An Experimental Study of Bicriteria Models for Robust Timetabling

Dimitrios Gkortsilas
Dept. of Computer Engineering & Informatics
University of Patras, 26500 Patras, Greece
gkortsilas@ceid.upatras.gr

Christos Zaroliagis
Dept. of Computer Engineering & Informatics
University of Patras, 26500 Patras, Greece
zaro@ceid.upatras.gr

Abstract

We conduct an experimental study for a fundamental case of the timetabling problem in a public railway network under disruptions. Three bicriteria optimization problems, modeling the robustness of the timetable towards delays, are experimentally evaluated against various waiting time rules at stations. Our results constitute the first proofs-of-concept for these models.

1 Introduction

The construction of a timetable is one of the most important phases in a public transportation network and has been extensively studied since quite some time; see e.g., [4, 5, 6, 8]. The timetabling problem asks for determining the departure and arrival times of public transportation media (e.g., trains) in order to serve, in a timely fashion, the customer demand. Typically the goal is to find an optimal solution, that is, a timetable that minimizes the overall traveling time of passengers. On the other hand, quite often there are disruptions to the normal operation of a public transportation network (e.g., due to signaling problems, maintenance work, weather conditions, accidents, etc) and this affects the timetable. As a consequence, there is a recent shift of focus towards robustness issues [2, 3], that is, providing a timetable that is robust to disruptions rather than an optimal timetable for an ideal case.

Many notions of robustness have been introduced and used for the timetabling problem (see e.g., [2]). Most of them use a given level of robustness that has to be determined beforehand. In this paper, we concentrate on a new robustness approach presented recently in [7]. In that paper, the robust timetabling problem is studied as a bicriteria optimization problem, where its first objective is the objective of the nominal (undisrupted) problem, while its second objective is a suitably defined function measuring the robustness of the provided solution. The new approach has the advantage that one can use any robustness function as a second objective. Based on common practical waiting time rules at train stations, three such robustness measures are proposed in [7], leading to three bicriteria optimization problems. The theoretical study in [7] uses as building block a fundamental case with two different trains, one arriving and the other departing at/from a given station, and a transfer activity between them.

In this work, we provide the first implementation and experimental evaluation of this fundamental case that forms the building block for other more involved cases. The purpose of our study is to provide a proof-of-concept for the three bicriteria optimization problems for the fundamental case, modeling the robustness of the timetable towards delays against various waiting time rules at stations. Our experimental study is based on real data taken from the German railway network and constitutes the first proof-of-concept for these models.

The rest of the paper is organized as follows. In Section 2, we review the definition of the timetabling problem, as well as the robustness measures and the corresponding three optimization problems as introduced in [7]. In section 3, we review the fundamental case studied in [7] along with the specific formulation of the optimization problems. In section 4, we present the results of our experimental study.
2 Preliminaries

We follow the exposition in [7]. A timetable is defined with the use of the so-called event-activity network, i.e., a directed (connected) graph \( G = (E, A) \) whose node set \( E \) represents events (departures, arrivals), while its edge set \( A \subseteq E \times E \) represents activities (transfer, stop, drive). A timetable \( \Pi \in \mathbb{R}^{[E]} \) assigns a time \( \Pi_i \) to each event \( i \in E \). Each activity \( a = (i, j) \), linking two events \( i, j \in E \), is associated with a lower \( l_a \) and an upper time bound \( u_a \) that have to be respected. A timetable is called feasible if \( \Pi_j - \Pi_i \in [l_a, u_a] \), for all \( a = (i, j) \in A \). A path \( P \) in \( G \) is a sequence of events \( (i_1, \ldots, i_n) \) such that either \((i_k, i_{k+1}) \in A \) (forward activity) or \((i_{k+1}, i_k) \in A \) (backward activity), for \( 1 \leq k < n \). The forward (resp. backward) activities of \( P \) are denoted as \( P^+ \) (resp. \( P^- \)). A cycle \( C \) is a path with \( i_1 = i_n \). The slack times of a timetable are defined as \( s_a = \Pi_j - \Pi_i - l_a \), \( \forall a \in A \). The slack time \( s_a \) is the available additional time for activity \( a \) and it is used to reduce delays.

Let \( m_a = u_a - l_a \), and let \( u_a \) be the number of passengers traveling along an activity \( a \). There are two equivalent definitions of the timetabling problem that aims at minimizing the overall traveling time of passengers (for details, see [7]).

- Timetabling using variables \( \Pi_i, i \in E \)
  \[
  \min F(\Pi) = \sum_{a=(i,j) \in A} u_a (\Pi_j - \Pi_i) \\
  \text{s.t. } l_a \leq \Pi_j - \Pi_i \leq u_a \quad \forall a = (i, j) \in A, \quad \Pi_i \in \mathbb{N}.
  \]

- Timetabling using variables \( s_a, a \in A \)
  \[
  \min F(s) = \sum_{a \in A} u_a s_a \\
  \text{s.t. } 0 \leq s_a \leq m_a \\
  \sum_{a \in C^-} s_a - \sum_{a \in C^+} s_a = - \sum_{a \in C^-} l_a + \sum_{a \in C^+} l_a
  \]

The robustness of a timetable is its sensitivity to unforeseen delays. Before defining robustness formally, we need to see what happens when some delay occurs. Delays typically affect transfer activities. When a transfer activity \( a = (i, j) \) takes place, there are two possibilities: either the outgoing train waits for the delayed incoming train so that the transfer is maintained, or the train departs on time and no delay is transferred. The problem of determining which transfers must be maintained and which must not is known as the delay management problem. As it is customarily assumed, we consider this problem solved, i.e., that we are given beforehand some waiting time rules (WTRs). We consider three typical such rules.

Let \( i \) represent an arrival event of train \( A \) at a station, let \( j \) represent a departure event of another train \( B \) at the same station, let \( a = (i, j) \) be a transfer activity from train \( A \) to train \( B \), and let \( a_{\text{next}} = (j, k) \) be the next driving activity of train \( B \). Assume that train \( A \) arrives at \( i \) with a delay \( \delta_i \). The next three WTRs determine whether train \( B \) waits for train \( A \), or departs on time.

**WTR1:** Train \( B \) is not allowed to have a delay at its next station. The transfer is maintained if and only if \( \delta_i \leq s_a + s_{a_{\text{next}}} \).

**WTR2:** Train \( B \) can wait \( n \) minutes, where \( n \) is fixed. The transfer is maintained if and only if \( \delta_i \leq s_a + n \).

**WTR3:** Train \( B \) is not allowed to have a delay of more than \( m \) minutes at its next station. The transfer is maintained if and only if \( \delta_i \leq s_a + s_{a_{\text{next}}} + m \).
We are now ready to define robustness. We assume that only one WTR is used within a public transportation system and that the timetabling problem is described using variables $s_i$. In particular, let a fixed WTR be given, let $s \in \mathbb{R}^{[A]}$ be a timetable, and consider a set of source-delayed events $E_{\text{delayed}}$ with delays $\delta_i \leq \delta$, for all $i \in E_{\text{delayed}}$, for some given $\delta$. Three robustness functions are defined.

- $R(s)$ measures the robustness of a timetable if all of its transfers are maintained whenever all source delays are smaller than or equal to some value $R$.
- $R_{\text{no}}(s, \delta)$ is the maximum number of passengers who miss a transfer if all source delays are smaller than $\delta$.
- $R_{\text{del}}(s, \delta)$ is the maximum sum of all passengers’ delays\(^1\), if all source delays are smaller than $\delta$.

The first definition of robustness computes the maximum source delay for which no transfer is missed. Clearly, this robustness measure needs to be maximized. The other two definitions of robustness evaluate how badly the passengers are affected by source delays. In these cases, the robustness measures need to be minimized.

The above definitions lead naturally to three bicriteria optimization problems that incorporate robustness to the timetabling problem. The first objective function is common for all models and represents the total traveling time of the passengers ($F(s)$). The second objective function is one of the robustness functions just defined ($R(s), R_{\text{no}}(s, \delta), R_{\text{del}}(s, \delta)$). To summarize, the three bicriteria optimization problems are as follows:

\[
(P) \quad \begin{align*}
\min & \quad F(s) \\
\text{s.t.} & \quad R(s) \leq R
\end{align*}
\]

\[
(P_{\text{no}}) \quad \begin{align*}
\min & \quad F(s) \\
\text{s.t.} & \quad R_{\text{no}}(s, \delta) \leq \delta
\end{align*}
\]

\[
(P_{\text{del}}) \quad \begin{align*}
\min & \quad F(s) \\
\text{s.t.} & \quad R_{\text{del}}(s, \delta) \leq \delta
\end{align*}
\]

Note that each problem can be considered for all three WTRs. In all problems we seek for Pareto optimal (or non-dominated) solutions, i.e., timetables $s$ that there does not exist another timetable $s'$ which is not worse in one of the two objectives and strictly better in the other one.

A timetable is called weak Pareto (or weakly non-dominated) if there does not exist another timetable which is strictly better in both objectives. A Pareto solution is always weak Pareto, but the reverse is not true in general.

3 A Fundamental Case

The simplest case – which forms the building block for more involved scenarios – is to model a single transfer between two trains [7]; see Fig. 1. We assume that train 1 travels from station $F$ to station $M$, train 2 travels from station $M$ to station $K$, there is one possible transfer activity at station $M$, and a delay of size $\delta$ occurs at station $F$.

Train 1 reaches station $M$ with a delay of $[\delta - s_1]$. Passengers transferring to train 2 will arrive with a delay of $[\delta - s_1 - s_2]$. Let us see how the transfer is maintained w.r.t. each WTR.

- Using WTR1, train 2 can wait at most $s_3$ minutes. Hence, the transfer is maintained if and only if $\delta - s_1 - s_2 \leq s_3 \iff \delta \leq s_1 + s_2 + s_3$.

\(^1\)If a passenger misses a transfer, then the delay is assumed to be $T$ minutes, provided that the timetable is repeated after $T$ minutes and hence the passenger can use the transfer of the next period.
Using WTR2, train 2 can wait at most $n$ minutes. Hence, the transfer is maintained if and only if $\delta - s_1 - s_2 \leq n \Leftrightarrow \delta \leq s_1 + s_2 + n$.

Using WTR3, train 2 can wait at most $m$ minutes and is not allowed to have a delay at its next station. Hence, the transfer is maintained if and only if $\delta - s_1 - s_2 \leq s_3 + m \Leftrightarrow \delta \leq s_1 + s_2 + s_3 + m$.

The above suggest that the first robustness function is defined as:

$$R(s_1, s_2, s_3) = \begin{cases} s_1 + s_2 + s_3, & \text{for WTR1} \\ s_1 + s_2 + n, & \text{for WTR2} \\ s_1 + s_2 + s_3 + m, & \text{for WTR3} \end{cases}$$

To determine the other two robustness functions, $R_{no}$ and $R_{del}$, we have to take into account the number of passengers $w_{FM}, w_{MK}, w_{FK}$ traveling among stations $F$, $M$, and $K$ (which is assumed given). For $R_{no}$ only passengers traveling from $F$ to $K$ can miss the transfer. Hence, for any WTR, $R_{no}$ becomes:

$$R_{no}(s, \delta) = \begin{cases} w_{FK}, & \text{if } \delta > R(s_1, s_2, s_3) \\ 0, & \text{if } \delta \leq R(s_1, s_2, s_3) \end{cases}$$

For $R_{del}$, observe that passengers from $F$ to $M$ gain a delay of $[\delta - s_1]^+$, passengers from $F$ to $K$ gain a delay of $T$ if they miss the transfer, or they get the same delay as the passengers from $M$ to $K$, which is $[\delta - s_1 - s_2 - s_3]^+$.

$$R_{del}(s, \delta) = \begin{cases} w_{FM}[\delta - s_1]^+ + Tw_{FK}, & \text{if } \delta > R(s_1, s_2, s_3) \\ w_{FM}[\delta - s_1]^+ + (w_{FK}+w_{MK})[\delta - s_1 - s_2 - s_3]^+, & \text{if } \delta \leq R(s_1, s_2, s_3). \end{cases}$$

The first objective function $F(s)$ concerns the minimization of the passengers' traveling time and is therefore common to all three problems $(P), (P_{no}), (P_{del})$.

$$F(s_1, s_2, s_3) = (w_{FM} + w_{FK})s_1 + w_{FK}s_2 + (w_{FK}+w_{MK})s_3.$$

After determining the objective functions, we now turn to the specific problem formulations. The first problem $(P)$ becomes:

$$\begin{align*}
\min \ & F(s_1, s_2, s_3) \\
\max \ & R(s_1, s_2, s_3) \\
\text{s.t.} \ & 0 \leq s_i \leq m_i, \text{ for } i = 1, 2, 3
\end{align*}$$

As it is proved in [7], for the solutions $(s_1, s_2, s_3)$ of $(P)$ it holds that $s_1 \in \{0, m_1\}$, $s_2 \in \{0, m_2\}$, and $s_3 \in \{0, m_3\}$ – the specific values depend in the used WTR. The interpretation of these solutions is that the slack should be put on the transfer and not on the driving activities.
For the second problem \((P_{no})\), a binary variable \(z\) is used with \(z = 0\) if train 2 waits for train 1, and \(z = 1\) otherwise. Then, \((P_{no})\) becomes:

\[
\begin{align*}
& \min F(s_1, s_2, s_3) \\
& \min z w_{FK} \\
& \text{s.t. } \delta z + R(s_1, s_2, s_3) \geq \delta \\
& \quad 0 \leq s_i \leq m_i, \text{ for } i = 1, 2, 3 \\
& \quad z \in \{0, 1\}.
\end{align*}
\]

Depending on the particular WTR used, we have the following. If the transfer is maintained, \(z = 0\) and \(R_{no} = 0\). If the transfer is missed, \(z = 1\) and \(R_{no} = w_{FK}\). As it is proved in [7], the Pareto solutions are of the form \((F(s), 0)\), if the transfer is maintained, and of the form \((0, w_{FK})\), otherwise (in the latter case the traveling time of passengers is zero, because they missed the transfer and hence do not travel). The interpretation of these solutions is that either distribute no slack at all, or distribute the minimum amount of slack to maintain the transfer.

For the third problem \((P_{del})\), we also use the binary variable \(z\) and recall that the sum of all delays of the passengers need to be taken into account. Then, \((P_{del})\) becomes:

\[
\begin{align*}
& \min F(s_1, s_2, s_3) \\
& \min (\delta - s_1) w_{FM} + z T w_{FK} + (1 - z) (w_{FK} + w_{MK}) (\delta - s_1 - s_2 - s_3) \\
& \text{s.t. } \delta z + R(s_1, s_2, s_3) \geq \delta \\
& \quad s_1 + s_2 + s_3 \leq \delta \\
& \quad 0 \leq s_i \leq m_i, \text{ for } i = 1, 2, 3 \\
& \quad z \in \{0, 1\}.
\end{align*}
\]

This model is quadratic and the linear formulation w.r.t. the three WTRs is as follows. For WTR1, if \(z = 0\), then the first constraint implies that \(R(s_1, s_2, s_3) \geq \delta \iff s_1 + s_2 + s_3 \geq \delta\); this in combination with the second constraint \((s_1 + s_2 + s_3 \leq \delta)\) gives that \(\delta - s_1 - s_2 - s_3 = 0\). If \(z = 1\), then the term \((1 - z) (w_{FK} + w_{MK}) (\delta - s_1 - s_2 - s_3)\) is zero. For WTR2 and WTR3, an auxiliary variable \(q\) is used. In summary, \((P_{del})\) simplifies to the following (left for WTR1, right for WTR2 and WTR3) models:

\[
\begin{align*}
& \min F(s_1, s_2, s_3) \\
& \min (\delta - s_1) w_{FM} + z T w_{FK} + (w_{FK} + w_{MK}) q \\
& \text{s.t. } \delta z + R(s_1, s_2, s_3) \geq \delta \\
& \quad s_1 + s_2 + s_3 \leq \delta \\
& \quad 0 \leq s_i \leq m_i, \text{ for } i = 1, 2, 3 \\
& \quad z \in \{0, 1\}.
\end{align*}
\]

\[
\begin{align*}
& \min F(s_1, s_2, s_3) \\
& \min (\delta - s_1) w_{FM} + z T w_{FK} + (w_{FK} + w_{MK}) q \\
& \text{s.t. } g + \delta z + s_1 + s_2 + s_3 \geq \delta \\
& \quad \delta z + R(s_1, s_2, s_3) \leq \delta \\
& \quad s_1 + s_2 + s_3 \leq \delta \\
& \quad 0 \leq s_i \leq m_i, \text{ for } i = 1, 2, 3 \\
& \quad z \in \{0, 1\} \\
& \quad q \geq 0.
\end{align*}
\]

It can be proved that the two linear formulations are equivalent to the original model [7].

4 Implementation and Experiments

Our implementations were conducted using C++ (compiler gcc version 4.1.2) and CPLEX 10.1. For our experiments we used real-world data from the German Railways [1]. In particular, we considered the case of passengers traveling from Frankfurt (F) to Kaiserslautern (K) with an intermediate transfer at Mannheim (M), using ICE (intercity express) and IC (intercity) trains.
We used the timetable of August 2010 (for one particular day) and considered the capacity of ICE trains to 415 passengers and that of IC trains to 1000 passengers. The overall 23 routes and the train connections are shown in Table 1. The number of passengers \( w_{FM}, w_{FK}, w_{MK} \) traveling in each activity (F→M, F→K, and M→K, resp.) was a random value in \([1, 415]\) if at least one ICE train was involved, and a a random value in \([1, 1000]\) otherwise. We also assumed that when a transfer is made from train 1 to train 2, the latter train had the same or bigger capacity than the former. Each bicriterion optimization problem had to be solved for every

<table>
<thead>
<tr>
<th>Routes (id's)</th>
<th>Departure</th>
<th>Arrival</th>
<th>Departure</th>
<th>Arrival</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frankfurt(Main)Hbf</td>
<td>Mannheim Hbf</td>
<td>Mannheim Hbf</td>
<td>Kaiserslautern Hbf</td>
</tr>
<tr>
<td>1</td>
<td>06:50</td>
<td>07:28</td>
<td>07:56</td>
<td>08:59</td>
</tr>
<tr>
<td>2</td>
<td>07:50</td>
<td>08:28</td>
<td>08:55</td>
<td>09:59</td>
</tr>
<tr>
<td>3</td>
<td>09:13</td>
<td>10:20</td>
<td>10:26</td>
<td>11:28</td>
</tr>
<tr>
<td>4</td>
<td>10:05</td>
<td>10:42</td>
<td>10:55</td>
<td>11:59</td>
</tr>
<tr>
<td>5</td>
<td>10:50</td>
<td>11:28</td>
<td>11:56</td>
<td>12:59</td>
</tr>
<tr>
<td>7</td>
<td>12:05</td>
<td>12:42</td>
<td>12:56</td>
<td>13:59</td>
</tr>
<tr>
<td>8</td>
<td>13:10</td>
<td>14:20</td>
<td>14:26</td>
<td>15:28</td>
</tr>
<tr>
<td>9</td>
<td>14:05</td>
<td>14:42</td>
<td>14:55</td>
<td>15:59</td>
</tr>
<tr>
<td>10</td>
<td>14:50</td>
<td>15:28</td>
<td>15:56</td>
<td>16:59</td>
</tr>
<tr>
<td>11</td>
<td>15:13</td>
<td>16:20</td>
<td>16:26</td>
<td>17:28</td>
</tr>
<tr>
<td>12</td>
<td>16:05</td>
<td>16:42</td>
<td>16:56</td>
<td>17:59</td>
</tr>
<tr>
<td>13</td>
<td>17:13</td>
<td>18:20</td>
<td>18:26</td>
<td>19:28</td>
</tr>
<tr>
<td>14</td>
<td>18:05</td>
<td>18:42</td>
<td>18:56</td>
<td>19:59</td>
</tr>
<tr>
<td>15</td>
<td>18:50</td>
<td>19:28</td>
<td>19:56</td>
<td>20:59</td>
</tr>
<tr>
<td>16</td>
<td>19:13</td>
<td>20:20</td>
<td>20:26</td>
<td>21:26</td>
</tr>
<tr>
<td>17</td>
<td>20:05</td>
<td>20:42</td>
<td>20:50</td>
<td>21:33</td>
</tr>
<tr>
<td>20</td>
<td>22:05</td>
<td>22:42</td>
<td>23:12</td>
<td>00:18</td>
</tr>
<tr>
<td>21</td>
<td>23:00</td>
<td>23:41</td>
<td>00:31</td>
<td>01:43</td>
</tr>
<tr>
<td>22</td>
<td>23:13</td>
<td>00:20</td>
<td>00:31</td>
<td>02:07</td>
</tr>
<tr>
<td>23</td>
<td>00:18</td>
<td>01:10</td>
<td>01:16</td>
<td>02:19</td>
</tr>
</tbody>
</table>

Table 1: Timetable for a single day (August 2010) for ICE and IC trains.

WTR and there are 23 routes to be considered in each case, leading to a vast amount of results. Due to space constraints, we report here on selected results representing the most interesting cases (concentrating on the strictest rule WRT2 and on WRT2).

We start with (P) using WTR1. The data used and the computed slack times are shown in Table 2 and concern routes 5-10 of Table 1. The results are reported in Fig. 2, with the horizontal axis representing function \( R \) (that needs to be maximized) and the vertical axis representing function \( F \) (that needs to be minimized). The legend in Fig. 2 shows the values of \( F \) and \( R \). The results confirm the theoretical solutions discussed in Section 3. Observe that (see Table 2) in every triple \((s_1, s_2, s_3)\) the slack variables get either their lower bound or their upper bound. In all triples except the first one, variables \( s_2 \) get their maximum slack, which confirms the theoretical observation that slack should be put on transfer activities.

Pareto solutions for \((P_{a})\) using WTR2 are shown in Table 3. The data shown are: \( n \) (number of minutes regarding WTR2), \( \delta \) (source delay in minutes), \( w_{FM}, w_{FK}, w_{MK} \) (number of passengers for the corresponding activities), the values of \( F \) and \( R \) when the transfer is missed (\( z = 1 \)), and when is maintained (\( z = 0 \)). In the latter case, the slack times are also shown with their upper bounds. The data correspond to routes 20-22. There are only two non-dominated solutions and hence the results confirm the theoretical solutions discussed in Section 3. In
Table 2: Input data and computed slack times for \((P)\) for routes 5-10.

<table>
<thead>
<tr>
<th>Routes</th>
<th>(w_{FM})</th>
<th>(w_{FK})</th>
<th>(w_{MK})</th>
<th>(s_1)</th>
<th>(s_2)</th>
<th>(s_3)</th>
<th>((s_1, s_2, s_3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>93</td>
<td>77</td>
<td>105</td>
<td>0.7</td>
<td>0.28</td>
<td>0.6</td>
<td>{(0,0.0),(0.28,0),(7,28,0),(7,28,6)}</td>
</tr>
<tr>
<td>6</td>
<td>55</td>
<td>13</td>
<td>55</td>
<td>0.3</td>
<td>0.6</td>
<td>0.1</td>
<td>{(0,0.0),(0,6,0),(3,6,1)}</td>
</tr>
<tr>
<td>7</td>
<td>158</td>
<td>242</td>
<td>307</td>
<td>0.3</td>
<td>0.14</td>
<td>0.5</td>
<td>{(0,0.0),(0,14,0),(3,14,0),(3,14,5)}</td>
</tr>
<tr>
<td>8</td>
<td>80</td>
<td>19</td>
<td>235</td>
<td>0.2</td>
<td>0.6</td>
<td>0.7</td>
<td>{(0,0.0),(0,6,0),(2,6,0),(2,6,7)}</td>
</tr>
<tr>
<td>9</td>
<td>183</td>
<td>150</td>
<td>457</td>
<td>0.8</td>
<td>0.14</td>
<td>0.3</td>
<td>{(0,0.0),(0,14,0),(8,14,0),(8,14,3)}</td>
</tr>
<tr>
<td>10</td>
<td>539</td>
<td>107</td>
<td>350</td>
<td>0.7</td>
<td>0.28</td>
<td>0.8</td>
<td>{(0,0.0),(0,28,0),(0,28,8),(7,28,8)}</td>
</tr>
</tbody>
</table>

Figure 2: Pareto solutions for \((P)\) concerning routes 5-10 using WTR1.

routes 20-22 the time provided by WTR2 reduces the delay \(\delta\) but does not absorb it, so either the minimum amount of slack is given in order to maintain the transfer or there is no slack at all and the transfer is missed.

Table 3: Input data and Pareto solutions for \(P_{no}\) regarding routes 20-22.

<table>
<thead>
<tr>
<th>Routes</th>
<th>(n)</th>
<th>(\delta)</th>
<th>(w_{FM})</th>
<th>(w_{FK})</th>
<th>(w_{MK})</th>
<th>(F)</th>
<th>(R)</th>
<th>(s_1)</th>
<th>(s_2)</th>
<th>(s_3)</th>
<th>(m_1)</th>
<th>(m_2)</th>
<th>(m_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>8</td>
<td>42</td>
<td>100</td>
<td>50</td>
<td>210</td>
<td>1800</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>30</td>
<td>2</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>21</td>
<td>1</td>
<td>28</td>
<td>88</td>
<td>77</td>
<td>96</td>
<td>2695</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>7</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>22</td>
<td>11</td>
<td>13</td>
<td>400</td>
<td>219</td>
<td>100</td>
<td>438</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>11</td>
<td>0</td>
</tr>
</tbody>
</table>

Finally, we report results for \((P_{del})\) using WTR1. The data used and the computed slack times are shown in Table 4, and concern routes 11-16. The results are reported in Fig. 3, with the horizontal axis representing function \(R\) and the vertical axis representing function \(F\) (both need to be minimized). Note that there are solutions with \(F = 0\). This means that the transfer was missed and the sum of all passengers’ delays is \(R\). When the transfer is maintained, the values of \(R\) are lower, but \(F\) increases as we are dealing with two conflicting functions. The results suggest that we should distribute slack equal to the delay when the transfer is maintained in order to absorb it. When the transfer is missed, either distribute no slack at all or distribute an amount of slack to minimize the maximum sum of all passengers’ delays.
<table>
<thead>
<tr>
<th>Routes</th>
<th>$\delta$</th>
<th>$w_{FM}$</th>
<th>$w_{FK}$</th>
<th>$w_{MK}$</th>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$s_3$</th>
<th>$(s_1, s_2, s_3, \epsilon)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>5</td>
<td>237</td>
<td>73</td>
<td>40</td>
<td>0.2</td>
<td>0.6</td>
<td>0.3</td>
<td>$(0.5,0.0),(2.3,0.0),(0.0,0.1),(2.0,0.1)$</td>
</tr>
<tr>
<td>12</td>
<td>19</td>
<td>607</td>
<td>302</td>
<td>41</td>
<td>0.3</td>
<td>0.14</td>
<td>0.3</td>
<td>$(2.14,3.0),(3.14,2.0),(0.0,0.1),(3.0,0.1)$</td>
</tr>
<tr>
<td>13</td>
<td>7</td>
<td>78</td>
<td>108</td>
<td>61</td>
<td>0.2</td>
<td>0.6</td>
<td>0.2</td>
<td>$(0.6,1.0),(1.6,0.0),(2.5,0.0),(0.0,0.1),(2.0,0.1)$</td>
</tr>
<tr>
<td>14</td>
<td>18</td>
<td>109</td>
<td>350</td>
<td>200</td>
<td>0.10</td>
<td>0.14</td>
<td>0.10</td>
<td>$(4.14,0.0),(10.8,0.0),(0.0,0.1),(2.0,0.1)$</td>
</tr>
<tr>
<td>15</td>
<td>25</td>
<td>300</td>
<td>309</td>
<td>102</td>
<td>0.10</td>
<td>0.28</td>
<td>0.10</td>
<td>$(0.25,0.0),(10.15,0.0),(0.0,0.1),(10.0,0.1)$</td>
</tr>
<tr>
<td>16</td>
<td>10</td>
<td>50</td>
<td>80</td>
<td>152</td>
<td>0.3</td>
<td>0.6</td>
<td>0.7</td>
<td>$(3.6,1.0),(0.0,0.1),(3.0,0.1)$</td>
</tr>
</tbody>
</table>

Table 4: Input data and computed slack times for $(P_{del})$ regarding routes 11-16.

Figure 3: Weak Pareto solutions for $(P_{del})$ for the routes 11-16 using WTR1.

References