pupils' strategies based on the use of the tool for automatic area measurement and on the use of simulations of pupils' sensory-motor actions

Figure 6. Pupils' strategies based on the use of tools for automatic transformations and on the use of tool for automatic area measurement

Figure 7. Pupils' strategies based on the use of tools that support the operation of area measurement using spatial units

Figure 8. An example of constructing a personal grid to enclose the given shapes

Figure 9. Pupils' strategies based on the use of tools that support the operation of area measurement using spatial units and on the use of tools for automatic transformations

Figure 10. Pupils' strategies based on area formulae

Figure 11. Pupils' strategies based on area formulae and on the simulations of pupils' sensory-motor actions

Figure 12. Pupils' strategies based on the use of the tool for automatic area measurement and on area formulae

Figure 13. Pupils' strategies based on the use of tools that support the operation of area measurement using spatial units and on area formulae

Figure 14. Two examples of strategies based on the operation of area measurement and on the area formulae

Table I. Categories of pupils strategies in the context of the C.AR.ME microworld

Table II. The use of the tools for visual and active area measurement

Table III. Pupils' strategies across the units of area measurement

Table IV. Pupils' measurement strategies across the categories

File	**Draw**	**Edit**	**Measuring**	**Automatic Transformations**	**Tools**	**Help**
Open	Dot Square Grid	Select Part	Areas	Square	Square unit	
Open Last	Dot Triangle Grid	Select All	Angles	Rectangles	Rectangular unit	
Save Last	Draw Polygons	Cut	Segments	Families of Rectangles	Student unit	
Save As	Draw Segments	Paste		Parallelograms	Unit iteration	
Print	End Draw Polygons	Draw an angle of Rotation		Families of Parallelograms	Counting of units	
Exit	Clear	Rotate		Triangles	Square Grid	
		Draw an axis of Symmetry		Families of Triangles	Rectangular Grid	
		Symmetry about axis		Show numerical elements	Student Grid	
		Erasers				

Figure 1

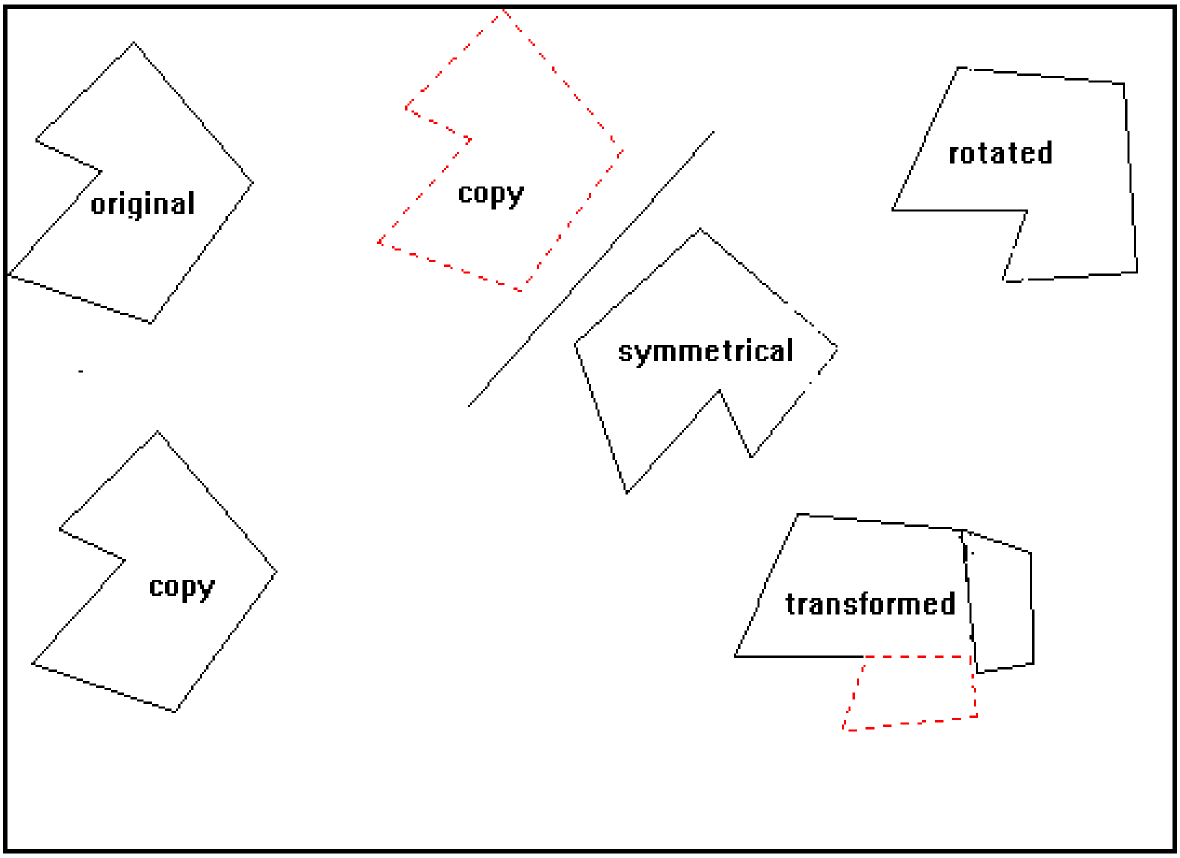

Figure 2

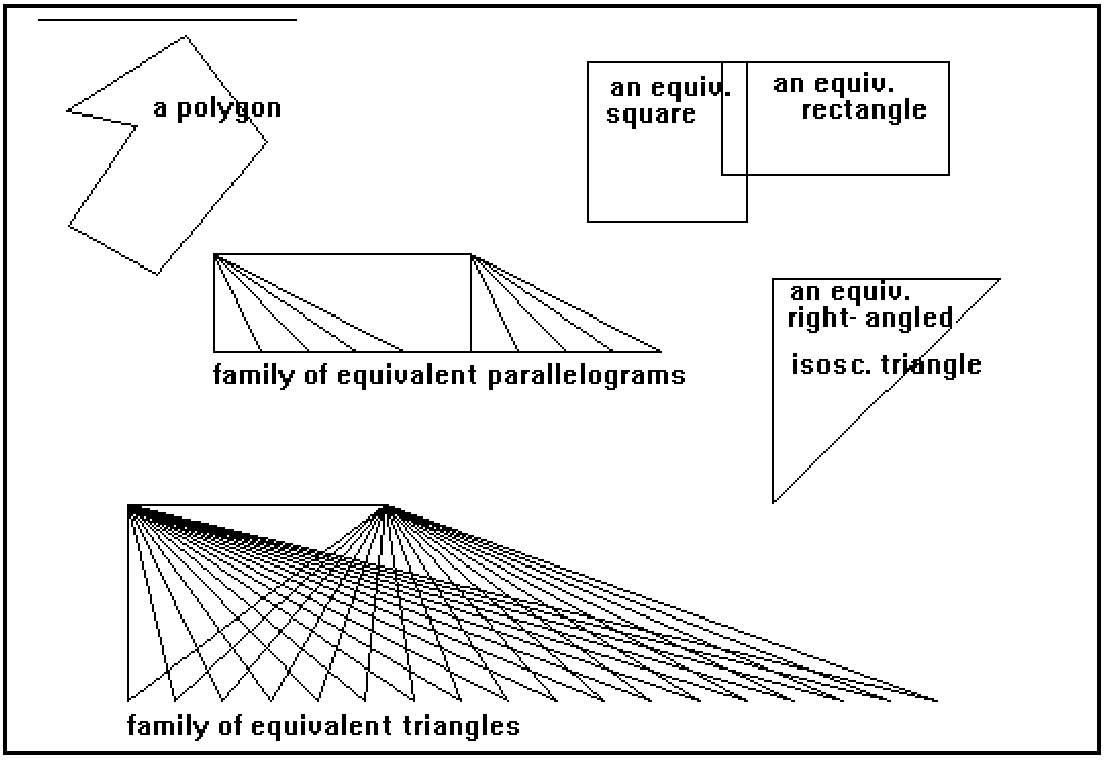
a polygon
an equiv.
square
an equiv.
rectangle
family of equivalent parallelograms
an equiv.
right- angled
isosc. triangle
family of equivalent triangles

Figure 3

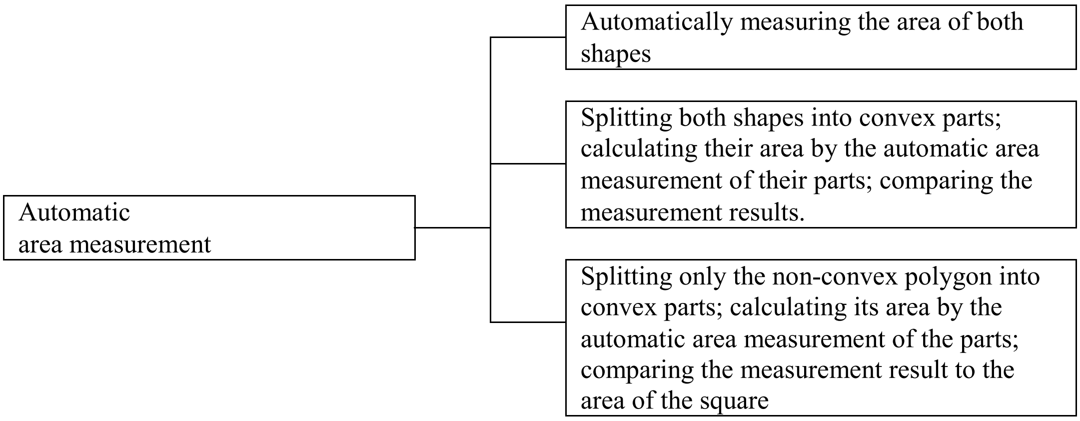
Automatic area measurement
Automatically measuring the area of both shapes
Splitting both shapes into convex parts; calculating their area by the automatic area measurement of their parts; comparing the measurement results.
Splitting only the non-convex polygon into convex parts; calculating its area by the automatic area measurement of the parts; comparing the measurement result to the area of the square

Figure 4

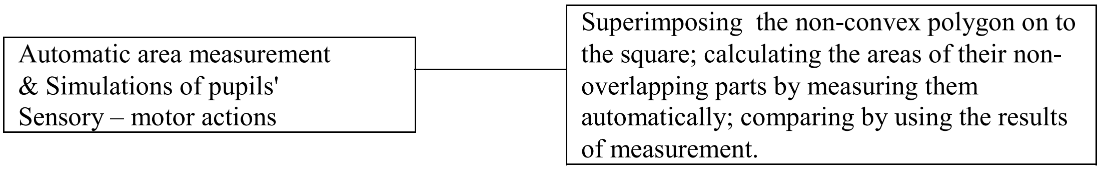
Automatic area measurement & Simulations of pupils' Sensory – motor actions
Superimposing the non-convex polygon on to the square; calculating the areas of their non-overlapping parts by measuring them automatically; comparing by using the results of measurement.

Figure 5

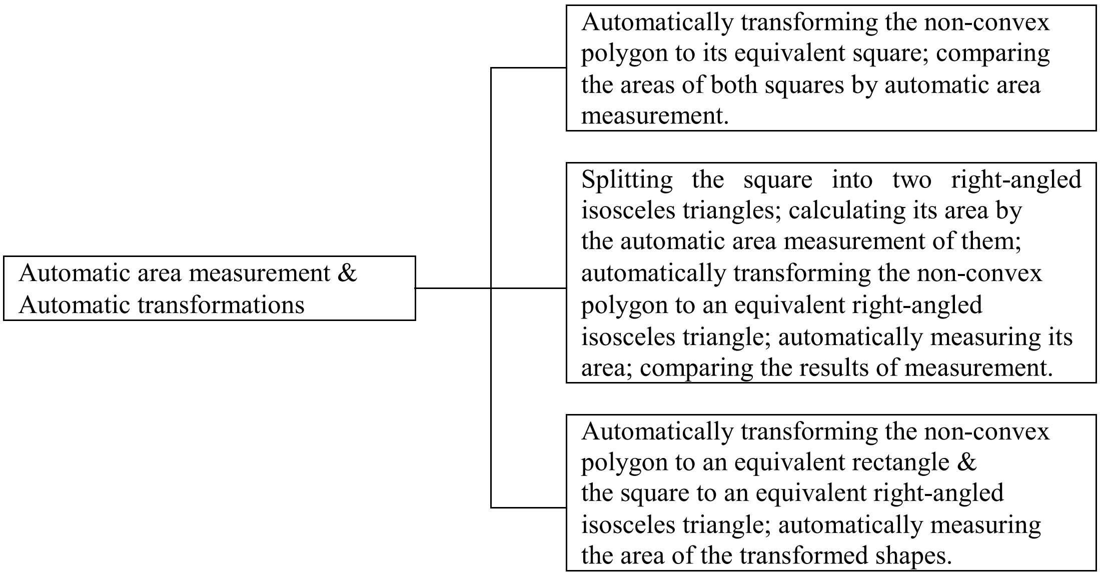
Automatic area measurement & Automatic transformations
Automatically transforming the non-convex polygon to its equivalent square; comparing the areas of both squares by automatic area measurement.
Splitting the square into two right-angled isosceles triangles; calculating its area by the automatic area measurement of them; automatically transforming the non-convex polygon to an equivalent right-angled isosceles triangle; automatically measuring its area; comparing the results of measurement.
Automatically transforming the non-convex polygon to an equivalent rectangle & the square to an equivalent right-angled isosceles triangle; automatically measuring the area of the transformed shapes.

Figure 6

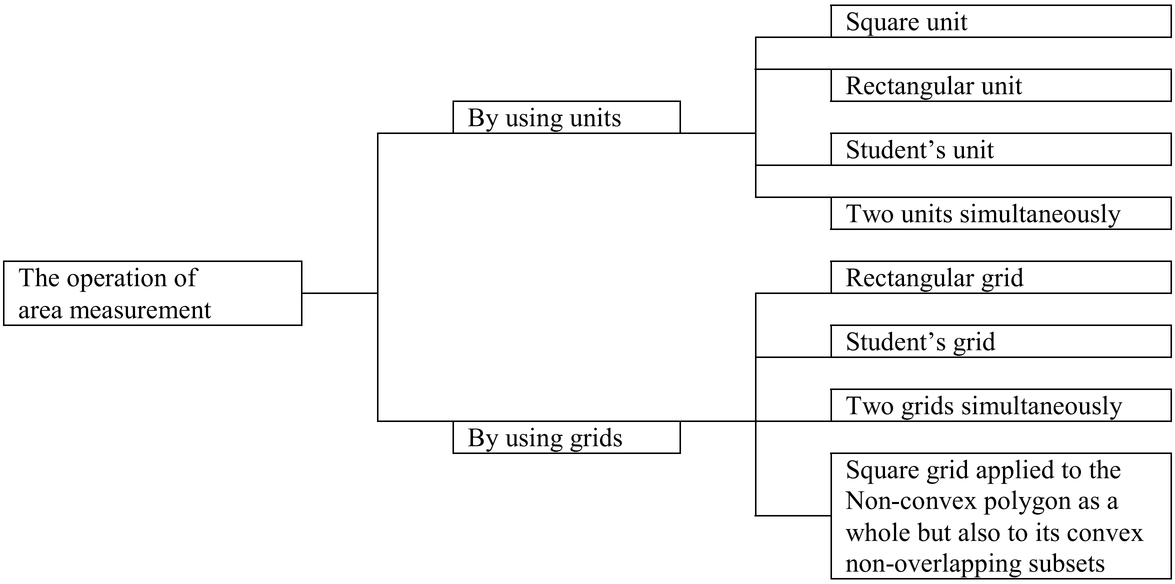
The operation of area measurement
By using units
By using grids
Square unit
Rectangular unit
Student's unit
Two units simultaneously
Rectangular grid
Student's grid
Two grids simultaneously
Square grid applied to the Non-convex polygon as a whole but also to its convex non-overlapping subsets

Figure 7

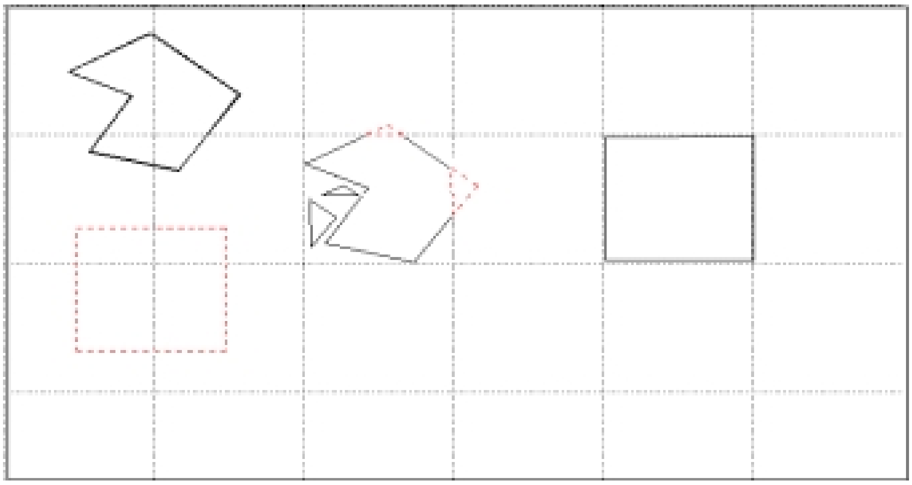

Figure 8

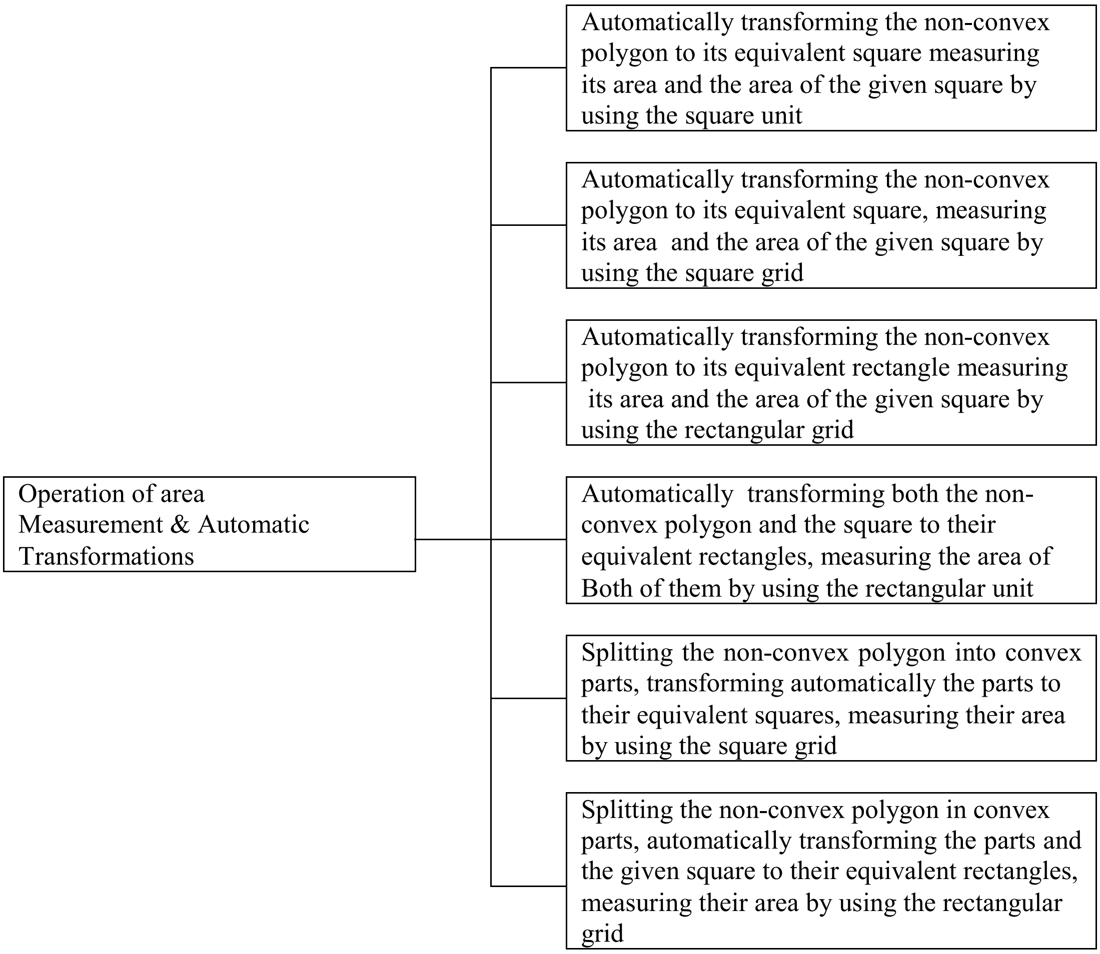
Operation of area Measurement & Automatic Transformations
Automatically transforming the non-convex polygon to its equivalent square measuring its area and the area of the given square by using the square unit
Automatically transforming the non-convex polygon to its equivalent square, measuring its area and the area of the given square by using the square grid
Automatically transforming the non-convex polygon to its equivalent rectangle measuring its area and the area of the given square by using the rectangular grid
Automatically transforming both the non-convex polygon and the square to their equivalent rectangles, measuring the area of Both of them by using the rectangular unit
Splitting the non-convex polygon into convex parts, transforming automatically the parts to their equivalent squares, measuring their area by using the square grid
Splitting the non-convex polygon in convex parts, automatically transforming the parts and the given square to their equivalent rectangles, measuring their area by using the rectangular grid

Figure 9

Area formulae

Splitting the non-convex polygon to a trapezium and a square;
measuring automatically the bases and the height of the trapezium as well as the side of the square
Calculating the area of the non convex polygon by using the area formulae of its parts.
Transforming the polygon to a square by using the area formulae

Figure 10

Area formulae & simulations of pupils' sensory-motor-actions

Splitting the non-convex polygon into its convex & non-overlapping sub sets. Automatically measuring the linear elements of the parts involved in the area formulae and calculating their area. Transforming the polygon into a square by using the area formulae. Superimposing this square to the initial one and comparing their areas.

Figure 11

Area formulae &
Automatic area measurement

Splitting the non-convex polygon to its non-overlapping & contiguous triangles; calculating its area as the sum of the automatic measurement of the triangles. Measuring automatically the length of the side of the square and calculating its area by the area formulae. Comparing the results of the calculations.

Figure 12

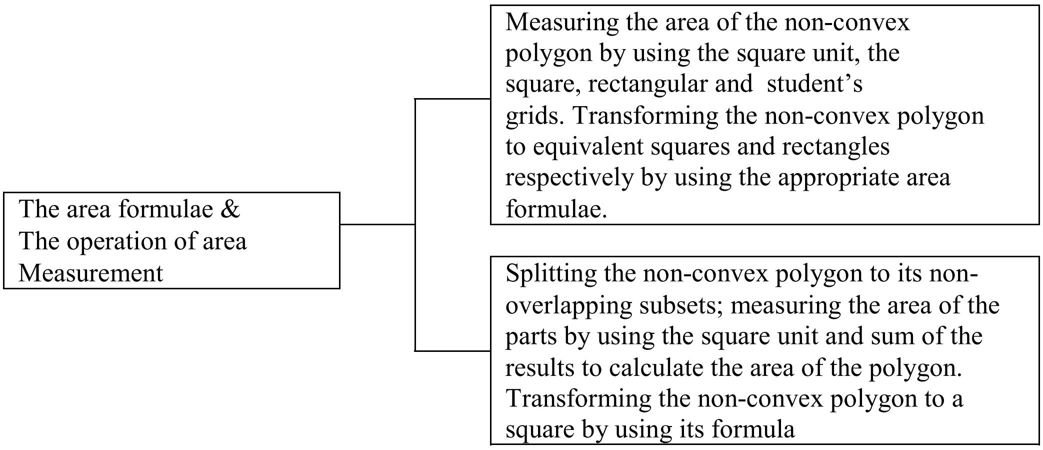
The area formulae &
The operation of area Measurement
Measuring the area of the non-convex polygon by using the square unit, the square, rectangular and student's grids. Transforming the non-convex polygon to equivalent squares and rectangles respectively by using the appropriate area formulae.
Splitting the non-convex polygon to its non-overlapping subsets; measuring the area of the parts by using the square unit and sum of the results to calculate the area of the polygon. Transforming the non-convex polygon to a square by using its formula

Figure 13

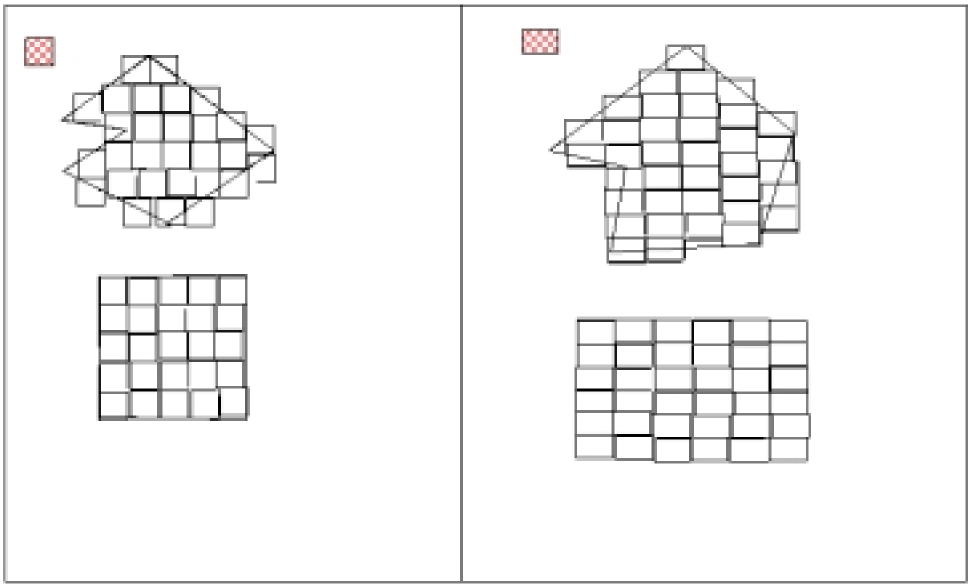

Figure 14

Categories of pupils' strategies	Number of pupils
C1 : Comparing and transforming areas by 'eye'.	1T, 1C
C2 : Comparing and transforming areas by using the perimeter of the shapes.	4T, 2C
C3 : Comparing areas by using the tool for automatic area measurement.	23 C
C4 : Comparing areas by using the tool for automatic area measurement in combination with the simulations of pupils' sensory-motor actions.	4 C
C5 : Comparing areas by using the tool for automatic area measurement in combination with the tools for automatic transformations.	5 C
C6 : Transforming and comparing areas by using the tools that support the operation of area measurement using spatial units.	19T, 25C
C7 : Comparing areas by using the tools that support the operation of area measurement using spatial units in combination with the tools for automatic transformations.	9 C
C8 : Transforming areas by using the area formulae.	1 T
C9 : Comparing areas by using the area formulae in combination with the simulations of pupils' sensory-motor actions.	1 C
C10: Comparing areas by using area formulae in combination with the tool for automatic area measurement.	1 C
C11: Transforming areas by using area formulae in combination with the tools that support the operation of area measurement using spatial units.	2 T

Table 1

C: The task of comparison, T: The task of transformation

	Task of transformation	Task of comparison	Sum
Units	Number of strategies	Number of strategies	Number of strategies
Square	15	18	33
Rectangular	3	4	7
Pupil's	5	1	6
Two units	1	0	1
Grids			
Square	16	20	36
Rectangular	2	4	6
Pupil's	2	2	4
Two grids	1	0	1

Table II

Pupils' strategies across the units	Number of pupils
Moving from square unit to square grid	12
Moving from square grid to square unit	3
Using more than one unit	10
Using more than one grid	8
Using only one unit	15 / 4 from those used the square unit
Using only one grid	18 / 11 from those used the square grid
No units used	5
No grids used	4
No units or grids used	1

Table III

Pupils' strategies across categories															
	The task of comparison									The task of transformation					Sum str
Pupils	**C1**	**C2**	**C3**	**C4**	**C5**	**C6**	**C7**	**C9**	**C10**	**C1**	**C2**	**C6**	**C8**	**C11**	
P1	1		2			3						4			4
P2						1,2						2,3			4
P3							2,3								2
P4			1			2,3,4						6,7			6
P5			1,3		4	2					7				5
P6			3	7		1,2,4						1,4,7	5		9
P7			6			1,2,3,4,5						5,6			8
P8			3			1			4			5,6,7			6
P9						1,3						2,3,4			5
P10					4		2,5					3			4
P11			1			3,5									3
P12		2	1		5	3,4						5			6
P13			1			2	4					1,3			5
P14			1	2		4,5	7					2		5	7
P15						1,2						1,7,2,3			6
P16						1,2,3						5,6			5
P17			1			2,5	3								4
P18		3	1			2									3
P19			1,3			2,						2,3			5
P20			1			6	4				2				4
P21			1		2	3				1					4
P22			1				2,3				4				4
P23			1				2								2
P24			1,3		7	5,6					2				6
P25			1,7	3		5,6						2,4,5,6,7			10
P26						2,3	5							2,4,5,6	7
P27			1					2				2,3			4
P28			1			4,5,6						5			5
P29			1,4	2		5,6						4			6
P30			1			2,5						3,4			5
Sum Strateg	**1**	**2**	**28**	**4**	**5**	**49**	**12**	**1**	**1**	**1**	**4**	**40**	**1**	**5**	**154**
Sum Pupils	**1**	**2**	**23**	**4**	**5**	**25**	**9**	**1**	**1**	**1**	**4**	**19**	**1**	**2**	

Table IV

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