

1

(The Stable Marriage Problem . SMP)

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(matching algorithm)

- (, internet sites)
- (,)
- (internet - load balancing, 😊) /



DAVID GALE & LLOYD SHAPLEY
COLLEGE ADMISSIONS AND THE STABILITY OF MARRIAGE
THE AMERICAN MATHEMATICAL MONTHLY, VOL. 69, NO. 1 (1962)

(Matching Problem)

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⋮

(matching)

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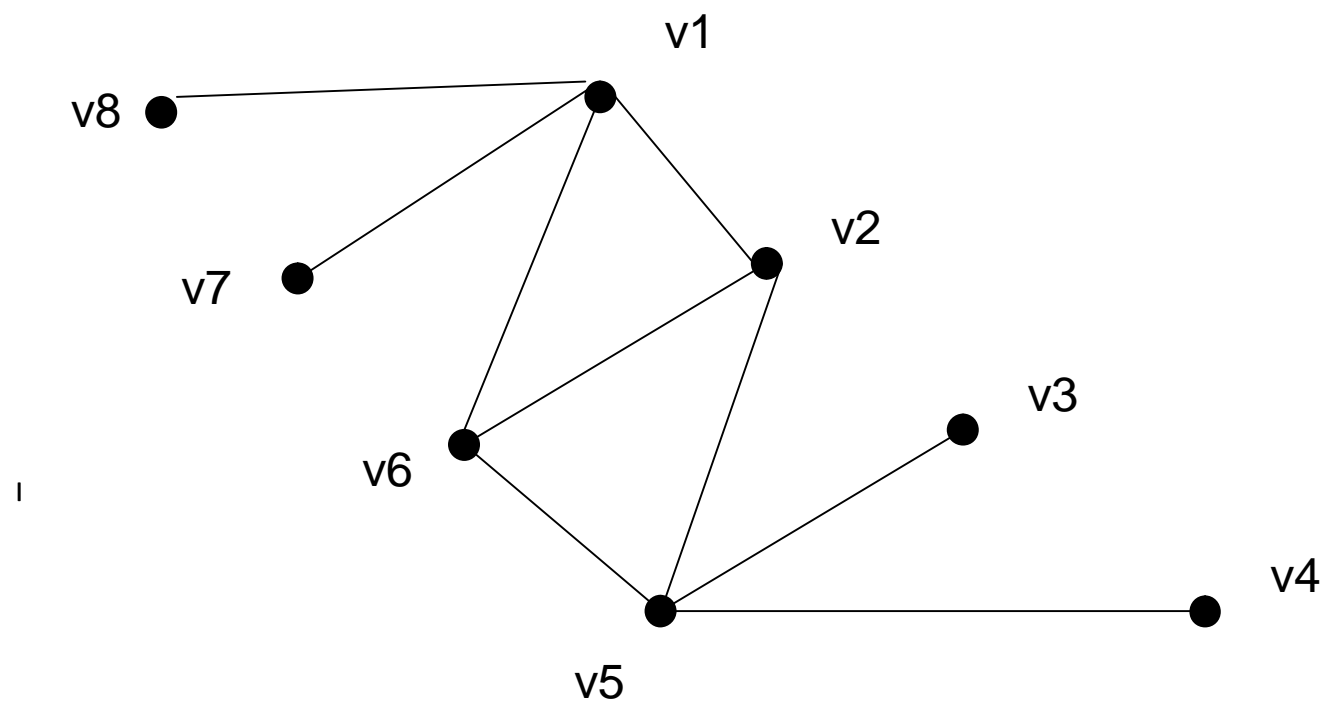
(matching):

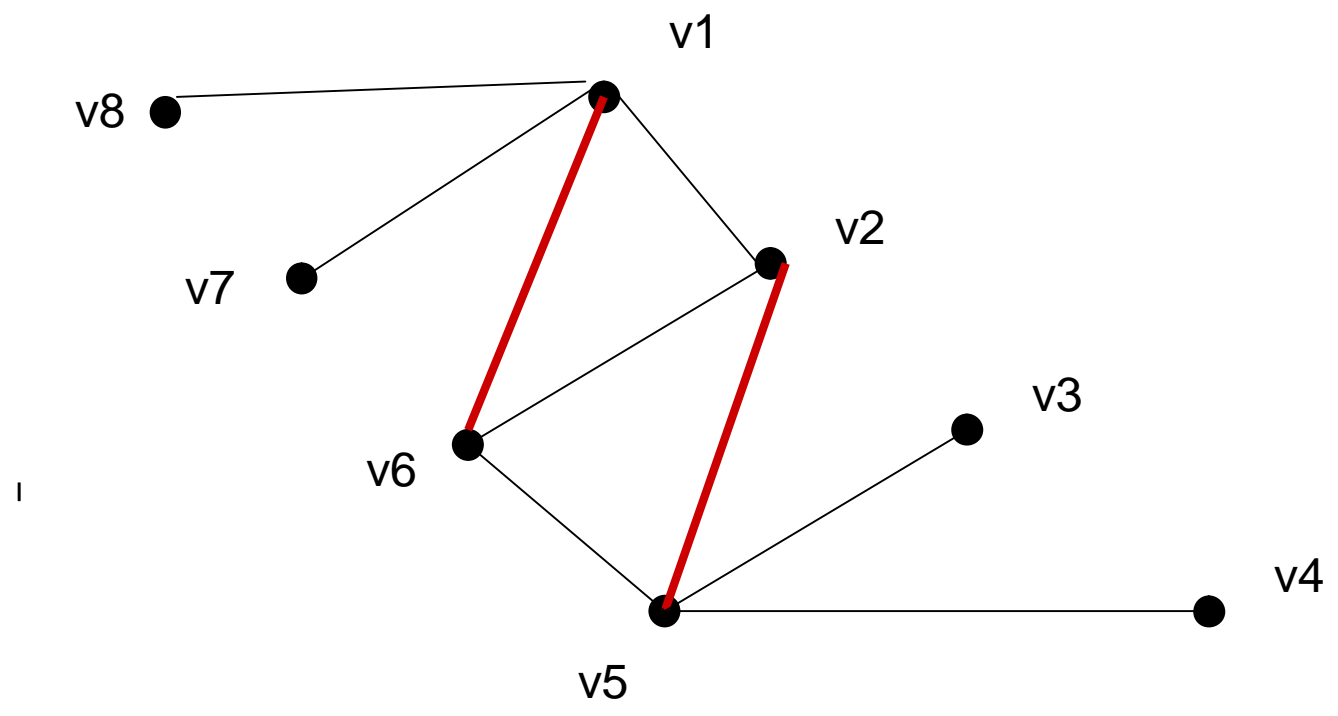


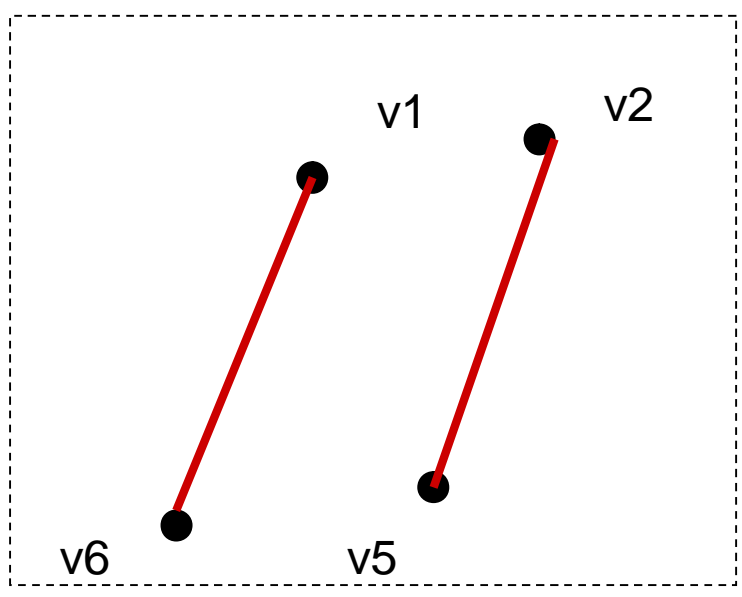
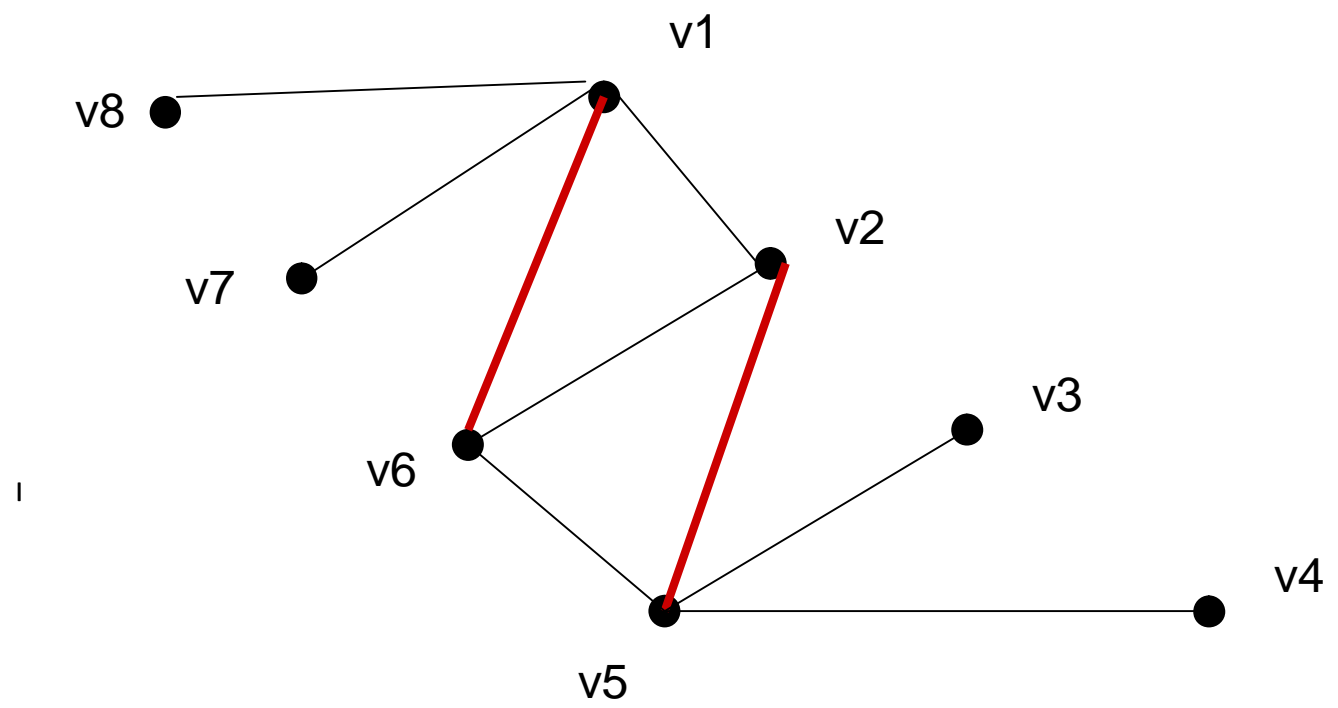
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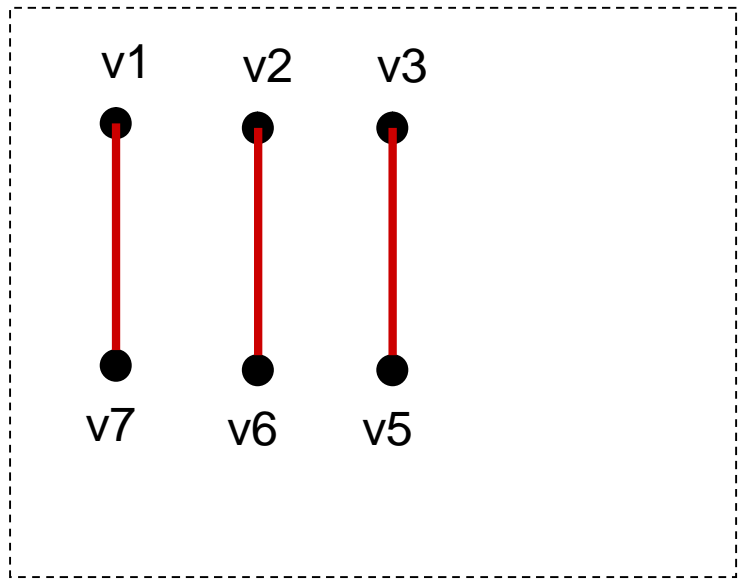
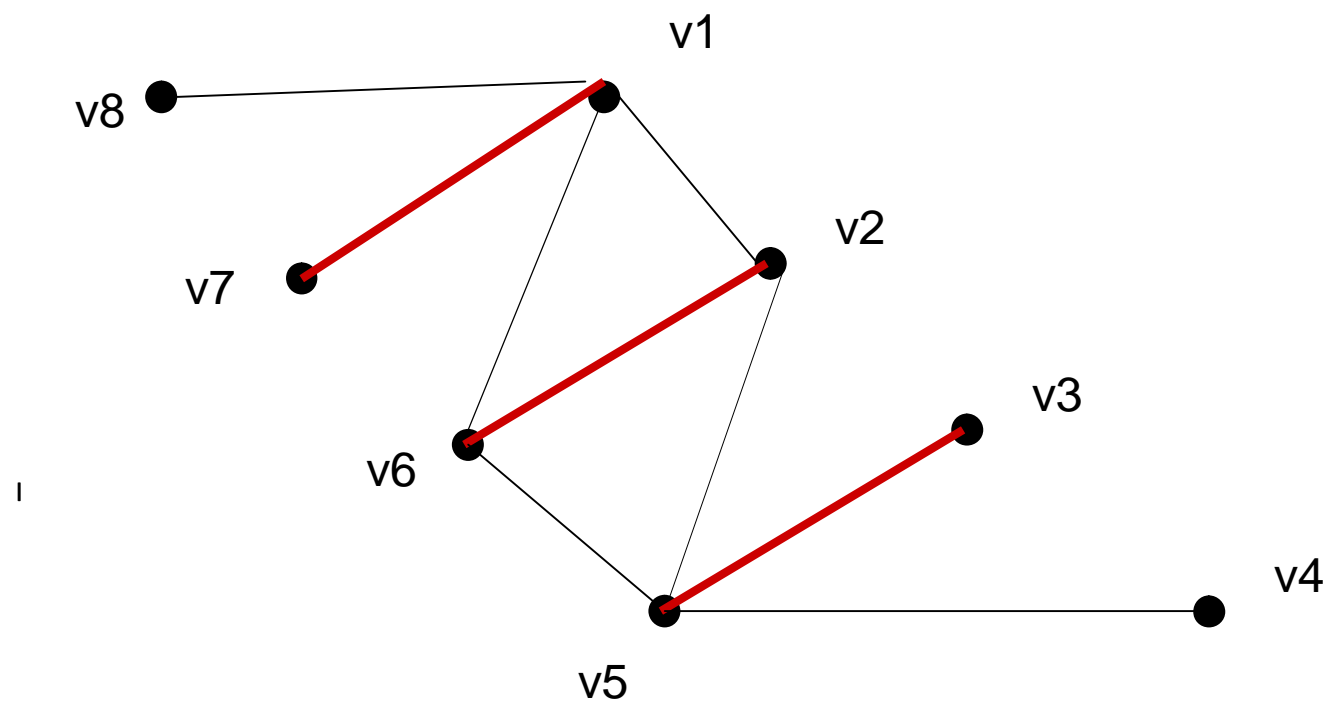




$\Leftrightarrow 2$

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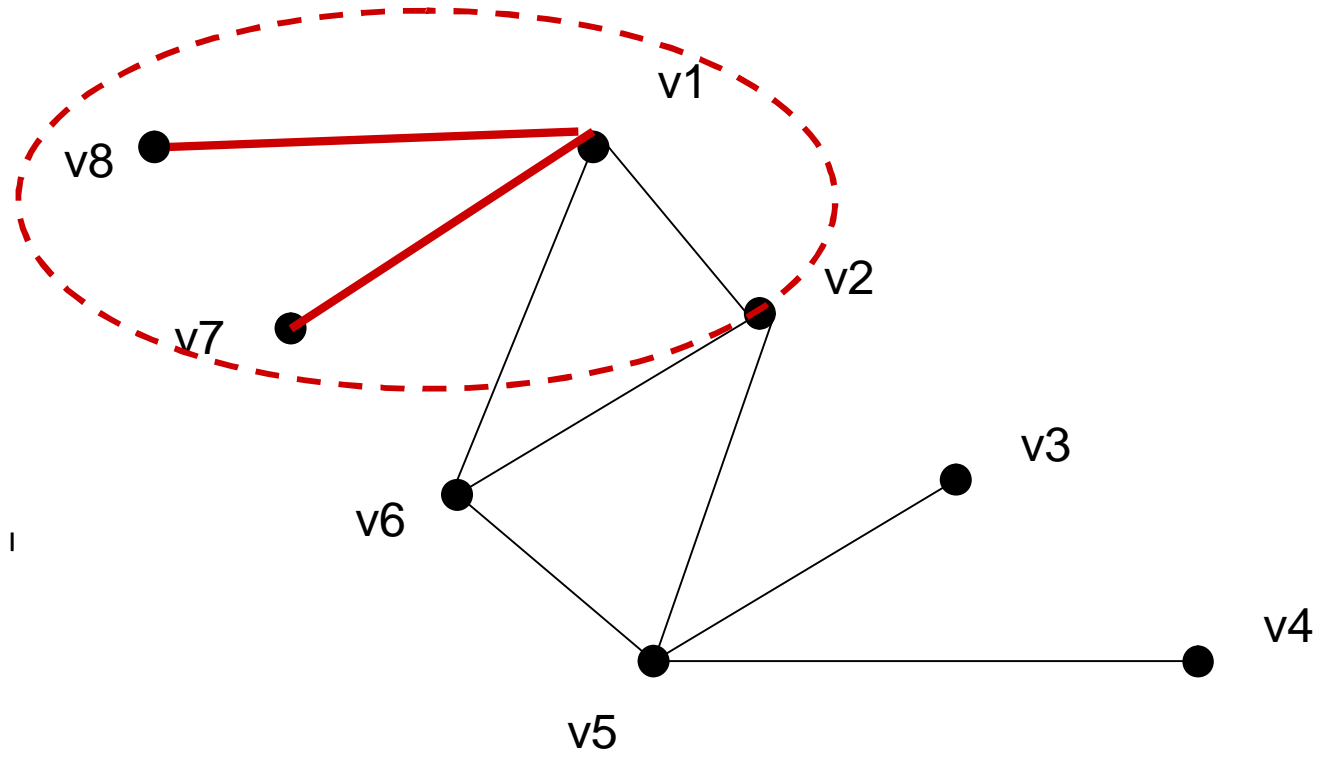


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v6

v5

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v3

v4

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(perfect matching)

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(perfect matching)

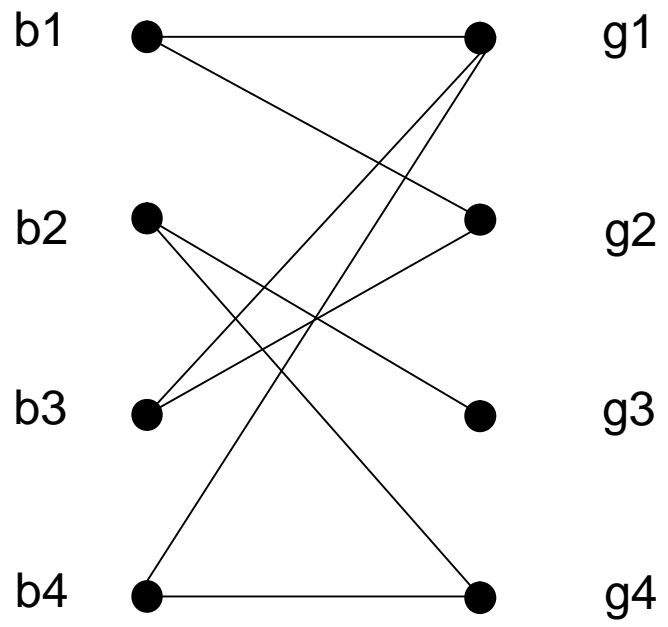
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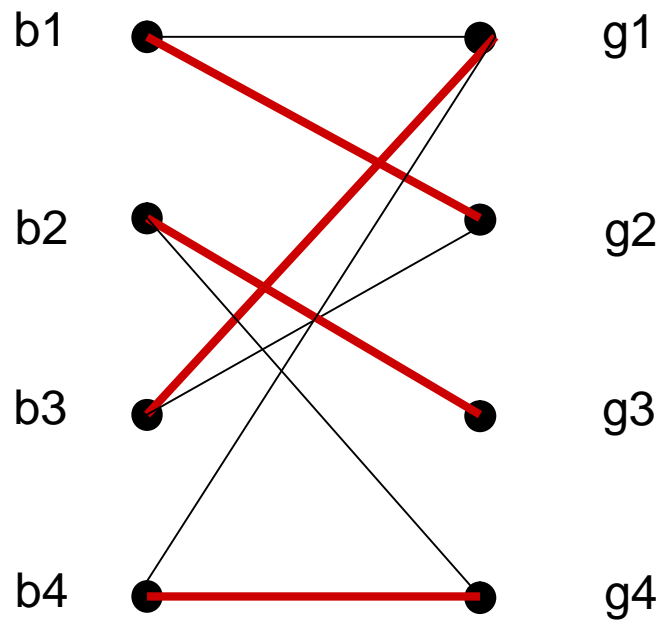
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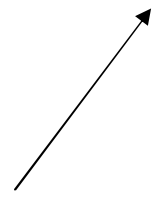
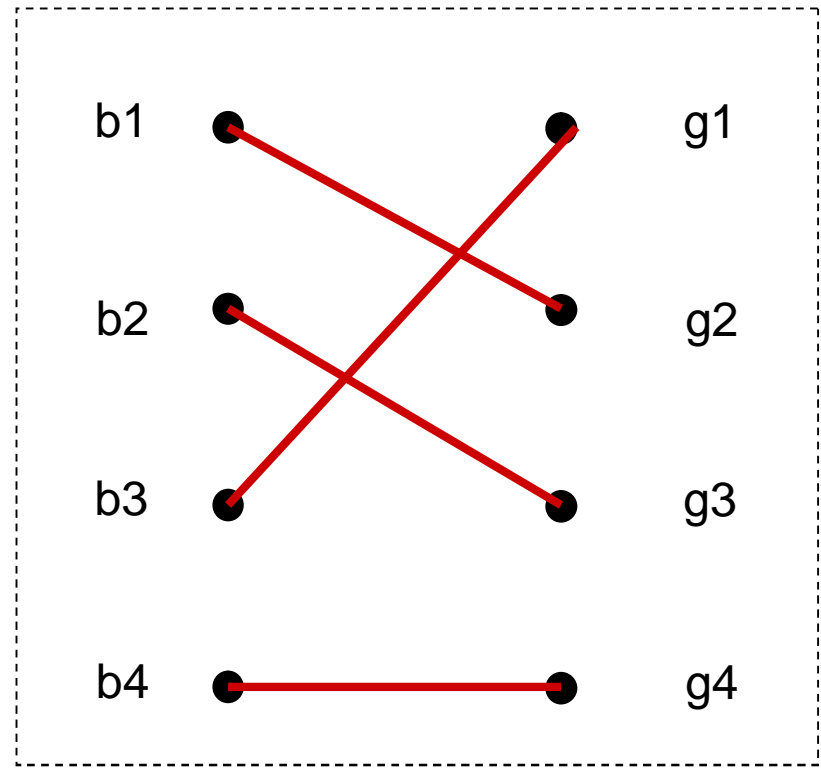
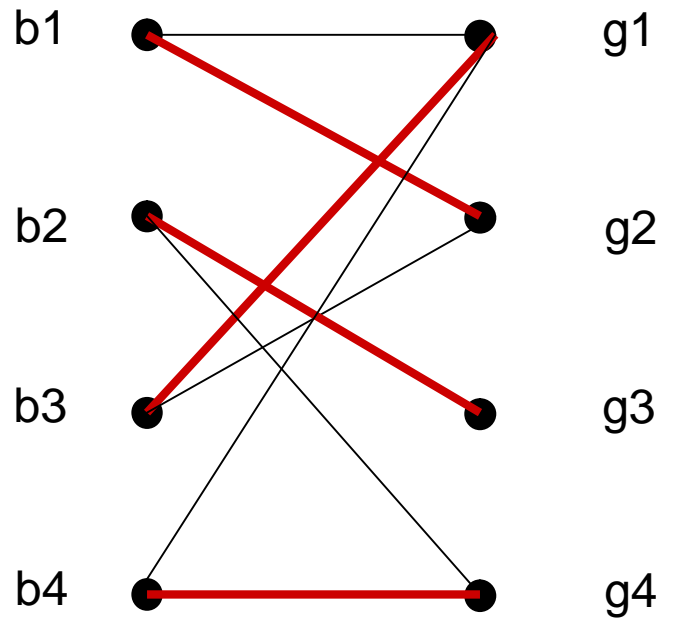
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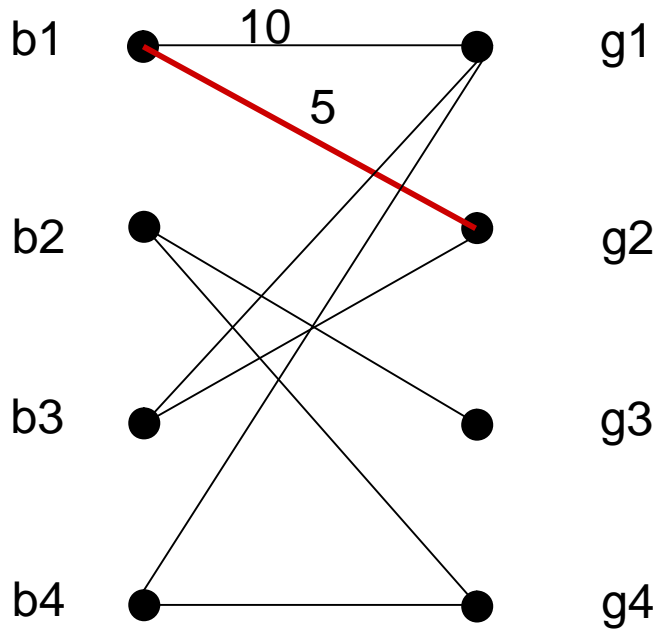
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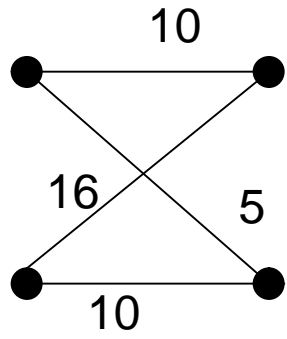
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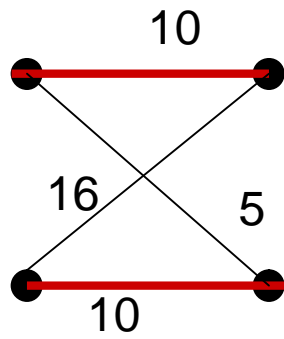
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(NP-complete)

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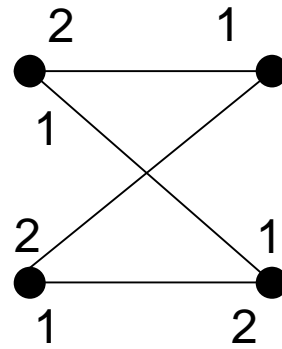
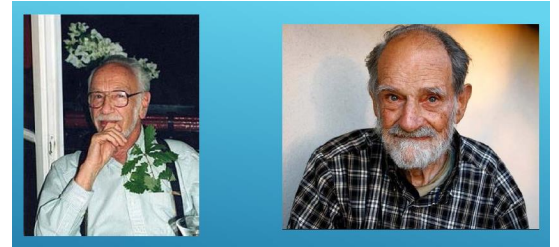
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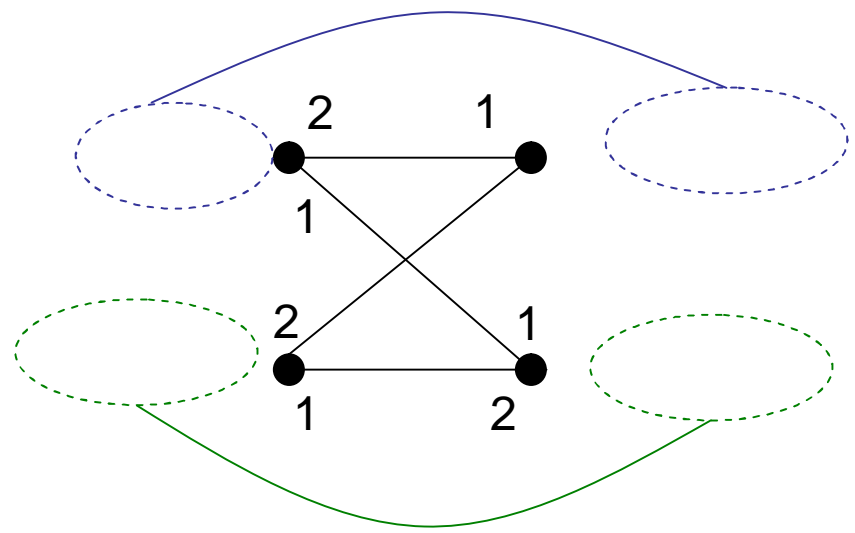
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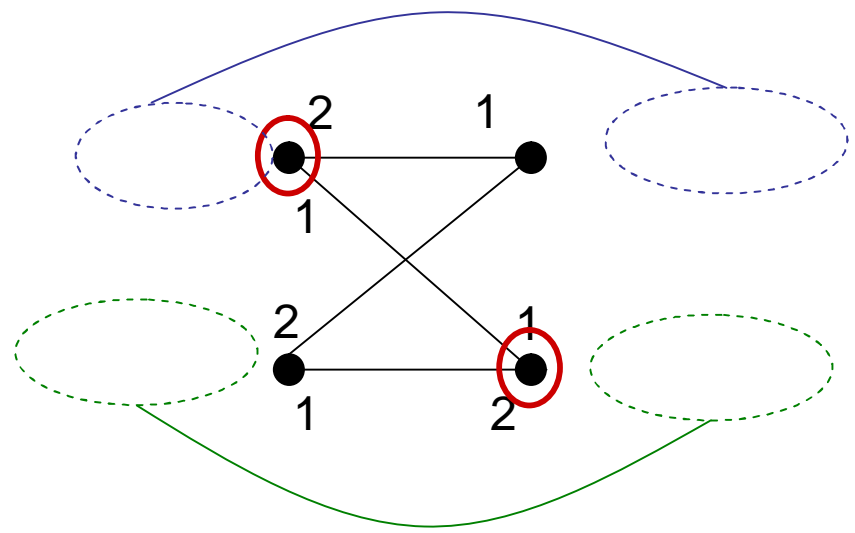
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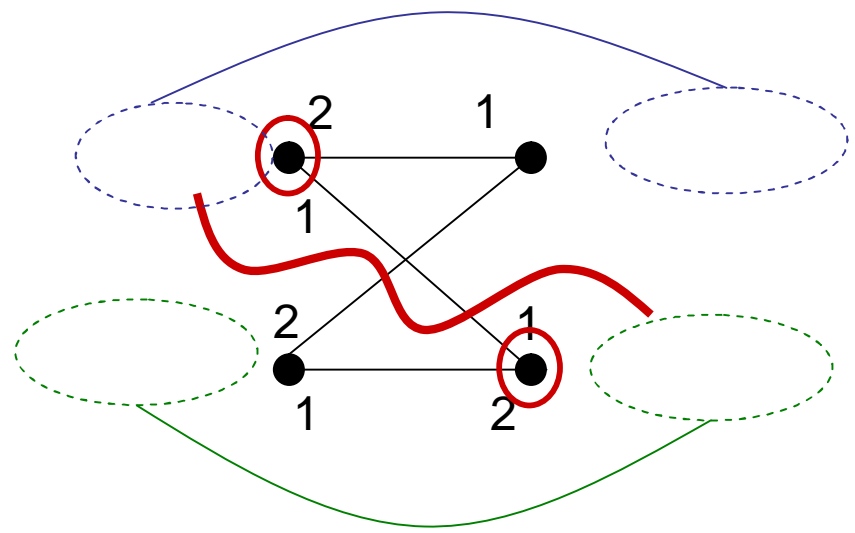
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» (rogue couple)

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(rogue couples)

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(matching) ,

x
» (rogue

y,
couple)

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(stable)

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(rogue couples)

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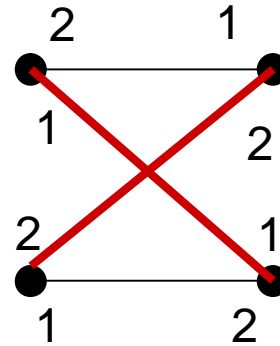
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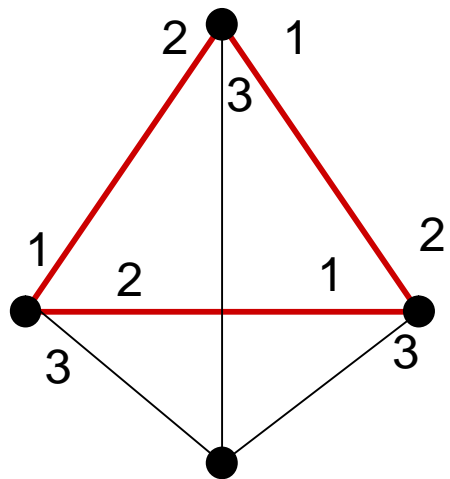
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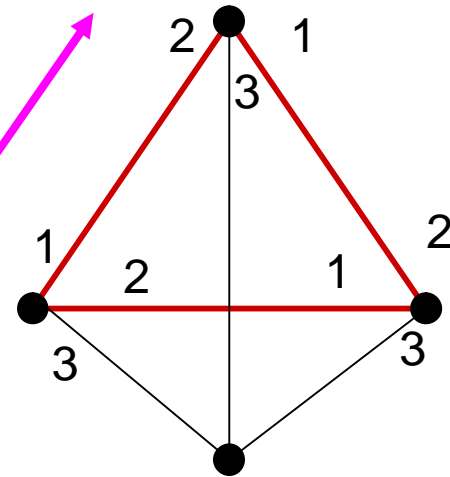
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(The Stable Marriage Problem . SMP)

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(The Stable Marriage Problem . SMP)

1: C, B, E, A, D

2: A, B, E, C, D

3: D, C, B, A, E

4: A, C, D, B, E

5: A, B, D, E, C

A: 3, 5, 2, 1, 4

B: 5, 2, 1, 4, 3

C: 4, 3, 5, 1, 2

D: 1, 2, 3, 4, 5

E: 2, 3, 4, 1, 5

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(The Stable Marriage Problem . SMP)

1: C, B, E, A, D

2: A, B, E, C, D

3: D, C, B, A, E

4: A, C, D, B, E

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C: 4, 3, 5, 1, 2

D: 1, 2, 3, 4, 5

E: 2, 3, 4, 1, 5

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(greedy algorithm)

(The Stable Marriage Problem . SMP)

1: **C**, B, E, A, D

2: **A**, B, E, C, D

3: **D**, C, B, A, E

4: ~~A, C, D~~, **B**, E

5: ~~A, B, D~~, **E**, C

A: 3, 5, 2, 1, 4

B: 5, 2, 1, 4, 3

C: 4, 3, 5, 1, 2

D: 1, 2, 3, 4, 5

E: 2, 3, 4, 1, 5

:

(greedy algorithm)

(The Stable Marriage Problem . SMP)

1: **C**, B, E, A, D

2: **A**, B, E, C, D

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4: ~~A, C, D~~, **B**, E

5: ~~A, B, D~~, **E**, C

A: 3, 5, 2, 1, 4

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:

(greedy algorithm)

(The Stable Marriage Problem . SMP)

1: **C**, B, E, A, D

2: **A**, B, E, C, D

3: **D**, C, B, A, E

4: ~~A, C, D~~, **B**, E

5: ~~A, B, D~~, **E**, C

A: 3, 5, 2, 1, 4

B: 5, 2, 1, 4, 3

C: 4, 3, 5, 1, 2

D: 1, 2, 3, 4, 5

E: 2, 3, 4, 1, 5



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(greedy algorithm)

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\Rightarrow

(The Stable Marriage Problem . SMP)

1: C, B, E, A, D

2: A, B, E, C, D

3: D, C, B, A, E

4: A, C, D, B, E

5: A, B, D, E, C

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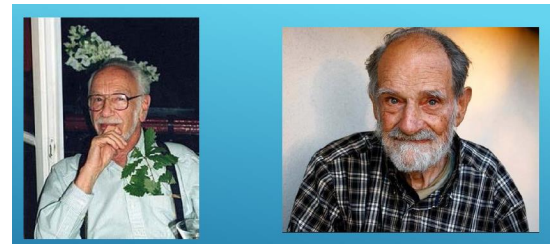
A: 3, 5, 2, 1, 4

B: 5, 2, 1, 4, 3

C: 4, 3, 5, 1, 2

D: 1, 2, 3, 4, 5

E: 2, 3, 4, 1, 5



(The Stable Marriage Problem . SMP)

1: C, B, E, A, D

A: 3, 5, 2, 1, 4

2: A, B, E, C, D

B: 5, 2, 1, 4, 3

3: D, C, B, A, E

C: 4, 3, 5, 1, 2

4: A, C, D, B, E

D: 1, 2, 3, 4, 5

5: A, B, D, E, C

E: 2, 3, 4, 1, 5

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(The Stable Marriage Problem . SMP)

1: C, B, E, A, D

A: 3, 5, 2, 1, 4

2: A, B, E, C, D

B: 5, 2, 1, 4, 3

3: D, C, B, A, E

C: 4, 3, 5, 1, 2

4: A, C, D, B, E

D: 1, 2, 3, 4, 5

5: A, B, D, E, C

E: 2, 3, 4, 1, 5

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(The Stable Marriage Problem . SMP)

1: C, B, E, A, D

A: 3, 5, 2, 1, 4

2: A, B, E, C, D

B: 5, 2, 1, 4, 3

3: D, C, B, A, E

C: 4, 3, 5, 1, 2

4: A, C, D, B, E

D: 1, 2, 3, 4, 5

5: A, B, D, E, C

E: 2, 3, 4, 1, 5

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(The Stable Marriage Problem . SMP)

1: C, B, E, A, D

2: A, B, E, C, D

3: D, C, B, A, E

4: A, C, D, B, E

5: A, B, D, E, C

A: 3, 5, 2, 1, 4

B: 5, 2, 1, 4, 3

C: 4, 3, 5, 1, 2

D: 1, 2, 3, 4, 5

E: 2, 3, 4, 1, 5

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2

(The Stable Marriage Problem . SMP)

1: C, B, E, A, D	2,4,5	A: 3, 5, 2, 1, 4
2: A, B, E, C, D	--	B: 5, 2, 1, 4, 3
3: D, C, B, A, E	1	C: 4, 3, 5, 1, 2
4: A, C, D, B, E	3	D: 1, 2, 3, 4, 5
5: A, B, D, E, C	--	E: 2, 3, 4, 1, 5

(The Stable Marriage Problem . SMP)

1: C, B, E, A, D	2,4, 5	A: 3, 5, 2, 1, 4
2: A, B, E, C, D	--	B: 5, 2, 1, 4, 3
3: D, C, B, A, E	1	C: 4, 3, 5, 1, 2
4: A, C, D, B, E	3	D: 1, 2, 3, 4, 5
5: A, B, D, E, C	--	E: 2, 3, 4, 1, 5

(The Stable Marriage Problem . SMP)

1: C, B, E, A, D	2,4, 5	A: 3, 5, 2, 1, 4
2: A , B, E, C, D	--	B: 5, 2, 1, 4, 3
3: D, C, B, A, E	1	C: 4, 3, 5, 1, 2
4: A , C, D, B, E	3	D: 1, 2, 3, 4, 5
5: A, B, D, E, C	--	E: 2, 3, 4, 1, 5

(The Stable Marriage Problem . SMP)

1: C, B, E, A, D

5

A: 3, 5, 2, 1, 4

2: B, E, C, D

2

B: 5, 2, 1, 4, 3

3: D, C, B, A, E

1,4

C: 4, 3, 5, 1, 2

4: C, D, B, E

3

D: 1, 2, 3, 4, 5

5: A, B, D, E, C

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E: 2, 3, 4, 1, 5

(The Stable Marriage Problem . SMP)

1: C, B, E, A, D

5

A: 3, 5, 2, 1, 4

2: B, E, C, D

2

B: 5, 2, 1, 4, 3

3: D, C, B, A, E

1,4

C: 4, 3, 5, 1, 2

4: C, D, B, E

3

D: 1, 2, 3, 4, 5

5: A, B, D, E, C

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E: 2, 3, 4, 1, 5

(The Stable Marriage Problem . SMP)

1: **C**, B, E, A, D

5

A: 3, 5, 2, 1, 4

2: B, E, C, D

2

B: 5, 2, 1, 4, 3

3: D, C, B, A, E

1, **4**

C: 4, 3, 5, 1, 2

4: C, D, B, E

3

D: 1, 2, 3, 4, 5

5: A, B, D, E, C

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E: 2, 3, 4, 1, 5

(The Stable Marriage Problem . SMP)

1: B, E, A, D

5

A: 3, 5, 2, 1, 4

2: B, E, C, D

1,2

B: 5, 2, 1, 4, 3

3: D, C, B, A, E

4

C: 4, 3, 5, 1, 2

4: C, D, B, E

3

D: 1, 2, 3, 4, 5

5: A, B, D, E, C

--

E: 2, 3, 4, 1, 5

(The Stable Marriage Problem . SMP)

1: B, E, A, D

5

A: 3, 5, 2, 1, 4

2: B, E, C, D

1,2

B: 5, 2, 1, 4, 3

3: D, C, B, A, E

4

C: 4, 3, 5, 1, 2

4: C, D, B, E

3

D: 1, 2, 3, 4, 5

5: A, B, D, E, C

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E: 2, 3, 4, 1, 5

(The Stable Marriage Problem . SMP)

1: **B**, E, A, D

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A: 3, 5, 2, 1, 4

2: B, E, C, D

1, **2**

B: 5, 2, 1, 4, 3

3: D, C, B, A, E

4

C: 4, 3, 5, 1, 2

4: C, D, B, E

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D: 1, 2, 3, 4, 5

5: A, B, D, E, C

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E: 2, 3, 4, 1, 5

(The Stable Marriage Problem . SMP)

1: E, A, D

5

A: 3, 5, 2, 1, 4

2: B, E, C, D

2

B: 5, 2, 1, 4, 3

3: D, C, B, A, E

4

C: 4, 3, 5, 1, 2

4: C, D, B, E

3

D: 1, 2, 3, 4, 5

5: A, B, D, E, C

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E: 2, 3, 4, 1, 5

4

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(The Stable Marriage Problem . SMP)

1: E, A, D

5

A: 3, 5, 2, 1, 4

2: B, E, C, D

2

B: 5, 2, 1, 4, 3

3: D, C, B, A, E

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C: 4, 3, 5, 1, 2

4: C, D, B, E

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D: 1, 2, 3, 4, 5

5: A, B, D, E, C

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E: 2, 3, 4, 1, 5

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1: C, B, E, A, D

2: A, B, E, C, D

3: D, C, B, A, E

4: A, C, D, B, E

5: A, B, D, E, C

(The Stable Marriage Problem . SMP)

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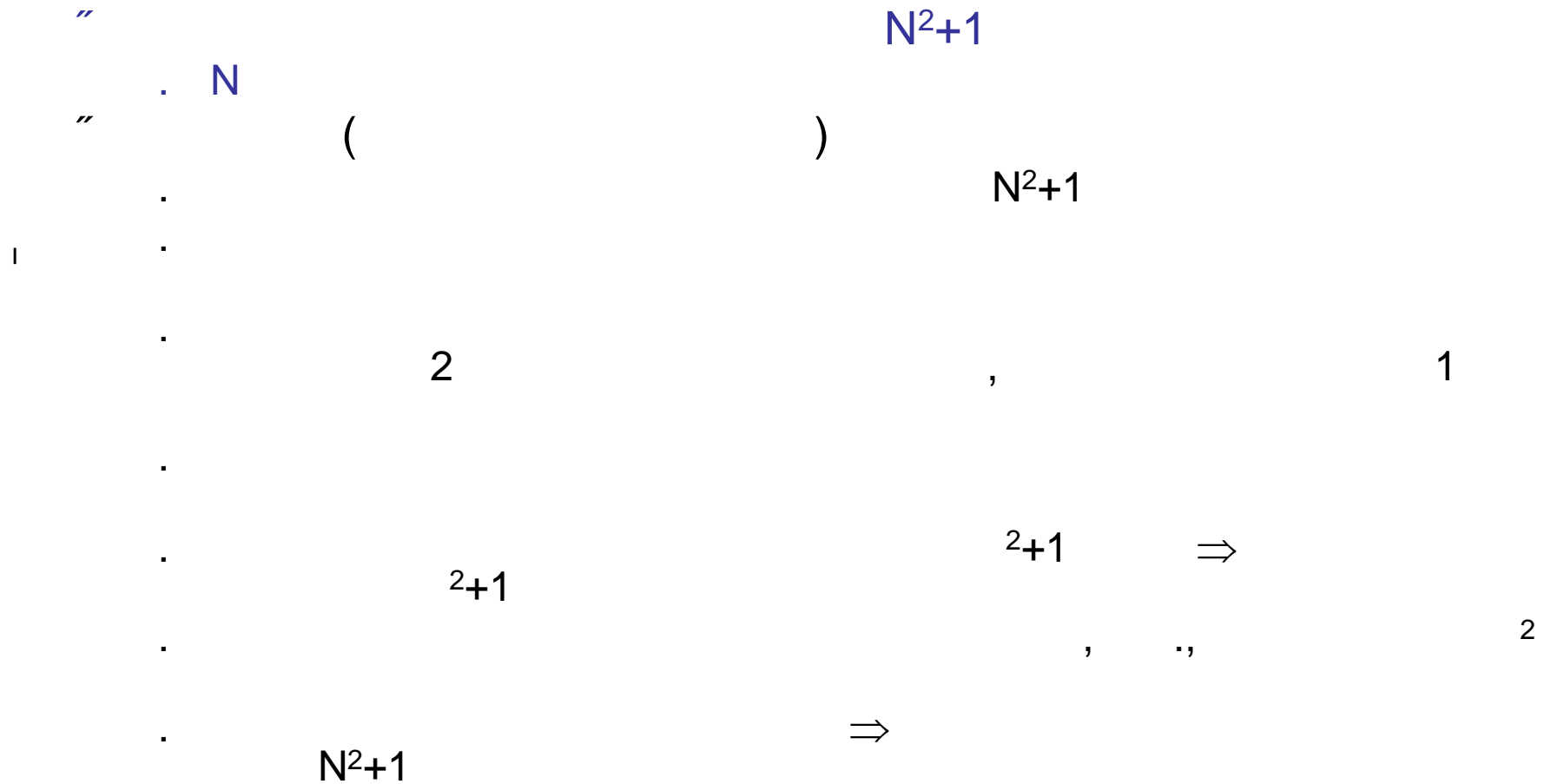
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(The Stable Marriage Problem . SMP)



(The Stable Marriage Problem . SMP)

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(The Stable Marriage Problem . SMP)

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" () P d+1;

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» HG P B d+1
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d+1 B d

(The Stable Marriage Problem . SMP)

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(The Stable Marriage Problem . SMP)

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(The Stable Marriage Problem . SMP)

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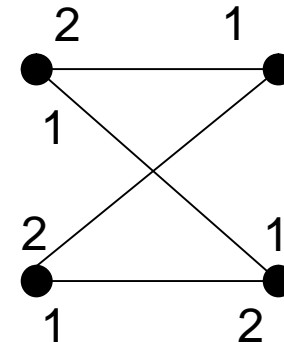
l . \Rightarrow S q

" P (realm of possibility)

$$\{Q \mid \exists M \in S \{P, Q\} \in M\}$$

$\Rightarrow 0$

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(The Stable Marriage Problem . SMP)

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(optimal mate):

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(pessimal mate):

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(optimal mate)

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(pessimal mate)

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(The Stable Marriage Problem . SMP)

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(optimal

mate)

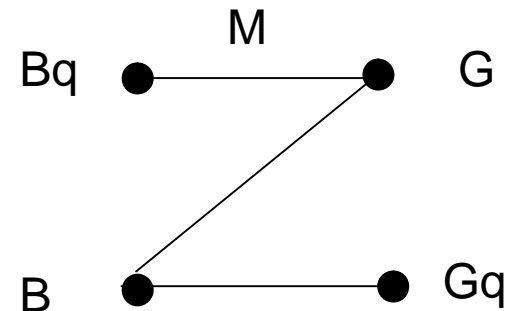
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(pessimal mate)

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q

To
 $G(B, G; \dots)$
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(The Stable Marriage Problem . SMP)

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. Online dating

(load balancing)

Web

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web servers

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servers

(The Stable Marriage Problem . SMP)

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group
visibility

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groups

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groups

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