On Least Expected Transmissions Multicasting in Wireless Networks

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Abstract— **In this paper we propose an optimal algorithm for the problem of least expected transmissions multicasting in wireless networks. The algorithm starts by transforming the network graph into an expanded graph that captures the wireless broadcast advantage (WBA) while simplifying point-to-multi-point transmissions in the original graph into point-to-point transmissions in the auxiliary expanded graph. Using an appropriate function to calculate the weights of the expanded graph links we also capture the wireless unreliable transmission (WUT) characteristics of the wireless medium. By solving the minimum Steiner tree problem on the expanded graph we obtain the optimal solution of the initial problem. Since the optimal algorithm is of non-polynomial complexity, we proceed to propose a heuristic algorithm. Simulation results show that the proposed heuristics have performance close to that of the optimal algorithm, at least for the instances for which we were able to track optimal solutions, while outperforming other heuristic multicast algorithms.**

Keywords— wireless multihop networks; multicasting; wireless broadcast advantage; wireless unreliable transmission;

I. Introduction

Wireless multihop networks consist of nodes connected in an autonomous manner, without the presence of any fixed infrastructure or centralized coordination. Communication among the nodes takes place in a hop-by-hop fashion, so that each node, in addition to sending and receiving the packets originating from and destined to it, also acts as a relay for forwarding other packets propagating through the network.

The wireless medium poses challenges that are different than those found in the wireline world when designing network protocols, but at the same time, its unique properties, appropriately exploited, also open up new opportunities for efficient performance. A primary characteristic of the wireless medium is its innate broadcast character, referred to as the wireless broadcast advantage (WBA) [1]. Instead of the wireline point-to-point links, in wireless networks all transmissions are point-to-multipoint: when a node transmits, all nodes within a transmission range, can hear the transmission. Nevertheless, in the early stages of research in wireless multihop networks and until recently, researchers were simply transferring ideas from the wireline field, ignoring this fundamental characteristic of the wireless medium and the benefits that could be obtained from the WBA.

Another basic characteristic of the wireless medium is that it is somewhat unreliable and lossy, and has time-varying state due to fading, mobility or interference. In this work we will refer to this as the wireless unreliable transmission (WUT) property. In unicast point-to-point routing the WUT can be accounted for through the Expected Number of Transmissions (ETX) metric [2]. In the case of point-to-multipoint transmission, the probability of successful reception may vary significantly across the multiple receivers. Actually, it has been observed that the probabilities of correct reception at each end are independent of each other [3], and thus, recently, variants of the ETX metric have been proposed for multicasting [4].

Multicasting is a primitive communication task that appears in many applications, and as such has attracted significant research interest [7]-[10]. Specifically, in wireless multihop networks, multicasting is employed in the context of data gathering (sensor networks), network state/topology information dissemination (mesh or ad hoc networks), event notification (vehicular networks), video distribution and general one-to-many content delivery in urban backbone networks, and other applications. The above comprise the use cases for the problem studied in this work. Note that the problem of finding the minimum cost multicast tree (the so called minimum Steiner tree) in traditional wireline networks with point-to-point links is NP-complete even when all links have the same cost [5] [6].

Ruiz et al [7] realized that in wireless multihop networks, calculating the minimum Steiner tree does not yield the solution with the minimum number of transmissions when the WBA and the WUT properties are considered. This led to proposing new multicasting heuristics for such networks. Zhao et al [8] proposed a metric, called Expected Multicast Transmissions (EMT), which captures both of these properties through the expected number of transmissions required by a node to successfully transmit a packet to a subset of its neighbors under certain probabilistic assumptions. A heuristic algorithm for constructing the minimal EMT schedule, without however obtaining an optimal solution is also proposed.

In this work we propose an optimal algorithm for obtaining the multicast schedule with the least expected transmissions; we will refer to this as the Minimum Expected Multicast Transmissions (MinEMT) problem. The proposed MinEMT algorithm takes into account both the WBA and the WUT properties. To the best of our knowledge, the optimal solution to this problem has not been reported before. Since the proposed optimal algorithm is non-polynomial, we also present a heuristic that achieves most of the performance gains of the optimal algorithm with considerably less computational effort. The performance of the proposed optimal and heuristic algorithms was evaluated using simulations and compared to that of other heuristic algorithms.

The paper is organized as follows. In Section II we propose a graph transformation that captures the WBA property. In Section III we formally describe the minimum expected transmissions multicast problem and present an optimal algorithm to solve it. To tackle complexity issues, heuristic algorithms are proposed in Section IV. The simulations follow in Section V. Finally, Section VI concludes the paper.

II. WBA Graph Expansion

A wireless multihop network is defined as a graph $G=(V,E)$, where V is the set of nodes and E the set of edges connecting them. We will denote by $N=|V|$ the number of nodes in the system. In our model, an edge $(v_i,v_j) \in E$ exists, if the probability p_{ij} of node v_j correctly capturing a transmission from node v_i and acknowledging it is above a certain threshold p_t, in which case we consider v_j to be *within the transmission range* of v_i. We will say that a *reliable point-to-point transmission* over edge (v_i,v_j) has been completed when v_j has correctly received and acknowledged the packet sent to it by v_i. Note that the existence of (v_i,v_j) does not indicate a reliable delivery of a packet over it in the first transmission (this is defined by p_{ij}), and that the definition of p_{ij} incorporates the reception of the ack as well.

Since the wireless medium is a broadcast channel (WBA property), a transmission from v_i can be heard by a set V_i of receivers that are within v_i's transmission range. In other words, $v_j \in V_i$ if $(v_i,v_j) \in E$ (or, equivalently, if $p_{ij} > p_t$). When node v_i wants to intentionally and reliably (as opposed to opportunistically) transmit a packet to a set $R \subseteq V_i$ within its transmission range, we will refer to it as a *reliable single hop point-to-multipoint transmission*, or *single hop multipoint transmission* for short, and will denote it by (v_i,R). A single hop multipoint transmission is completed when all the nodes in the intended set R have correctly received and acknowledged the packet. This may entail a number of retransmissions carried out by the underlying MAC protocol with an appropriate acknowledgement mechanism (e.g., IEEE 802.11).

Each edge (v_i,v_j) in E is characterized by a weight, denoted by w_{ij}. We also assume that we are given (or have a way to find, given w_{ij}'s definition) a function f that computes the weight $w_{i,R}$ of the single hop multipoint transmission (v_i,R) by the weights w_{ij} for all edges (v_i,v_j), $v_j \in R$, that is,

$$w_{i,R}=f(w_{ij} \mid v_j \in R). \quad (1)$$

The weight w_{ij} can be viewed as a cost metric for the point-to-point transmission (v_i,v_j), while $w_{i,R}$ as a corresponding metric for the single hop multipoint transmission (v_i,R).

Since single hop multipoint transmissions are harder to visualize and work with than point-to-point transmissions, it will be useful to introduce a transformation of the original network graph G into an auxiliary graph G', in which single hop multipoint transmissions in G will correspond to point-to-point transmissions in G'.

The auxiliary expanded graph $G'=(V',E')$ is obtained from the original network graph G in the following way. In the beginning, we set $G'=G$. For each node v_i, for each subset $R \subseteq V_i$ with more than two nodes ($|R| \geq 2$), we create a *virtual node* $v_{i,R}$, which is inserted in the expanded graph G' and is connected as follows:

- through a directed virtual edge $(v_i,v_{i,R})$ with weight $w_{i,R}$ calculated by Eq. (1), and
- through directed virtual edges $(v_{i,R},v_j)$ for all $v_j \in R$ of zero weight.

In other words, for each set of receivers R within v_i's transmission range V_i we create a virtual node $v_{i,R}$ that acts as an intermediate node to connect v_i to all nodes in set R. Virtual node $v_{i,R}$ represents the option that v_i transmits until all nodes in R receive and acknowledge successfully, that is, it represents a reliable single hop multipoint transmission (v_i,R). The weight of the single hop multipoint transmission $w_{i,R}$ obtained by Eq. (1) is assigned to the virtual edge $(v_i,v_{i,R})$ in G'. Then zero weight edges are used to connect virtual node $v_{i,R}$ to all receiver nodes in R. The (reliable) single hop multipoint transmission (v_i,R) in the original graph G is equivalent to the (reliable) point-to-point transmission $(v_i,v_{i,R})$ in the expanded graph G'. The edges between v_i and each node in R in G are maintained in G' to represent the unicast point-to-point transmissions from v_i to each of these nodes.

The original graph G representing the wireless network is thus transformed into the *expanded graph G'*, in which we do not have to consider multipoint transmissions, since WBA is already accounted for. Instead, in graph G', we only have to consider unicast point-to-point transmissions.

A. Graph transformation and WUT property

The previously described graph transformation captures the WBA property by introducing all possible single hop multipoint transmissions. A key attribute of this graph transformation is that on the expanded graph we have to consider only point-to-point transmissions (as in wireline networks) as opposed to point-to-multipoint transmissions (WBA property). This transformation can be used to solve a number of different optimization problems in wireless networks, using different definitions of function f in Eq (1) to calculate the cost of a single hop multipoint transmission by the costs of the point-to-point links that comprise it.

To capture the WUT property, we assume that the reliable transmission over an edge (v_i,v_j) is a random experiment defined by a Bernoulli trial with probability of success p_{ij}. If some of the recipient nodes fail to receive the data or the ACK message is lost, the sender will retransmit until the ACK is received successfully. Thus, the success probability p_{ij} corresponds to both the forward and the reverse (ACK) transmission being successful. The number X_{ij} of transmissions before a successful reliable transmission has been completed over (v_i,v_j) follows a geometric distribution, and its expected value is defined as the weight of edge (v_i,v_j):

$$w_{ij}=\text{ETX}(v_i,v_j)=\text{E}\{X_{ij}\}= 1/p_{ij}.$$

The success probability p_{ij} captures a number of diverse transmission characteristics and physical parameters, such as the distance and the obstacles and possible mobility, but it also depends on traffic parameters, such as the load that determines the interfering traffic overheard by v_i and v_j. The ETX metric has been widely adopted, as it is related to the actual achievable throughput over an error-prone link [2]. ETX can be calculated by collecting statistics on the control/data packet transmissions or using a periodic transmission of special purpose control packets to estimate it.

As already mentioned, a packet transmission from node v_i is heard by a set of possible receivers V_i. We assume that the probabilities of successful reception of a packet and its corresponding acknowledgement are independent for the different receivers [3]. Consider now the case where we want to perform a reliable single hop multipoint transmission from node v_i to a set of receivers $R \subseteq V_i$. For each node $v_j \in V_i$ we denote by $f_{ij}=1- p_{ij}$ the complementary failure probability. The expected number of transmissions for a successful single hop multipoint transmission (that is, for the successful reception of a packet and its successful acknowledgement by all nodes in R) is denoted by $w_{i,R}=\text{EMT}(v_i,R)$, where EMT stands for Expected Multicast Transmissions. According to [8], we have:

$$w_{i,R}=\text{EMT}(v_i,R)=\sum_{c=1}^{|R|}(-1)^{c-1}\cdot\sum_{S:S\subseteq R \text{ and } |S|=c}\frac{1}{1-\prod_{j\in S} f_{ij}} \quad (2)$$

From the above, $\text{EMT}(v_i,R) \geq \text{ETX}(v_i,v_j)$, for all $v_j \in R$, and also $\text{EMT}(v_i,R) \leq \sum_{j:v_j\in R}\text{ETX}(v_i,v_j)$. Note that if the set R consists of a single receiver, e.g. $R=\{v_j\}$, then $\text{EMT}(v_i,R)=\text{ETX}(v_i,v_j)$. Thus, the definition of the EMT metric for single hop multipoint transmissions includes the simple point-to-point transmissions as a subcase, and both point-to-point and multipoint transmissions can be treated in a unified way.

Eq. (2) enables us to calculate the weight of a single hop multipoint transmission from the weights of the point-to-point links that comprise it. Thus, Eq. (2) is a special case of Eq. (1) and can be used in the graph transformation described earlier to obtain the weights of the auxiliary expanded graph *G'*.

Figure 1 presents an example of the expansion of a graph that utilizes the ETX metric. The original graph is depicted in Figure 1a. In the expanded graph, depicted in Figure 1b, the edge weights are omitted for the purpose of clarity. The bold edges connecting the virtual nodes with original nodes signify zero weights. Each virtual node in the expanded graph is labeled according to the receiver nodes of the single hop multipoint transmission it represents.

1) Complexity of the graph transformation

An issue that needs attention is the complexity of the transformation performed on the original graph *G* to obtain the expanded graph *G'*. Recall that node v_j is said to be within the transmission range of v_i if the probability p_{ij} of v_j correctly capturing and acknowledging a transmission from node v_i is above a certain threshold q. Let $d_i=|V_i|$ be the node degree of v_i. The number of virtual nodes that are added in *G'* corresponding to node v_i is equal to $2^{d_i}-d_i-1$. This is because a virtual node is added for each subset of nodes of V_i except for the empty and single element subsets. The number of virtual edges can also be calculated accordingly. Based on the above, the size of the expanded graph depends, in general, exponentially on the node degrees of the original graph and thus the transformation is non-polynomial. Note, however, that the important in practice case of regular network topologies like line-arrays, 2-dim grids and other topologies with constant node degree or node degree less than logarithmic to the number of nodes, do not exhibit such an issue. Also, a virtual node (v_i,R) corresponding to receiving set R may not have to be created if the cost of the links leading to it have weight $w_{i,R}$ that is above a certain threshold. For example, we can use the threshold p_t and remove nodes from the expanded graph *G'* corresponding to subsets R for which $w_{i,R} > 1/p_t$. By changing the threshold p_t that defines which nodes are within reach, we control the node degrees and thus the complexity of the algorithm. To be more generic, in Section IV.A we will give a truncated graph transformation of polynomial complexity for general network topologies.

III. Minimum Expected Multicast Transmissions (MinEMT) Algorithm

We are given the network graph $G=(V,E)$ and the success probabilities p_{ij} for each $(v_i,v_j)\in E$. The weights of the edges are defined as $w_{ij}=\text{ETX}(v_i,v_j)=1/p_{ij}$. We are also given a source node v_1 and a multicast group $M\subseteq V$ whose nodes have to receive a copy of the packet located at node v_1. The objective is to find the multicast schedule that minimizes the expected number of transmissions that have to take place to carry a copy of the packet to all nodes in the multicast group *M*.

A multicast schedule is defined as a sequence of single hop multipoint transmissions $T_k=(v_k,R_k)$, each consisting of the transmitter v_k and the set of neighbors R_k that have to (reliably) receive the transmission. Thus, at each T_k step the transmitting node v_k keeps retransmitting until all nodes in the set R_k of receivers successfully receive and acknowledge the packet. The expected number of transmissions $\text{EMT}(v_k,R_k)$ required to accomplish this is given by Eq. (2). Note that T_k may just be a simple point-to-point transmission since the definition of single hop multipoint transmission includes this as a special

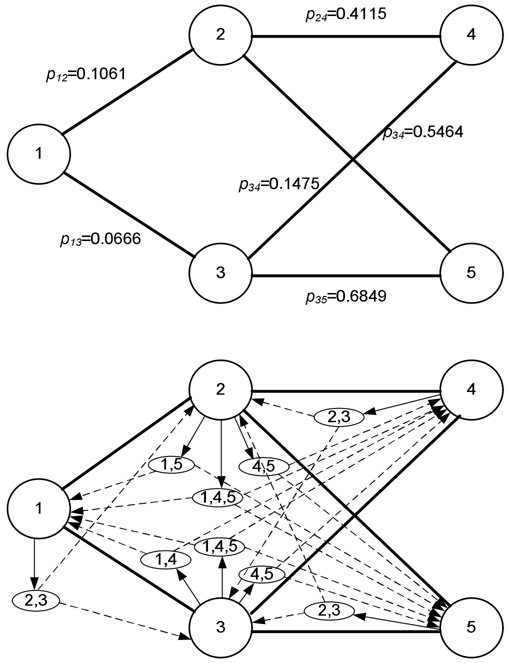

Fig. 1. An example of the network graph expansion step. The original graph presented in (a) is expanded to the graph shown in (b). The virtual nodes in the expanded graph are named according to the original nodes they lead to with zero weight edges (dashed lines).

case. A feasible multicast schedule is obtained by having the transmitter v_k at step k be among the nodes that have received the packet up to step k-1, formally, $v_k \in \bigcup_{z=1:k-1} R_z$. The starting node is the source node v_1 where the packet is originally located. The schedule finishes at step K when all nodes in the multicast group *M* have received the transmission, $M \subseteq \bigcup_{z=1:K} R_z$. The variable K corresponds to the number of different transmitters used in the schedule.

The objective of the MinEMT problem is to find the schedule that minimizes the expected number of transmissions

$$\sum_{k=1:K} \text{EMT}(v_k, R_k)$$

required to get a copy of the packet to all nodes in the group *M*.

In our problem definition the number K of transmitting nodes is not an optimization criterion; instead, we are interested in the expected number of transmissions performed by these nodes, which is captured by the EMT metric, which also represents the load caused to the network. Following this schedule, irrespectively of the success or failure of the transmissions, would give the lowest a priori average number of transmissions. Note that during the multicast and based on the actual successes and failures of the transmissions, other schedules could be calculated that could improve performance. This would however happen at the cost of rerunning the algorithm and coordinating the transmissions at each step. In the future we plan to examine such dynamic re-scheduling policies and their advantages and disadvantages.

Note that calculating the minimum Steiner tree in the network graph $G=(V,E)$ does not yield an optimal solution, as would be the case in a wireline network with point-to-point links. This is because the WBA and the consequent gains that can be obtained by multipoint transmissions are not exploited when solving the Steiner tree problem in the original graph.

A. Multicast Optimal Algorithm on the Expanded Graph (MOE)

We now describe an optimal algorithm to solve *minEMT* problem, as defined above. The algorithm is divided into two

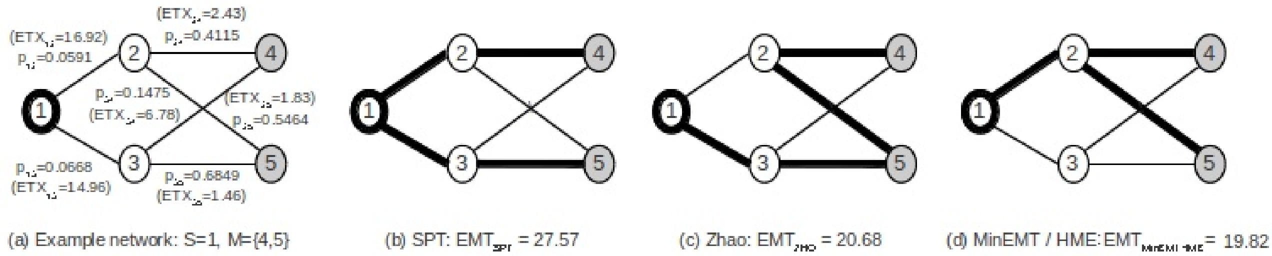

Fig. 2 (a) The example network with the probabilities of correct transmission pij and the corresponding weights wij=ETX(vi,vj). The source node is node 3 and the multicast set consists of nodes 1 and 2. The multicast tree (with bold edges) calculated by the (b) Shortest Path Tree (SPT) algorithm, (c) the heuristic algorithm of Zhao et al [7], and (d) the proposed optimal/heuristic algorithms, along with the total EMT cost of each solution

phases. In the first phase the network graph is expanded so as to capture the WBA and WUT properties. Then in the second phase the minimum Steiner tree on the expanded graph is found in order to obtain the MinEMT schedule.

In the first phase of the algorithm we use the graph transformation algorithm described in Section II to obtain the expanded graph *G'*, using as link weights the Expected Number of Transmissions (ETX) metric and Eq. (2) to calculate the Expected Multicast Transmissions (EMT) of a reliable single hop multipoint transmission.

In the second phase of the algorithm, the minimum Steiner tree is calculated in the auxiliary graph *G'*. The start node is source v_1, the destination nodes are the nodes in the multicast group *M*, and the Steiner nodes are the rest of the nodes in *G'*. Since the definition of the expanded graph *G'* captures the WBA and WUT properties, we only have to consider point-to-point (unicast) transmissions in *G'*, and, thus, the minimum Steiner tree of *G'* yields the optimal MinEMT solution.

After obtaining the minimum Steiner tree in the expanded graph *G'* [11] we translate the solution to the original graph *G* to obtain the optimal MinEMT schedule. A transmission over a virtual edge in *G'* is translated to the corresponding single hop multipoint transmission in *G*, while a transmission over an original and not a virtual edge in *G'* remains the same in *G*. Note that a single hop multipoint transmission requires the source to re-transmit the packet until all nodes described in that transmission receive and acknowledge it correctly.

Finding the minimum Expected Transmissions Multicast without taking into account the WBA is an NP-hard problem [5]. Considering the WBA we have to consider all possible single hop multipoint transmissions, which complicates even more the problem. Since finding the optimal solution is difficult, we propose in the following section a heuristic algorithm that can provide practical solutions to the problem.

IV. HEURISTIC ALGORITHM

Since the complete graph transformation depends exponentially on the nodes' degree, it is not practical for networks whose degree grows faster than logarithmically with the number of nodes. To reduce the complexity, we developed a heuristic variation to the optimal transformation. This *truncated graph transformation* uses a *connectivity threshold* D_t and considers for the expansion of a node the D_t links with the highest success probability, or equivalently the lowest EMT values. So for node v_i, the set V_i of receivers considered in the transformation has cardinality bounded by D_t, which can control the complexity. The algorithm that uses the truncated graph is polynomial if D_t is chosen to be at most logarithmic on the number of nodes. Note that all links that existed in the original graph *G* are included in the expanded graph *G'*, while the threshold D_t prunes links only for the expansion process.

We also propose a heuristic algorithm, called the Heuristic multicast schedule on the expanded graph (HME), which works on the expanded graph *G'* calculated using the full non-polynomial transformation or the previously described truncated graph transformation. On *G*' we calculate the all-pairs shortest paths using the Floyd-Warshall algorithm. For every node v_i a distance vector D_i with size $k=|M|$ is maintained, which keeps the shortest distances to every node in the multicast group *M*. The HME algorithm works as follows. At every step, a set *C* is maintained containing the nodes already covered by the algorithm. When $M \subseteq C$, the algorithm terminates. Initially, the set *C* contains only the source node v_1. Let D_C be the distance vector with the shortest distances from the covered nodes in *C* to the multicast group *M*. At every step, a node contained in the covered set *C* is selected for transmitting over one of its outgoing edges (edges of the original graph *G* or the virtual edges of *G'* that represent single hop multipoint transmissions). For all nodes v_i in *C*, and all outgoing edges of v_i [that is, for all $(v_i,v_j)\in E'$] we first calculate the gain vector $Q_{ij}=D_j-D_C$ defined as the difference of the distance vector of the new node v_j minus the distance vector of the covered set *C*. Then we calculate a scalar total gain q_{ij} by considering only the nodes belonging to the multicast set that are not yet covered in *C* and are closer to v_j, that is, the nodes for which $Q_{ij}>0$. More formally,

$$q_{ij} = \left(\sum_{z \in M \cap z \notin C \cap G_{ij}(z)>0} Q_{ij}(z) \right) - w_{ij}, \quad (3)$$

where w_{ij} is the weight of edge (v_i,v_j) and $Q_{ij}(z)$ corresponds to the z^{th} element (multicast node) of the gain vector Q_{ij}.

Having obtained the scalar total gains for all outgoing edges of all the nodes in *C* we select the outgoing edge that yields the highest gain. Depending on whether this corresponds to an original edge or a single hop multipoint transmission we update accordingly the data structures. If all the nodes in the multicast set have been covered ($M \subset C$) the algorithm terminates. Else, we proceed to the next step.

For the remaining of this paper we will refer to the above described algorithm as Heuristic Multicast Schedule on the Expanded graph (HME). The HME algorithm is a greedy heuristic that at each step expands the covered set of nodes with the transmission that takes it closer to the uncovered nodes in the multicast set. It uses the expanded graph that captures both the WBA and the WUT, so that the algorithm only has to consider point-to-point transmissions. Note that as the algorithm is executed, if one node is used for transmission at a certain point, this node will not be used again later for another transmission. That is, at each point we have only to consider the "front" of the covered set, a feature that is not formally presented in the above description but used to reduce its running time of the algorithm. Still, the algorithm has to check all the outgoing edges of a potentially large number of nodes, which can take exponential time. However, its complexity can be reduced to polynomial by employing the truncated graph transformation described above.

In Figure 2 we illustrate an example of finding the minimum expected transmissions for a multicast session. In particular we graph the multicast trees obtained by the Shortest Path Tree (SPT) algorithm, the heuristic proposed by Zhao et al in [8], which we will refer to as Zhao Heuristic algorithm on the original graph (ZHO), and our optimal MinEMT algorithm that runs on the expanded graph. The trees obtained are compared in terms of the expected number of transmissions (EMT) required from source-node 1 to the nodes 4 and 5 comprising the multicast group. The optimal MinEMT algorithm, by exploiting fully the WBA and WUT properties manages to save one transmission, compared to the schedule produced by the heuristic ZHO of Zhao. Interestingly, the HME heuristic algorithm, in this example, also succeeds in obtaining the optimal multicast tree.

V. SIMULATION RESULTS

In our experiments, we randomly generated networks with a given (i) number of nodes *N*, (ii) average node degree *D*, and (iii) average value *W* for the link weights (average number of transmissions required). To create a random network, we start with the given number of nodes *N* and create an all-nodes-to-all-nodes edge matrix with random numbers as it entries using a uniform distribution. We keep only the edges that result in a network with the requested connectivity degree *D*, discarding the edges with the smallest randomly created numbers. Having obtained the topology, we assign weights to edges according to an exponential distribution with mean equal to *W*. In each experiment we also select the source node and the multicast set with a given cardinality |*M*|, from a uniform distribution.

The proposed optimal MinEMT and the heuristic HME algorithms are comparatively evaluated against a heuristic that is based on Shortest Path Tree (SPT), and the heuristic of Zhao et al [8] (ZHO algorithm). SPT routes a packet from its source to the nodes in the multicast set over their shortest paths. To have a fair comparison, when a node is utilized in more than one shortest path we consider this as a single hop multipoint transmission and calculate the corresponding EMT value. ZHO runs on the original graph and employs the EMT metric to obtain a multicast tree in a greedy fashion.

Unless stated otherwise, the default parameters used in our experiments are: *N*=50 nodes, *D*=3, *W*=1.5 transmissions, |*M*|=10 nodes. For each reported experiment we created 10 random networks, sources and multicast sets, and averaged the results over these 10 experiments. For each problem instance we use the related algorithms to calculate the Expected Multicast Transmissions (EMT) value of the solution, without exchanging packets in the network.

Figure 3 presents the average number of transmissions and the average running time of the examined algorithms as the number of nodes *N* ranges from 25 up to 100 nodes with a step of 25 nodes. As expected, the average number of transmissions and the average running times of the algorithms increases with the number of nodes. The Shortest Path Tree (SPT) algorithm exhibits the worst performance, which was expected given the unsophisticated nature of the algorithm that does not account for the WBA and the WUT properties, but since it is quite simple, its execution time is considerably smaller than that of the other algorithms. The optimal MinEMT algorithm was able to track optimal solutions for all the experiments with 25 nodes and more than half of the experiments with 50 nodes, exhibiting the best performance in both cases. Even for higher values of *N*, where we are not sure if it has found the optimal solution in many instances, MinEMT still exhibits the best performance. However, as can be seen in Figure 3b, the average running time of MinEMT is quite high and it reaches the limit of 1 hour per experiment for *N*=100 nodes. Zhao's heuristic algorithm (ZHO) achieves good performance with relatively low running times. The heuristic HME algorithm that runs on the expanded graph performs significantly better than ZHO. The performance of HME comes quite close to that of the optimal MinEMT algorithm, especially as *N* increases and MinEMT is no longer able to track the optimal solutions. The running time of HME is the highest among the heuristic algorithms, but it scales well with the number of nodes *N*.

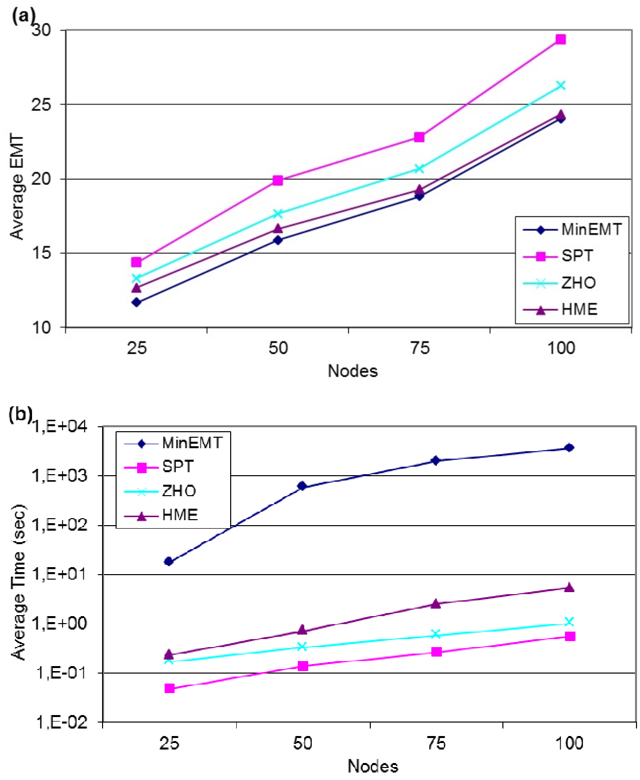

Fig. 3. (a) Average expected number of transmissions and (b) average running time as a function of the number of network nodes N.

Figure 4 shows the results obtained for different values of the average node degree *D*. The average number of transmissions decreases as *D* increases, for two main reasons: (i) the paths that lead to nodes in the multicast set become shorter, and (ii) single hop multipoint transmissions become more efficient. The SPT algorithm exhibits the worst average number of transmissions but the best running time. The MinEMT algorithm has the best performance for low values of the average node degree *D*, but as *D* increases it is no longer able to track optimal solutions within the 1 hour limit, and its performance deteriorates. This is because the optimal algorithm runs on the expanded graph that utilizes virtual nodes to capture all the possible single hop multipoint transmissions. As *D* increases the expanded graph becomes very large and the performance of the MinEMT algorithm deteriorates vastly. For *D*≥5 the MinEMT algorithm turns out to be worse than that of the other algorithms. ZHO has good average number of transmissions and average running time performance. The HME algorithm that runs on the expanded graph outperforms ZHO as well as MinEMT for *D*≥5, but its running time increases as *D* increases. This is because it runs on the expanded graph as minEMT. This bad scalability of HME with respect to the node degree *D* was expected, and this is the reason we introduced the truncated graph transformation.

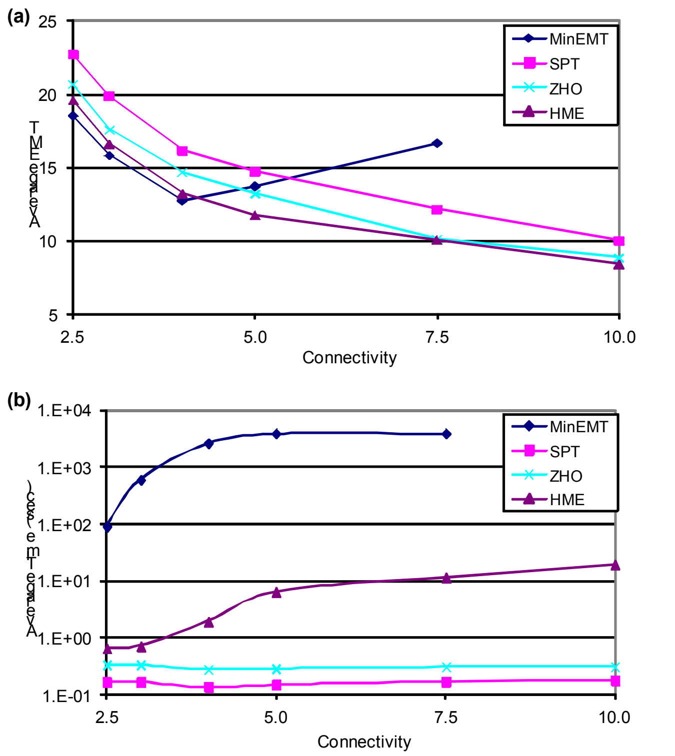

Fig. 4 (a) Average expected number of transmissions and (b) average running time as a function of the average connectivity degree.

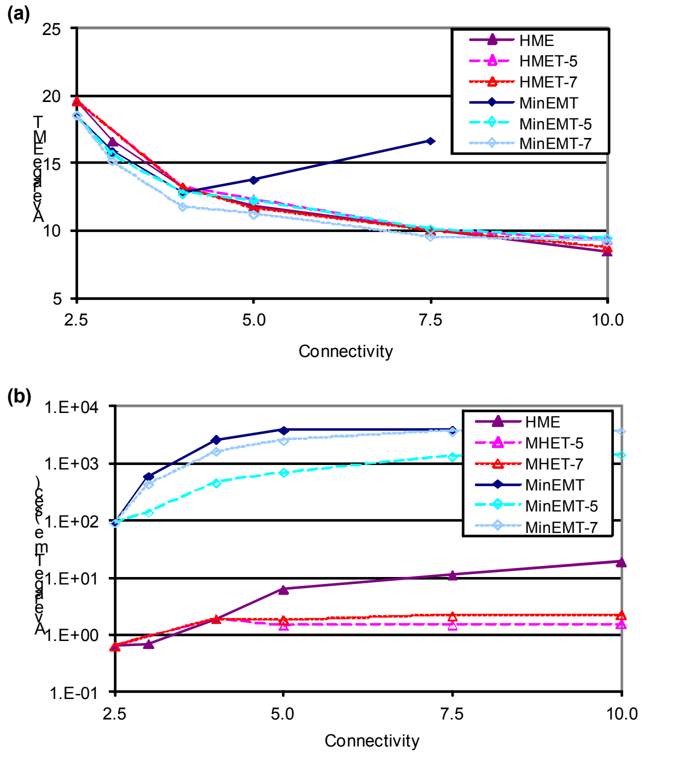

Fig 5. Evaluation of the truncated graph transformation: (a) Average expected number of transmissions and (b) average running time as a function of the average connectivity degree D for different connectivity degree thresholds D_t.

To further examine the effect of the nodes' degree, we show in Figure 5 the performance of the MinEMT and the HME algorithms using the complete and the truncated graph transformation with truncation values D_t equal to 5 and 7, (referred to as MinEMT-5, MinEMT-7, and HME-5, HME-7, respectively). We see that the truncated graph transformation improves vastly the solutions obtained by MinEMT in the given time (1 hour). As expected the running times are reduced when a smaller D_t threshold is used, indicating that we can control the running time while maintaining most of its performance benefits. Finally, we see that the performance of HME does not deteriorate when it runs on the truncated graph, while its running time is reduced. Since HME combined with the truncated graph transformation and appropriately chosen threshold D_t is of polynomial complexity it does not experience scalability problems and can be used to solve much larger problem instances than the ones presented here.

VI. CONCLUSIONS

We considered the multicasting problem in wireless multihop networks taking into account both the wireless broadcast advantage (WBA) and the wireless unreliable transmission (WUT) characteristic of the medium. We proposed a graph transformation that captures the WBA property, resulting in an expanded auxiliary graph where we have to consider only simple point-to-point transmissions. We used an appropriate weight function to capture the WTU characteristic on the expanded graph and we formulated the minimum expected multicast transmissions problem as the minimum Steiner tree problem on the expanded graph. To address the non-polynomial complexity of the optimal algorithm, we also proposed a truncated graph transformation and a heuristic algorithm. Simulation results showed that the proposed heuristic has performance close to the optimal, while also outperforming other heuristics proposed in the literature.

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